

Calculation of the 1-axial stress conditions prior drilling the hole:

$$\sigma'_r = \frac{\sigma_x}{2}(1 + \cos(2\alpha))$$

$$\sigma'_\theta = \frac{\sigma_x}{2}(1 - \cos(2\alpha))$$

$$\sigma'_{r\theta} = -\frac{\sigma_x}{2}\sin(2\alpha)$$

Calculation of the stress distribution after KIRSCH\*:

$$\sigma_r = \frac{\sigma_x}{2}\left(1 - \frac{1}{r^2} + \frac{\sigma_x}{2}\left(1 + \frac{3}{r^4} - \frac{4}{r^2}\right)\cos(2\alpha)\right)$$

$$\sigma_\theta = \frac{\sigma_x}{2}\left(1 + \frac{1}{r^2} - \frac{\sigma_x}{2}\left(1 + \frac{3}{r^4}\right)\cos(2\alpha)\right)$$

$$\sigma_{r\theta} = -\frac{\sigma_x}{2}\left(1 - \frac{3}{r^4} + \frac{2}{r^2}\right)\sin(2\alpha)$$

Superposing both conditions as above and considering the linear-elastic material behavior the calculation for the relaxing strains can be expressed by:

$$\varepsilon_r = \frac{\sigma_x(1+\nu)}{2E}\left(\frac{1}{r^2} - \frac{3}{r^4}\cos(2\alpha) + \frac{4\nu}{r^2(1+\nu)}\cos(2\alpha)\right)$$

$$\varepsilon_\theta = \frac{\sigma_x(1+\nu)}{2E}\left(-\frac{1}{r^2} - \frac{3}{r^4}\cos(2\alpha) - \frac{4\nu}{r^2(1+\nu)}\cos(2\alpha)\right)$$

The consideration of the elastic constants:

$$A = -\frac{1+\nu}{2E}\left(\frac{1}{r^2}\right)$$

$$B = -\frac{1+\nu}{2E}\left(\left(\frac{4}{1+\nu}\right)\frac{1}{r^2} - \frac{3}{r^4}\right)$$

$$C = \frac{1+\nu}{2E}\left(-\left(\frac{4}{1+\nu}\right)\frac{1}{r^2} + \frac{3}{r^4}\right)$$

yields finally:

$$\varepsilon_r = \sigma_x(A + B\cos(2\alpha)) \text{ and}$$

$$\varepsilon_\theta = \sigma_x(-A + C\cos(2\alpha))$$

\*KIRSCH, G. et al: "Thermal Stress in a Viscoelastic-Plastic Plate with Temperature depending Yield Stress" Journ. Appl.Mech., 27 (1898), pp.205-207