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Fatigue Behavior of Welded Joints Part 1 — Statistical Methods for Fatigue Life Prediction

A model is investigated where fatigue life and fatigue limit are treated as random variables

BY T. LASSEN, Ph. DARCIS, AND N. RECHO

ABSTRACT. Statistical models for fatigue life prediction for welded joints are discussed and fitted to experimental data for fillet-welded steel joints where cracks emanate from the weld toe. The models are based on an S-N approach where the number of cycles N to failure is assumed to be directly correlated to the applied nominal stress range ΔS. The models assume the existence of a fatigue limit given as a stress range below which no failure will take place. Emphasis is laid on the modeling of the fatigue life close to this limit where the service stresses for welded details often occur. Experimental data in this stress regime are sparse and do not fit the knee point of the conventional bi-linear S-N curve found in rules and regulations. Consequently, an alternative model where both the fatigue life and the fatigue limit are simultaneously treated as random variables is investigated. The model parameters for this random fatigue-limit model (RFLM) are determined by the maximum likelihood method, and confidence intervals are obtained by the profile likelihood method. The advantage of the model is that it takes into consideration the variation in fatigue limit found from specimen to specimen and that run-out results are easily included. The median S-N curve obtained from the model coincides with the conventional bi-linear curves in the high-stress regime (stress ranges higher than 110 MPa), but predicts longer lives as the stress range decreases below 100 MPa. The model gives a nonlinear S-N curve for a log-log scale in the fatigue-limit area; the fatigue life is gradually increasing and is approaching a horizontal line asymptotically instead of the abrupt knee point of the bilinear curve. The nonlinear curve is more in accordance with experimental data. At stress ranges below 100 MPa, the predicted fatigue lives are between 2 to 10 times longer than predictions made by the bilinear F-class curve. The conclusion is that the rule-based S-N curves may be overly pessimistic in the stress regime where service stresses frequently occur. A more correct statistical model based on a random fatigue-limit model results in S-N curves that give decreased dimensions for a given fatigue design factor under constant amplitude loading.

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Introduction and Objectives

Welded steel joints are vulnerable to fatigue damage when subjected to repetitive loading. Fatigue failure may occur even under modest in-service stresses. Furthermore, fatigue lives exhibit considerable scatter even under constant amplitude loading in controlled laboratory conditions. This phenomenon makes statistical methods indispensable and fatigue life has to be predicted at given probability levels of failure for a given welded detail under defined environment and loading conditions. The most common approach is to assume that the nominal applied stress range is the key parameter for the fatigue life, and that other loading parameters such as the mean stress has a minor effect for as-welded joints. The relation between stress range and number of cycles is determined by test series where welded details are subjected to constant amplitude loading at various stress range levels and where the number of cycles to failure is registered. The traditional S-N curves given in rules and regulations are bilinear for a log-log scale between N and ΔS. The background for these curves is a simple linear regression analysis in the finite life region. Data for the curve is obtained by accelerated constant amplitude tests at relatively high stress levels. The slope of the mean curve is determined along with the coefficient of variation in fatigue life. The fatigue limit is subsequently analyzed separately to obtain a fatigue limit (cut-off stress) for the resulting regression line. The fatigue limit is often treated as a fixed parameter where the curve becomes horizontal, typically at 10^7 cycles. The procedure and calculations are quite straightforward, apart from test specimens that are stopped at a given number of cycles before fatigue fracture occurs. There has also been debate as to whether the fatigue limit exits; if tests last long enough, the specimens may eventually fail. The objective of the present work was to investigate the ability of the bilinear S-N curves (BS 5400, Eurocode 3, AISC, Refs. 1–3) to describe the fatigue life behavior in general and in particular near the fatigue-limit regime. The validity of the upper part of these curves in the finite life area is not questioned, as these curves have been corroborated by a huge amount of data. The dilemma is that the

Keywords

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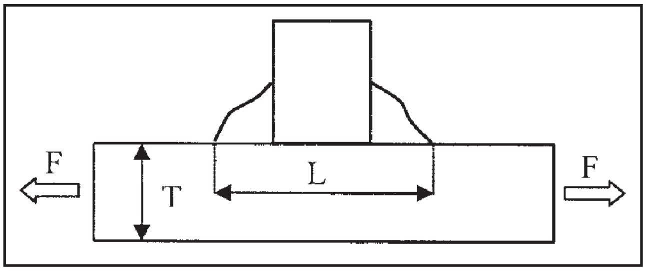

Fig. 1 — Plate with fillet welded attachments (Typical F-class in BS 5400 and category 71 in Eurocode 3, Refs. 1, 2).

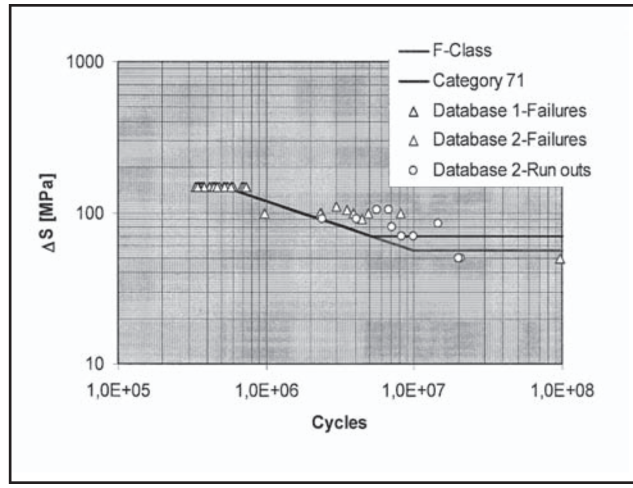

Fig. 2 — Bilinear rule-based median S-N curves together with test data.

conventional S-N curves are widely used in a stress regime where the curve changes slope to a horizontal line as described above and where very few data exist to corroborate this abrupt shift in slope. Small alterations of the position of the knee point of the curve in the fatigue-limit area will have a strong bearing on fatigue life predictions and, as a consequence, fatigue design and final dimensions of welded details. If the ability of the bilinear curve to describe fatigue life behavior is dubious, other statistical models should be applied to obtain more reliable predictions. In the present work, an investigation is made using the random fatigue-limit model (RFLM) suggested by Pascual and Meeker (Ref. 4). In this model, the scatter in finite fatigue life and in the fatigue limit is integrated into one joint model and treated simultaneously. This results in a smooth nonlinear S-N curve for a log-log scale. The model has so far been applied with success to describe the fatigue life behavior of epoxy laminate panels and smooth specimens with metal base material (Refs. 4, 5). It has so far not been applied to welded joints. To study this problem, the present study has endeavored to collect experimental data in the actual stress regime for fillet welded joints — Fig. 1. For this type of joint, fatigue cracks emanate from the weld toe and through the plate thickness. The purpose of the study is to compare the conventional S-N curves found in rules and regulations and with the alternative approach based on the random fatigue-limit model for given series of test data. The analysis and discussion are limited to constant amplitude behavior in a dry air environment in the high cycle fatigue regime.

Statistical Models for Fatigue Life

General Considerations for the S-N Approach

The basic assumptions for the S-N curves of a given welded detail are as follows:

- There is a strong dependence between the applied stress range ΔS and fatigue life given as the number of cycles N to failure.
- The standard deviation in fatigue life is not constant and increases as the stress range decreases.
- There is a fatigue limit, and it is assumed that below this threshold stress the joints will never fail.

The S-N Approach Based on BS 5400 and Eurocode 3

In rules and regulations, the relation between number of cycles N and the stress range ΔS is assumed to be linear for a log-log scale:

$$\log(N) = \log A - mLog(\Delta S) + \varepsilon \quad \Delta S > \Delta S_0 \quad (1)$$

where log denotes the logarithm to base 10. The parameters A and m characterize the fatigue quality for the joint in question, whereas ε is the error term due to the inherent scatter. ΔS_0 is the fatigue limit, and it is assumed that no failure occurs under this threshold value. Hence, the ruled-based S-N curves are bilinear for a log-log scale. The upper curve will have a slope 1/m, whereas the lower line will be horizontal. Examples are given in Fig. 2, which shows the F-class curve and the Category 71 curve taken from BS 5400 and Eurocode, respectively. These curves are discussed below. The parameters are determined by linear regression analysis in the finite life regime. The standard deviation for the logarithm of fatigue life is assumed constant for all stress levels. As a consequence, the coefficient of variation (COV) for fatigue life will be constant, and the standard deviation will increase toward lower stress levels. The COV in fatigue life is as high as 0.5 and this makes scatter a major issue. The S-N approach is based entirely on constant amplitude (CA) experimental fatigue life. A linear regression analysis is carried out, and the mean curve and standard deviation for the fatigue life are obtained. The design curve is drawn at the median value minus two standard deviations (BS 5400, Ref. 1), alternatively at minus 1.5 standard deviations if the standard deviation has a 75% confidence level (Eurocode 3, Ref. 2). At a given stress level, the fatigue life is assumed to obey a log normal distribution. This implies that the resulting mean logarithmic curve corresponds to a failure probability of p = 0.5, i.e., the median fatigue life. In the present work we work with the median S-N curve unless otherwise stated. The S-N curve taken from BS 5400, Ref. 1, reads

$$N = \begin{cases} A\Delta S^{-m} & \Delta S > \Delta S_0 \\ \infty & \Delta S \le \Delta S_0 \end{cases} \qquad (2)$$

The parameters for the median F-class curve are logA = 12.238 and m = 3. The COV is 0.54. The fatigue limit is 56 MPa and the corresponding fatigue life is 10^7 cycles. The F-class gives almost the same predictions as category 71 (Eurocode 3, Ref. 2), except that the latter curve becomes horizontal at 5×10^6 instead of 10^7 cycles. This difference pinpoints the uncertainty in the stress region near the knee-point of the curves and this is why we have collected test results in this region.

The curves are drawn in Fig. 2 together with collected data points. The data will be presented in detail in the next section. As can be seen from the figure, almost all the data are to the right-hand side of the curves near the knee points. The F-class curve is almost identical to the F-class given in AISC (Ref. 3). A general problem with the S-N curves is that they are based on data compiled without regard to material quality, thickness, and loading ratio. A reanalysis of the data where these aspects are taken into account when defining the test population is given in Ref. 6. More homogeneity classes are chosen and the scat-

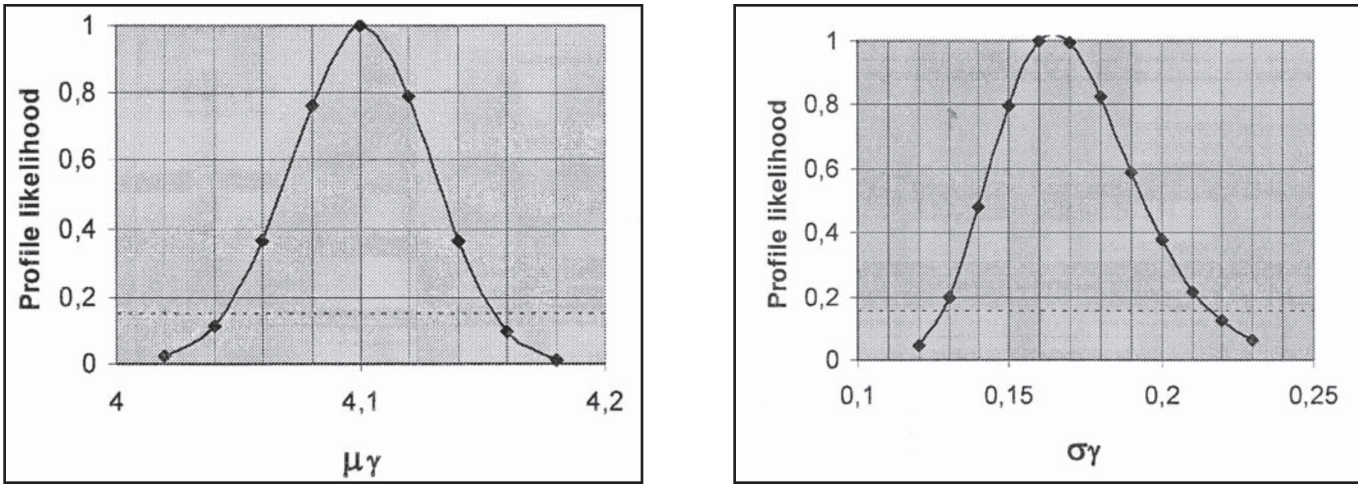

Fig. 3 — Profile likelihood with 90% confidence interval for the median and standard deviation of the fatigue limit.

ter in fatigue life decreases significantly.

The present F-class and category 71 are comparable with category 20 (Cruciform joint, Ref. 6) and category 31A (lateral attachment on a flange plate, Ref. 6). The former class predicts somewhat less fatigue life than the F-class, the latter somewhat longer. The difference is not important for the present study, but one should be aware of the fact that the scatter may be reduced by a stricter definition of the test population.

S-N Curves Based on a Random Fatigue Limit Model

Due to the uncertainty and large scatter in fatigue life in the knee-point region, an S-N curve based on a random fatigue-limit approach has been proposed (Refs. 4, 5). In fact, the sparse data available indicate that there is a variation in fatigue limit from specimen to specimen. Consequently, the distribution for the fatigue limit should be sought and incorporated into the statistical model for the fatigue life. It should not be treated separately as done for the bilinear curves. The S-N curve obtained from the RFLM will not have an abrupt change from an inclined straight line to a horizontal line, but a gradual change in slope as stress ranges get very low. Our hypothesis is that this nonlinear curve for a log-log scale is more consistent with observed fatigue life data for welded details at low stress ranges. The governing equation is

$$\ln(N)=\beta_0-\beta_1\ln(\Delta S-\gamma)+\varepsilon,\ \ \Delta S>\gamma \tag{3}$$

where ln denotes the natural logarithm and $\gamma = \Delta S_0$ is the fatigue limit. β_0 and β_1 are fatigue curve coefficients. As can be seen, Equation 3 is fundamentally different from Equation 1. Let $V = \ln(\gamma)$ and assume that V has a probability density function (PDF) given by

$$f_v(v)=\frac{1}{\sigma_\gamma}\phi_v\left(\frac{v-\mu_\gamma}{\sigma_\gamma}\right) \tag{4}$$

with location parameter and scale parameter μ_γ and σ_γ, respectively. $\phi_v(\cdot)$ is the normal PDF. The normal distribution was chosen because it gave the best fit to fatigue data in Ref. 4, and due to the fact that it is the usual assumption for fatigue life distribution in rules and regulations. Let $x = \ln(\Delta S)$ and $W = \ln(N)$. Assuming that, conditional on a fixed value of $V<x$, $W|V$ has a PDF

$$f_{w|v}(w)=\frac{1}{\sigma}\phi_{w|v}\left(\frac{w-\left[\beta_0-\beta_1\ln\left(exp(x)-exp(v)\right)\right]}{\sigma}\right) \tag{5}$$

with the location parameter $\beta_0 - \beta_1 \ln(exp(x) - exp(v)$ and scale parameter σ. The marginal PDF of W is given by

$$f_w(w)=\int_{-\infty}^{x}\frac{1}{\sigma\sigma_\gamma}\phi_{w|v}\left(\frac{w-\left[\beta_0-\beta_1\ln\left(exp(x)-exp(v)\right)\right]}{s}\right)\phi_v\left(\frac{v-\mu_\gamma}{\sigma_\gamma}\right)dv \tag{6}$$

The marginal cumulative distribution function (CDF) of W is given by

$$F(w)=\int_{-\infty}^{x}\frac{1}{\sigma_\gamma}\Phi_{w|v}\left(\frac{w-\left[\beta_0-\beta_1\ln\left(exp(x)-exp(v)\right)\right]}{\sigma}\right)\phi_v\left(\frac{v-\mu_\gamma}{\sigma_\gamma}\right)dv \tag{7}$$

where $\Phi_{W|V}(\cdot)$ is the CDF of $W|V$. For given sample data w_i and x_i from various test specimens $i = 1, n$, the model parameters can be determined by the maximum likelihood (ML) function

$$L(Q)=\prod_{i=1}^{n}\left[f_w(w_i;x_iQ)\right]^{\delta_i}\left[1-F_w(w_i;x_iQ)\right]^{1-\delta_i} \tag{8}$$

where $\delta_i = 1$ if w_i is a failure and $\delta_i = 0$ if w_i is a censored observation (run out).

The vector Q contains the model parameters

$$Q=\left(\beta_0,\beta_1,\sigma,\mu_\gamma s_\gamma\right) \tag{9}$$

Once these parameters have been determined from optimization of Equation 8, the corresponding confidence intervals can be obtained by a profile likelihood method using profile ratio of the variables together with chi-square statistics. The integration of Equations 6 and 7 and the optimization of Equation 8 must be done numerically. Details are found in Refs. 4, 5. When the parameters are determined, we can calculate the fatigue life for a chosen probability p of failure using Equation 7. Hence, the median curve and quantiles curves for design purpose are obtained.

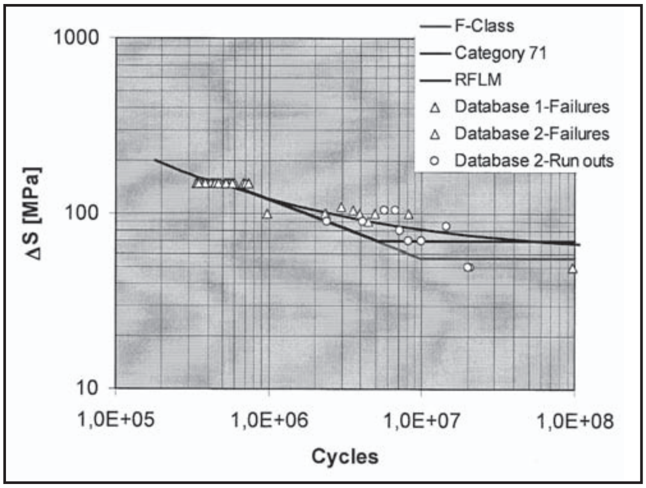

Fig. 4 — Bilinear and RFLM based median S-N curves together with test data.

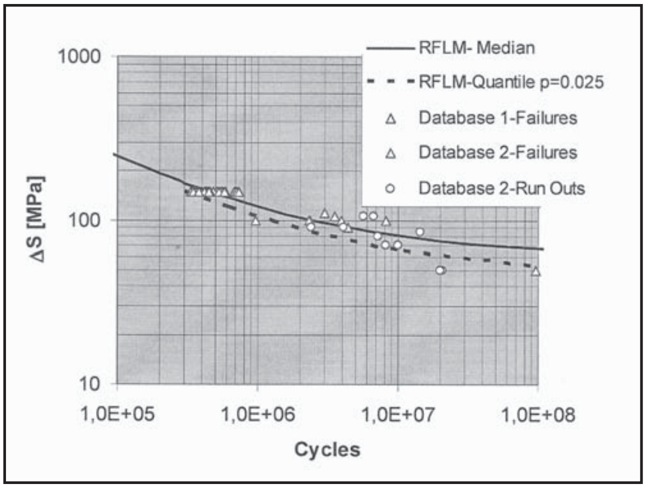

Fig. 5 — Median and quantile S-N curves based on the RFLM together with test data.

Table 1 — Parameters for the Random Fatigue-Limit Model

Parameter	Point Estimate	90% Confidence Interval	
β_0	22.4800	22.407	22.555
β_1	2.100	2.084	2.118
σ	0.14	0.089	0.240
μ_γ	4.100 (60.3 MPa)	4.044 (57 MPa)	4.154 (63.7 MPa)
σ_γ	0.16	0.120	0.216

Experimental Data for Model Calibration

Data for Fatigue Life at High Stress Levels (Database 1)

Database 1 contains experimental fatigue lives in the finite life regime where all tests are continued until fracture occurs. The test series consist of 34 non-load-carrying cruciform and T joint test specimens — Fig. 1. All the test specimens were fabricated from C-Mn steel plate 25 mm (1 in.) thick. The nominal yield stress was 345 MPa (50 ksi). The welding procedures were taken from normal offshore fabrication practice. The joints were proven free from cracks and undercuts. The specimens were tested under constant amplitude axial loading at ΔS = 150 MPa (21.7 ksi) with a loading ratio of R = 0.3. Experimental details are found in Ref. 7. The total fatigue lives for the 34 specimens have been plotted in Fig. 2. The median life of the series (N = 460,000 cycles) is only 12% less than the prediction of the F-class (N = 513,000 cycles).

Hence, the test series is of normal quality and comparable with population pertaining to the F-class and category 71. However, due to the homogeneity of the test series, the COV is as expected much smaller — COV = 0.22. In addition to recording the fatigue life, crack growth measurements were made during the course of each test. These data are important information for modeling the fatigue process, but not for the present fatigue life statistical analysis.

Data for Fatigue Lives at Low Stress Levels (Database 2)

Data points pertaining to the F-class S-N curve have the center of gravity at a stress range in the region of 120–150 MPa (17.4–21.7 ksi). This regime is well represented by database 1. In addition, we have in the present work assembled results at lower stress level from fatigue life test series in several large experimental investigations carried out in Europe (Ref. 8). The applied stress ranges are between 80 and 105 MPa (11.6–15.2 ksi) and the thicknesses of the plates range from 16 to 38 mm (0.63–1.5 in.). Other thicknesses are excluded to minimize the so-called thickness effect. All the selected specimens are as-welded and the loading ratio R is between 0 and 0.3. These data-points are also plotted in Fig. 2.

As can be seen, most of the data points have substantially longer lives than the predictions of the F-class curve, with only a few exceptions. Furthermore, some of the results are run outs. The scatter is considerably greater than for database 1, as expected. This is partly due to the fact that database 1 contains only one homogeneity test series, but mostly due to the fact that scatter increases at low stress levels as already discussed.

Comparison between the F-Class Curve, the RFLM Based Curve, and the Data

By applying Equation 4 to 9 for the databases presented above, the parameters for the RFLM are determined and given in Table 1. The 90% confidence intervals are also listed.

The confidence intervals are obtained from plots of the profile ratio as shown for μ_γ and σ_γ in Fig. 3. As can be seen, the point estimate for the fatigue limit is near 60 MPa. The BS 5400 fatigue limit of 56 MPa is well within the 90% confidence interval and is quite close to the point estimate of 60 MPa. The Eurocode fatigue limit of 70 MPa is far outside our confidence interval.

Although the present database is limited, it is a surprise that the Eurocode fatigue limit is so far outside the 90% confidence interval. The median curve (p = 0.5) for the fatigue life pertaining to the point estimates in Table 1 is drawn in Fig. 4 along with the F-class curve and the data points. As can be seen, the F-class curve, the RFLM curve, and the data points at a stress range of 150 MPa (Database 1) are in good agreement. At lower stress ranges the RFLM curve becomes nonlinear and predicts substantially longer fatigue lives than does the F-class. At a stress range of 80 MPa the RFLM predicts more than three times longer fatigue life, and at 70 MPa, the RFLM predicts close to 10 times longer fatigue life than the F-class.

It can also be seen from the figure that the RFLM is obviously more in accordance with the data-points. The quadratic sum of the error terms ε in Equation 3 will be much less than the corresponding sum

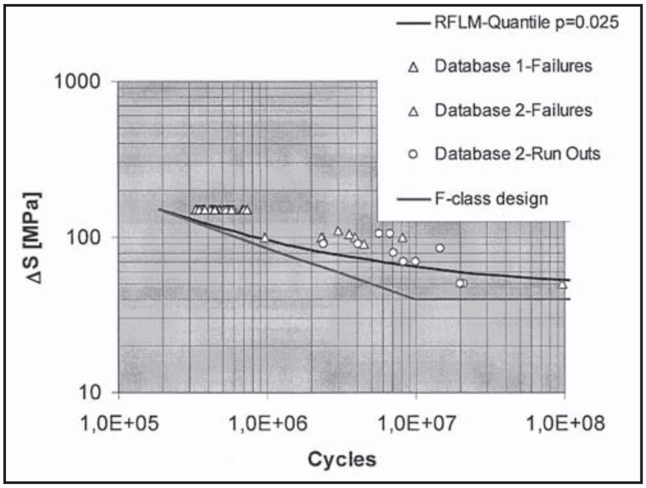

Fig. 6 — Comparison between the RFLM design curve (with β_0=22.60, σ = 0.5) and the F-class design curve.

pertaining to Equation 1. As the stress level approaches the F-class fatigue limit of 56 MPa, the RFLM curve has become so horizontal that it almost coincides with the F-class horizontal line. However, it is only for fatigue lives longer than 10^9 cycles that the two curves are for all practical purposes identical. This illustrates that care must be taken when comparing fatigue limits as we did above. The fatigue limit for the F-class will already appear at 10^7 cycles, whereas the RFLM curve will approach the fatigue limit of 60 MPa at 5 10^9 cycles. The reason for this major difference is that the RFLM is based on a joint distribution (random fatigue life, random fatigue limit), and this will push the point estimate for fatigue limit toward a higher number of cycles.

When comparing the RFLM curve with the category 71 curve, it can be seen from Fig. 4 that the discrepancy is less than was found from comparison with the F-class. In fact, the curves cross each other at 5 10^7 cycles at the fatigue limit of 70 MPa given for the category 71. Above this stress range the RFLM curve will predict longer lives than the category 71, whereas at lower stress ranges the reverse will be true. The quantile curve pertaining to p = 0.025 is shown in Fig. 5.

When comparing with the data points, it can be seen that it is only the two run outs at 2 10^7 cycles that are well below the curve, whereas two failure data points at 10^6 and 10^8 cycles are just beneath the curve. All these 4 points pertain to database 2. The curve obtained cannot be directly compared to the F-class design curve due to the fact that our database is more limited and has less scatter in the finite life region (database 1).

To force our model to be valid for the huge amount of data pertaining to the F-class in the finite life region, we adjust the parameters given in Table 1 so that the mean life and scatter for high stresses coincide with the F-class data. The parameter β_0 is increased from 22.48 to 22.60 and σ is increased from 0.14 to 0.50. The latter figure is a characteristic scatter for most categories of welded joints. The other parameters are kept constant as a first approximation. The obtained quantile S-N curve (p = 0.025) is drawn together with the F-class design curve (based on logarithmic mean minus two standard deviations).

As can be seen from Fig. 6, the fit between the two curves is amazingly good in the high-stress region, with the curve coinciding at stress ranges above 110 MPa. Both curves have now good safety margins to all the data points in test series 1, as expected. As for the median curves, the RFLM curve will predict substantially longer lives than the F-class below 100 MPa. In fact, the RFLM curve is only slightly lower in this stress region compared with the original quantile curve in Fig. 5. This is due to the fact that it is the parameters that characterize the random fatigue limit that mainly govern the curve in this area.

Conclusions

The statistical behavior of the fatigue life of fillet welded joints has been examined and modeled with reference to conventional S-N curves found in current rules and regulations.

An alternative statistical model based on a joint random fatigue life and a random fatigue limit has been applied. Constant amplitude fatigue life data near the “knee point” of the rule-based bilinear S-N curves are assembled to study and corroborate the model.

The model has been fitted to experimental fatigue lives and the obtained S-N curve is compared with the traditional bilinear S-N curves given in rules and regulations. The rule-based S-N curves and the RFLM based curve coincide for stress ranges above 110 MPa. For stress ranges below 100 MPa, the RFLM curve will predict fatigue lives that are from 2 to 10 times longer than the predictions made by the F-class S-N curve.

It appears that the nonlinear curve obtained from the RFLM has a much better ability to model fatigue life behavior in this stress region. The abrupt knee point of rule-based bilinear curves does not fit the experimental facts for the assembled data. The fatigue life behavior in this stress regime is obviously more complex than the conventional bilinear S-N curve can describe.

The discrepancy between the present RLFM curve and the F-class curve is important as it occurs in a stress region where the majority of the load cycles for a welded detail in service usually occur. The rule-based S-N curves seem overly pessimistic in this regime and this will have a strong bearing on practical fatigue life predictions, fatigue design and final dimensions of welded details.

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