Technical Note: Shear Stress Distribution at the Bond of Brazed Heat Transfer Panels

BY A. C. NUNES, JR.

Analyses of the distribution of stresses in an adhesion-bonded or riveted double lap joint have appeared in engineering journals from 1909[1-3] to 1938[4] when O. Volkerson's paper achieved sufficiently wide recognition to make the analysis general knowledge. Review papers by N. K. Benson[5] and O. Volkerson[6] discussed later developments of the mechanics of adhesion-bonded lap joints.

The analysis, which is also applicable to screw threads,[7] is based on the idea that the shear stress at the bonded interface transfers the tensile load from one lap member to the other; as the lap members distort according to their tensile loadings, the relative displacements on opposite sides of the bonded interface change so as to give rise to a variation in shear stress along the length of the bonded interface. For a double lap joint these bond shear stresses prove to be a maximum at the outer edges of the bond. The maximum shear stress applied to such a bond in a shear test is higher than the average shear stress for the entire contact surface by a stress concentration factor that depends on the amount of overlap.

The *double* lap joint analysis traditionally ignores bending stresses and end effects (except, of course, for the above mentioned stress concentration that forms the chief point of interest in the analysis); these stresses and end effects are present to some extent even in a physical double lap joint and to a much greater extent in a single lap joint where care has not been taken to make shear forces act through the bond plane. For more sophisticated (and more complicated) analyses the reader is referred to the above cited reviews, to apply the double lap joint analysis to a slotted plate bonded on the slotted side to another plate as shown in Fig. 1. Such a configuration provides passages that may be used for a cooling or heating fluid. Although the slots are taken as rectangular in cross section, adaptations of the analysis to more complicated shapes can be made without great difficulty.

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As shown in Fig. 1 the slotted plate has a slot height h with a backing height w above the slots; the webs between the slots are spaced at distance λ and have thickness t. The lower plate which supports the slotted surface has height w_s; and the overlap distance is ℓ. The elastic modulus of the slotted plate is taken as E, and that of the supporting plate is E_s. The average shear stress over the bonded surface is taken as $\bar{\tau}$, and in Fig. 1 the shear parallel to the slots is distinguished by a subscript p and that transverse to the slots by subscript t. The p and t distinctions being clear from the context in what follows, these subscripts will be omitted.

Free body force diagrams and assumed deformation modes are shown in Fig. 2. Labelling the webs between the slots by the subscript i varying from 1 for the first web to N for the last and taking the tensile force per unit width in the backing immediately ahead of the i^{th} web as P_i and the transverse local shear in the i^{th} web as τ_i force equilibrium requires that:

$$P_i - P_{i-1} = -\tau_i t \qquad (1a)$$

The same equation for the parallel shear situation, where the local shear is taken as a function of z, i.e., $\tau(z)$, where z is the distance from the beginning of overlap, and the local backing tension is $P(z)$ is:

$$\frac{\partial P}{\partial z} = -\tau \cdot \frac{t}{\lambda} \qquad (1b)$$

where it is understood that τ and P are local values.

The web stress-deformation relation relates τ_i to the displacements V_i in the slotted plate and v_i in its support in terms of the shear modulus G of the slotted plate:

$$-\tau_i = G\left(\frac{u_i - v_i}{h}\right) \qquad (2a)$$

The equivalent equation for parallel shear is:

$$-\tau = G\left(\frac{u - v}{h}\right) \qquad (2b)$$

Taking Poisson's ratio for the slotted plate as ν and ν_s for its support stress-deformation relations for the slot backings are:

$$u_{i+1} - u_i = \left(\frac{1 - \nu^2}{E}\right)\left(\frac{\lambda}{w}\right) P_i \qquad (3a)$$

and,

$$\frac{\partial u}{\partial z} = \left(\frac{1 - \nu^2}{E}\right)\left(\frac{1}{w}\right) P \qquad (3b)$$

in the transverse and parallel cases respectively, and for the support plates:

$$v_{i+1} - v_i = \left(\frac{1 - \nu_s^2}{E_s}\right)\left(\frac{\lambda}{w_s}\right) T_i \qquad (4a)$$

and

$$\frac{\partial v}{\partial z} = \left(\frac{1 - \nu_s^2}{E_s}\right)\left(\frac{1}{w_s}\right) T \qquad (4b)$$

The stress perpendicular to the plates is assumed zero as is also the strain in the plane of the plates and transverse to the direction of tension.

Overall equilibrium across the joint requires that for the transverse case:

$$T_i + P_i = P_o \qquad (5a)$$

and for the parallel case,

$$T + P = P_o - \frac{E}{1 - \nu^2}\left(\frac{ht}{2\lambda}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial z}\right) \qquad (5b)$$

where P_o is the total tension in half of the double lap joint. It should be noted that in the parallel case the web between slots carries some of the tensile load.

Because the boundary conditions for the tensions are known:

$$P_{i=o} = P_o \qquad (6a)$$

$$P_{i=N} = O \qquad (7a)$$

or in the parallel case,

$$P(o) = P_o \qquad (6b)$$

$$P(\ell) = 0 \qquad (7b)$$

Equations 1–5 are set up as a difference (transverse) and a differential (parallel) equation that may be solved for P_i and $P(z)$ respectively.

$$P_{i+2} - \left\{2 + \frac{Gt\lambda}{h}\left[\frac{1-\nu^2}{Ew} + \frac{1-\nu_s^2}{E_s w_s}\right]\right\} P_{i+1} + P_i = -\frac{Gt\lambda}{h}\frac{1-\nu_s^2}{E_s w_s} P_0 \qquad (8a)$$

$$\frac{\partial^2 P}{\partial z^2} - \frac{Gt}{h\lambda}\left\{\frac{1-\nu^2}{Ew} + \frac{1-\nu_s^2}{E_s w_s}\left[\frac{1 + \frac{ht}{2w\lambda}}{1 + \frac{E}{E_s}\left(\frac{1-\nu_s^2}{1-\nu^2}\right)\frac{ht}{2w_s\lambda}}\right]\right\} P = -\frac{Gt}{h\lambda}\frac{1-\nu_s^2}{E_s w_s}\left[\frac{1}{1 + \frac{E}{E_s}\left(\frac{1-\nu_s^2}{1-\nu^2}\right)\frac{ht}{2w_s\lambda}}\right] P_0 \qquad (8b)$$

By making the definitions:

$$a \equiv 1 + \frac{G\lambda t}{2h}\left[\frac{1-\nu^2}{Ew} + \frac{1-\nu_s^2}{E_s w_s}\right] \qquad (9a)$$

$$b \equiv \frac{G\lambda t}{h}\left[\frac{1-\nu_s^2}{E_s w_s}\right] \qquad (10a)$$

$$x \equiv a + \sqrt{a^2 - 1} \qquad (11a)$$

$$y \equiv a - \sqrt{a^2 - 1} \qquad (12a)$$

Solving eq 8a incorporating the boundary conditions eqs 6a and 7a:

$$\frac{P_i}{P_0} = \left[1 - \frac{b}{2(a-1)}\right]\left[\frac{x^{N-i} - y^{N-i}}{x^N - y^N}\right] + \frac{b}{2(a-1)}\left[1 - \frac{x^i - y^i}{x^N - y^N}\right] \qquad (13a)$$

and from eq 1a,

$$\frac{\tau_i}{\bar{\tau}} = N\left\{\left[1 - \frac{b}{2(a-1)}\right]\left[\frac{x^{N-i}(x-1) - y^{N-i}(y-1)}{x^N - y^N}\right] + \frac{b}{2(a-1)}\left[\frac{x^i(1-y) - y^i(1-x)}{x^N - y^N}\right]\right\} \qquad (14a)$$

Similarly by defining:

$$c = \frac{Gt}{h\lambda}\left[\frac{1-\nu^2}{Ew} + \frac{1-\nu_s^2}{E_s w_s}\left(\frac{1 + \frac{ht}{2w\lambda}}{1 + \frac{E}{E_s}\left(\frac{1-\nu_s^2}{1-\nu^2}\right)\frac{ht}{2w_s\lambda}}\right)\right] \qquad (9b)$$

$$d = \frac{Gt}{h\lambda}\left(\frac{1-\nu_s^2}{E_s w_s}\right)\left[\frac{1}{1 + \frac{E}{E_s}\left(\frac{1-\nu_s^2}{1-\nu^2}\right)\frac{ht}{2w_s\lambda}}\right] \qquad (10b)$$

Solving eq 8b incorporating the boundary conditions eqs 6b and 7b:

$$\frac{P}{P_0} = \left(1 - \frac{d}{c}\right)\left[\frac{e^{\sqrt{c}(l-z)} - e^{-\sqrt{c}(l-z)}}{e^{\sqrt{c}\,h} - e^{-\sqrt{c}\,h}}\right] + \frac{d}{c}\left[1 - \frac{e^{\sqrt{c}z} - e^{-\sqrt{c}z}}{e^{\sqrt{c}l} - e^{-\sqrt{c}l}}\right] \qquad (13b)$$

and from eq 1b,

$$\frac{\tau}{\bar{\tau}} = \sqrt{c}\,\ell\left\{\left(1 - \frac{d}{c}\right)\left[\frac{e^{\sqrt{c}(l-z)} + e^{-\sqrt{c}(l-z)}}{c^{\sqrt{c}l} - e^{-\sqrt{c}l}}\right] + \frac{d}{c}\left[\frac{e^{\sqrt{c}z} + e^{-\sqrt{c}z}}{e^{\sqrt{c}l} - e^{-\sqrt{c}l}}\right]\right\} \qquad (14b)$$

Equations 11b and 12b which would correspond to 11a and 12a do not exist. Equations 13b and 14b are so numbered to link them with 13a and 14a which correspond to them in the difference formulation of the transverse stress.

The maximum shear stresses τ_1 at $i = 1$ or $\tau(0)$ and $z = 0$ are:

$$\frac{\tau_1}{\bar{\tau}} = N\left\{\left[1 - \frac{b}{2(a-1)}\right]\left[1 - \frac{x^{N-1} - y^{N-1}}{x^N - y^N}\right] + \frac{b}{2(a-1)}\left[\frac{x-y}{x^N - y^N}\right]\right\} \qquad (15a)$$

and

$$\frac{\tau(0)}{\bar{\tau}} = \left\{\left[1 - \frac{d}{c}\right]\frac{\sqrt{c}\,\ell}{\tan h\sqrt{c}\,\ell} + \frac{d}{c}\frac{\sqrt{c}\,\ell}{\sin h\sqrt{c}\,\ell}\right\} \qquad (15b)$$

As an illustrative example let us suppose that w_s is so large that terms containing b and d may be neglected (i.e., a very stiff support) and let $w = h = t = \lambda/4 = 1$ unit and $\nu = 1/3$. Then $a = 5/3$, $x = 3$, $y = 1/3$, and $c = 1/12$. In the transverse dierction let the overlap be such that $N = 6$ (a distance of about $N\lambda$ or 24 units) and in the parallel direction let the overlap be 24 units also (i.e., $\ell = 24$). In the stress distributions shown in Fig. 3, it is apparent that the leading edges of the bond carry the bulk of the load and that bond failure or yield may be expected to begin at the leading edge when the average bond stress is much below the actual bond failure stress.

The value of a mechanical test lies in its applicability to an engineering design. However, it is unlikely that a shear test of the strength of a brazed bond will duplicate closely the in-service

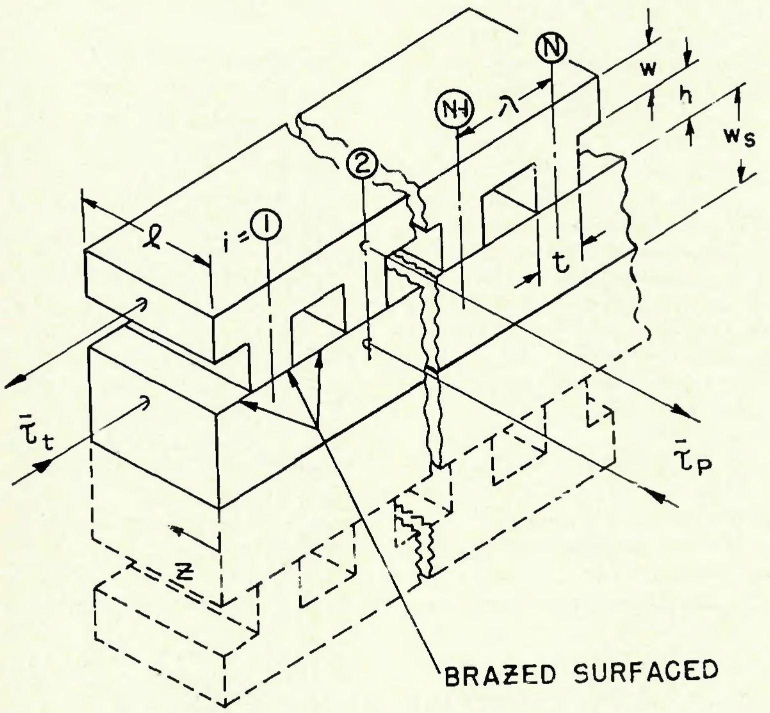

Fig. 1—Geometry of brazed lap joint showing parallel (τ_p) and transverse (τ_t) shears. A mirror image of the single lap joint is introduced to eliminate gross bending; the double lap joint is thus formed

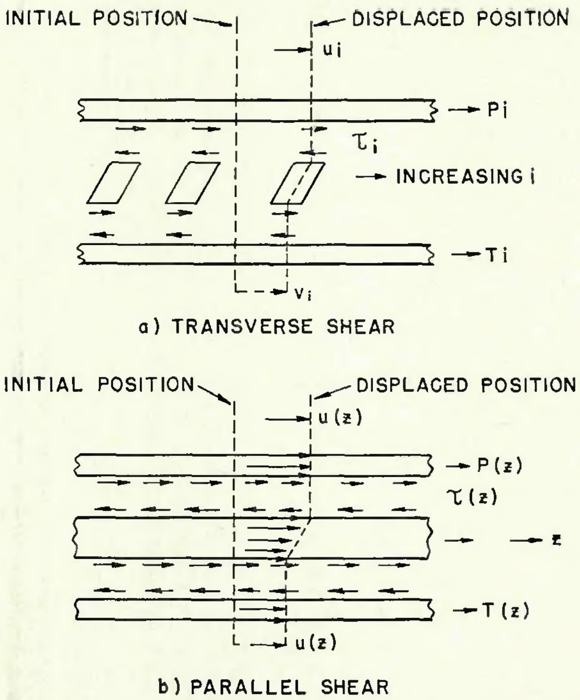

Fig. 2—Assumed deformation modes and force-stress sign conventions. Note that displacements are taken as positive in the direction of the coordinates; in actuality they will be in the opposite direction to that shown. The shear stress will be directed as shown

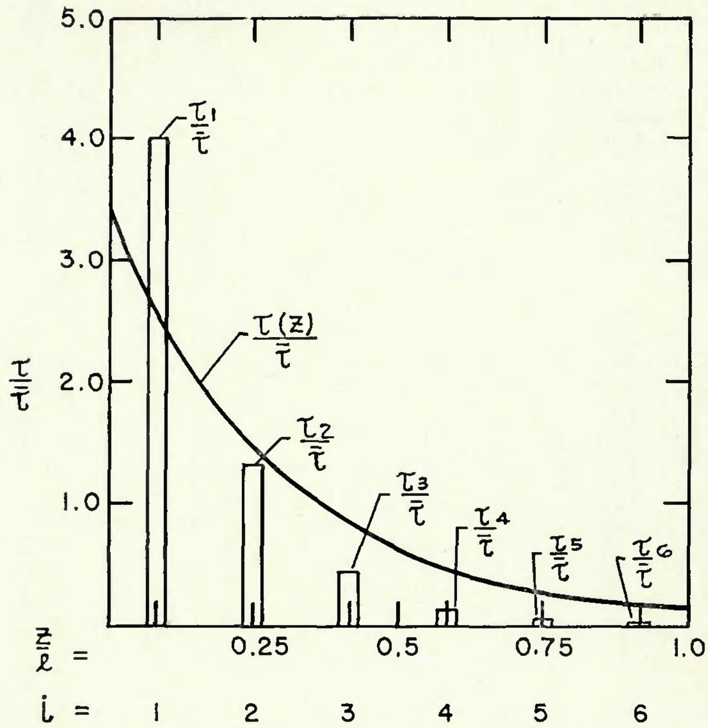

Fig. 3—Sample problem bond interface shear stress distribution

loading conditions on the bond. Thus, it is necessary to have a theory by which one can proceed from the experiment to a generality (bond shear strength) and back to an application. The above analysis enables one to approach closer to but, unfortunately, not to obtain an actual bond shear strength. Local distortions of the material away from the assumed distortional modes give rise to a fine structure in the bond stress distribution; this requires additional stress concentration factors to be taken into account.

An additional stress concentration factor can arise out of bending of the entire joint if a single lap joint is used instead of a double. Local bending of the webs between the grooves is important if the webs are long enough to be flexible; treating the webs as plates and suitably modifying the constants in eq 2a would allow a better estimate of the transverse shear stress distribution overall and locally over the web interface in this case. Transverse shear could produce a large tensile stress normal to the bond, due to web bending, if the webs were flexible enough and such a test might have value in determining the normal tensile strength of the bond. Complex distortions at the free surfaces of the loading edge of the bond again give rise to another stress concentration factor.

On the other hand, plastic deformation at the bond interface or in the webs tends to redistribute the bond stresses in such a way as to equalize them over the bond surface. If two very stiff surfaces are bonded and tested in shear, the relative displacements along the bond will tend to be unchanged with distance from the leading edge, and this type of test coupled with some tendency of the bond to shear without failure may be expected to give a reasonably meaningful indication of the bond shear strength. It is possible, however, that such a test specimen bond may be metallurgically different from the bond in the more flexible brazed heat transfer panels under discussion.

It is concluded from the above that the results of a shear test of the brazed heat transfer panel discussed above may be so strongly dependent upon the geometry of the test specimen and nature of its loading as to prove very difficult to interpret. Unless the test is made under actual service conditions so that no analytical work is necessary to span the gap between service and test conditions, it is suggested that test results be compared with other tests of apparently similar bonds between very stiff surfaces.

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See pages 92-s and 137-s for important announcements