Computer Analysis of Degree of Constraint of Practical Butt Joints

Computed and measured values agree reasonably well when utilizing the finite-element method, with constant strain element, to compute the degree of constraint for different joint configurations

BY K. MASUBUCHI AND N. T. ICH

ABSTRACT. This paper deals with a computer analysis of degree of constraint against transverse shrinkage of several practical butt joints. The so-called finite-element method was used for the stress analysis.

The degree of constraint, which characterizes how much a joint is restrained, is an important parameter for studying distortion and cracking tendency of the joint.

First, a study was made to determine whether or not the finite-element method can be used effectively for analyzing the degree of constraint. Computations using several different models were made of a rectangular plate containing a slit of a very thin elliptic shape, since an analytical solution for the degree of constraint had already been obtained by Masubuchi for an infinitely large plate. It was concluded that the constant strain element type with a relatively coarse grid system, say 105 nodes, be used in the computation of the degree of constraint.

Then a study was made to compute the degree of constraint of various joint configurations as follows:

1. Rectangular plate containing a straight slit, and a slit with circular holes at the ends.
2. H-type slit specimens.
3. Lehigh-type crack-susceptibility test specimen with and without sawcut.

Analytical results coincided well with experimental results obtained on some specimens.

Introduction

Welding is an efficient method of fabricating metal structures. Various products including space vehicles, ships, bridges, pressure vessels, have been built by welding.

One of the troublesome problems that accompany the construction of welded structures is shrinkage distortion. The more complex the structure is, the more involved the problem becomes. Shrinkage distortion can cause mismatch of joints which leads to the possibility of welding defects.

The correction of weld distortion is costly and in some cases impossible. It is therefore desirable to develop techniques to predict values of the shrinkage distortion in order that palliative measures can be devised that will neutralize or reduce the effect of shrinkage distortion.

Another problem associated with welding is cracking. Cracks may form as a result of the welding operation and can occur within the weld metal or in the heat-affected base metal.

The degree of constraint, which characterizes how much a joint is restrained, is an important parameter for studying distortion and weld cracking. As a joint is more highly restrained, or as the degree of constraint increases, distortion decreases and residual stresses increase. When the degree of constraint exceeds a certain limit for the particular materials and welding processes used, cracking may result.

A number of investigations have been made of the effects of the degree of constraint on distortion of simple laboratory weldments. For example, Kihara-Masubuchi[1-4] and Watanabe-Satoh[4-6] have obtained relationships between the degree of constraint, K, and the amount of transverse shrinkage, or shrinkage across the weld, of several types of butt joints including a slit-weld joint and a ring-type joint.

However, applications of such results to the analysis of distortion in actual structures have been rather limited. One of the major problems here is the difficulty in analytically determining the degree of constraint of a practical joint which may be restrained by various structural members. Due to mathematical complexity involved in the stress analysis, studies conducted so far have been limited to simple joints.

If a method was developed to analytically determine the degree of constraint of a practical joint, the experimental results could be utilized more effectively to predict distortion during welding fabrication of various structures. This type of analysis should be very useful for the control of weld distortion during fabrication of complex, critical structures such as space vehicles, aircraft, and super tankers, as well as other ordinary structures.

A similar situation exists in the study of weld cracking. Many cracking tests in various configurations have been developed for evaluating a tendency for weld cracking.[7] Most tests have used specimens with varying degrees of restraint such as the Varestraint specimen developed by Savage,[8] or a series of specimens with different degrees of constraint such as the Lehigh test developed by Stout.[9]

The problem, again, is the lack of a numerical study. Although experiments show that a test specimen can be welded without cracking, it is not possible to determine whether or not an actual structure can be welded without cracking. If there were an analytical method of determining the degree of constraint in both a cracking specimen and an actual structure, it would then be possible to use results from cracking tests for evaluating the cracking tendency in an actual structure.

Bearing in mind these needs for better analytical means for determining the degree of constraint in a practical joint, a study was conducted at the Massachusetts Institute of Technology on the use of a computer in solving this problem.[10] After an initial survey, it was decided that the so-called finite-element method be used.[11-14] The finite-element method is an approximate stress analysis technique which yields accurate solutions for complex problems by use of a high-speed computer. The

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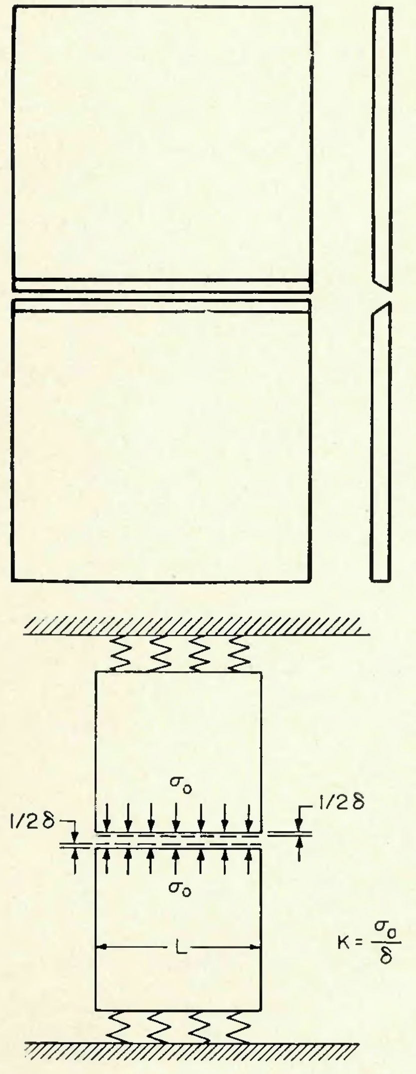


Fig. 1—Free joint and constrained joint: a (top)—free joint; b (bottom)—joint constrained by a set of springs

method, which was originally developed in the aircraft industry, has been applied to a wide variety of problems with gratifying success.

Technical Background

Residual Stresses and Distortion

Residual stresses and distortion are closely related phenomena. Residual stresses are those stresses that would exist in a body if all external loads were removed. Residual stresses in metal structures occur for many reasons during various manufacturing stages, including welding.

Because a weldment is locally heated by the welding heat source, complex strains occur in the weld metal and base-metal regions near the weld, both during heating and cooling. The strains produced during heating are accompanied by plastic upsetting. The stresses resulting from these strains combine and react to produce internal forces that cause bending, buckling, and rota-

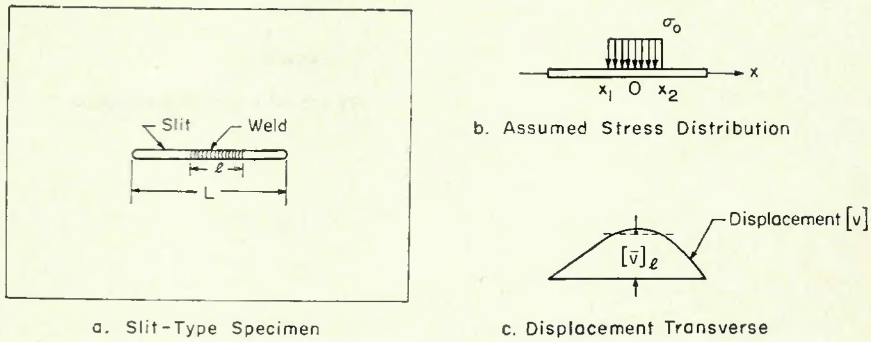


Fig. 2—Analysis of degree of constraint K of a slit-type weld

tion. It is these displacements that are called distortion.

The distortion found in fabricated structures is caused by three fundamental dimensional changes that occur during welding:

1. Transverse shrinkage that occurs perpendicular to the weld line.
2. Longitudinal shrinkage that occurs parallel to the weld line.
3. An angular change that consists of rotation around the weld line.

This paper discusses effects of degree of constraint on transverse shrinkage. However, it is theoretically possible to expand the same principle to cover other types of distortion.*

Degree of Constraint and Its Effect on Transverse Shrinkage

Welded joints may be classified into a free joint, as shown in Fig. 1a, and a constrained joint. Figure 1b shows a simple butt joint constrained by a set of springs. When tensile stress, σ_o, uniformly distributed along the weld (length L), or total load, $P = \sigma_o L$, is needed to cause transverse shrinkage δ, the degree of constraint, or the spring constant of the constaint, K, can be defined:

$$K = \frac{\sigma_o}{\delta} = \frac{P}{L\delta} \qquad (1)$$

Equation (1) gives the definition of the degree of constraint of a butt joint under the simplest condition.

In the fabrication of a complex structure, however, joints are seldom completely free. In addition, the degree of constraint is frequently not uniform along the weld, as it is shown in Fig. 1b, but vary along the weld.

As an example, Fig. 2 shows a slit weld, in which a slit is made in a plate and then the slit is welded. Welds similar to the slit weld are frequently made in repairs. Transverse shrinkage in the slit weld is restrained by the base metal surrounding weld; in other words, the joint is internally restrained. In a slit weld, the degree of constraint varies along the length of the slit, being highest at the ends. Consequently, the amount of transverse shrinkage is greater near the center of the joint and very slight near the ends of the joint.

Analysis of Degree of Constraint. Kihara and Masubuchi[1,2] analyzed the degree of constraint of a slit weld, as shown in Fig. 2. They defined the degree of constraint of a slit weld, K, when welding is done on a part of the slit between $x = x_1$ and x_2 (slit length L; weld length $l = x_1$ to x_2).

When uniform stress σ_o is applied along both edges of the part of the slit between $x = x_1$ and x_2, the distance between both edges decreases. The distribution of the transverse displacement, $[v]$, is as shown in Fig. 2c. By determining the average value of the lateral displacements between x_1 and x_2, $[\bar{v}_l]$, the degree of constraint, K, can be defined as follows:

$$K = \frac{\sigma_o}{[\bar{v}]_l} \qquad (2)$$

Based upon the theory of elasticity, and assuming that plate is infinitely large, Kihara and Masubuchi determined an analytical expression of K as follows:

$$K = \frac{\pi}{2}\frac{E}{L}\frac{l}{L}\frac{1}{\phi} \qquad (3)$$

where,

E = modulus of elasticity

$$\phi = \sum_{n=1}^{\infty}\left[\int_{\theta_1}^{\theta_2} \sin\theta \cdot \sin n\theta \cdot d\theta\right]^2$$

$$x_1 = \frac{L}{2}\cos\theta_1$$

$$x_2 = \frac{L}{2}\cos\theta_2$$

The degree of constraint, K, is determined by:

1. Young's modulus of the material, E.
2. Slit length, L.
3. Ratio of the weld length to the slit length, l/L.

*An interpretive report being prepared by Masubuchi[15] for the Welding Research Council discusses effects of degree of constraint on angular change as well as transverse shrinkage.

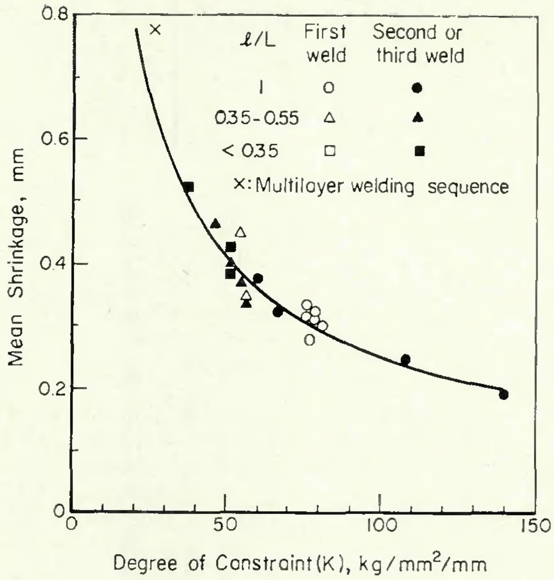
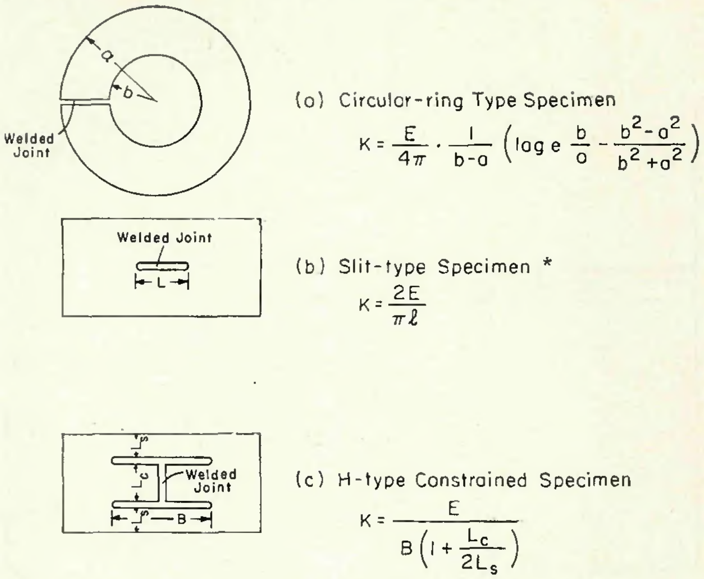


Fig. 3—Relationship between degree of constraint and transverse shrinkage in a slit-type specimen



* When weld is made along the entire slit length, or $l/L = 1$, the value o ϕ in eq (3) becomes $(\pi/2)^2$. Therefore,

$$K = \frac{\pi}{2} \cdot \frac{E}{L} \cdot \left(\frac{2}{\pi}\right)^2 = \frac{2E}{\pi L}$$

Fig. 4—Specimen types included in the analysis by Watanabe and Satoh[5]

4. A function, ϕ, determined by the location of the weld in the slit.

Kihara and Masubuchi conducted experiments on a number of specimens in low-carbon steel plates $^3/_4$ in. thick. Experiments were conducted on joints with various slit length, L (3 to 20 in.) and weld length, l, (l/L = 0.3 to 1.0). They used the parameter K defined by eq (3) to study the effects of degree of constraint on experimentally determined transverse shrinkage of a slit weld. Figure 3 shows a relationship between K values of joints and values of transverse shrinkage determined experimentally. Results obtained with a block welding sequence and a multilayer sequence are plotted; Fig. 3 shows that transverse shrinkage decreases as the degree of constraint increases.

Kihara and Masubuchi[16] analyzed the effect of the degree of constraint on the transverse shrinkage of a ring-type specimen as shown in Fig. 4a. Later, Watanabe and Satoh[5] found that results obtained by different investigators using different types of specimens can even be compared by using the degree of constraint. Figure 5 shows a relationship between the degree of constraint K and the ratio between the transverse shrinkage of a free weld, S_{tf}, and the transverse shrinkage of a constrained weld, S_t. Figure 5 also shows formulas used to calculate the degree of constraint of different specimens:

$$\frac{S_t}{S_{tf}} = \frac{1}{1 + 0.086\,K^{0.87}} \tag{4}$$

The shrinkage of a free butt weld, S_{tf}, is determined by the heat pattern of the weld. Watanabe and Satoh[5] have developed the following formulas for the value of S_{tf} in millimeters:

$$S_{tf} = C_1\left(\frac{A}{h^2}\right)\log_e \frac{W}{W_o} + C_2\left(\frac{A}{h^2}\right)^{1/2} \tag{5}$$

where,

A = sectional area of groove of butt joint, mm².
h = plate thickness, mm.
W = weight of deposited metal per unit length, gr/cm.
W_o = weight of deposited metal per unit length per welding of each pass, gr/cm.

When a joint is welded with ilmenite-type electrodes, the coefficient C_1 and C_2 have the values shown in Table 1.*

Experimental Determination of Degree of Constraint. The degree of constraint of a welded joint in a complex structure can be determined experimentally. Figure 6 shows a setup by Watanabe, et al.[17] to measure the degree

*Ilmenite-type electrodes are widely used in Japan.

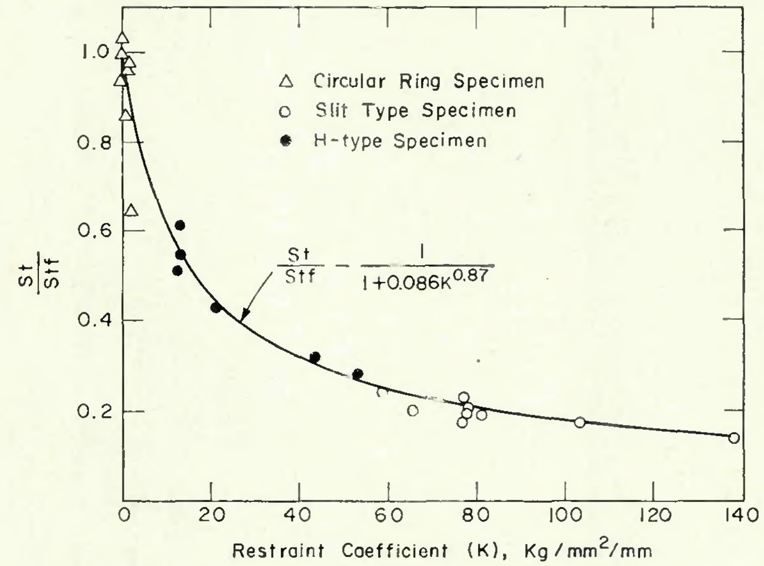


Fig. 5—Effect of external constraint on the transverse shrinkage of butt-welded joints

Table 1—Coefficient C_1 and C_2 Values

Welding current, amp	Welding speed, cm/sec	Electrode diameter, mm	C_1	C_2
120	0.3	3.2	0.0960	0.0416
150	0.3	4	0.1021	0.0584
210	0.3	5	0.1530	0.0745
260	0.3	6	0.1249	0.0690

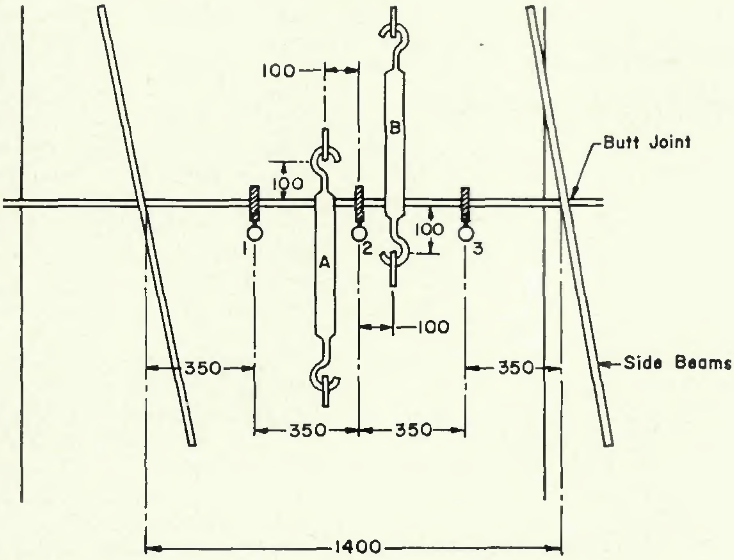
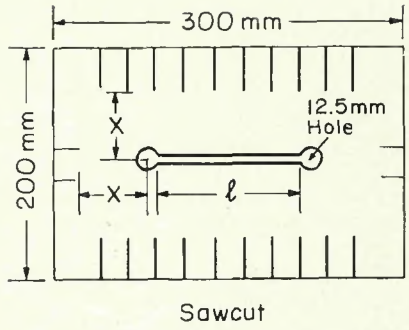


Fig. 6—Measurement of the degree of constraint of a butt joint between deck assemblies of an actual cargo ship (Watanabe et al.[17]). A and B—turnbuckles; 1, 2, 3—dial gauges; note that units are mm



ℓ mm	X mm	K, measured $kg/mm^2/mm$
75	40	34
	50	54
	70	60
	80	66
	90	68
	100	75
125	40	11
	50	21
	60	27
	70	34
	80	36
	100	44

Fig. 7—Lehigh-type restrained cracking specimens used by Watanabe and Satoh[6,17]

of constraint of a butt joint between two deck assemblies of an actual cargo ship. A and B are turnbuckles which were hooked to small steel pieces welded to the deck plate. Dial gages are numbered 1, 2, and 3. Changes in the gap between the deck plates were measured with the dial gages while the plate were pulled together by the turnbuckles. The tightening force was determined by strain gages mounted on the turnbuckles. The degree of constraint of the joint was then calculated by eq (1). Attempts were made successfully to estimate from laboratory weld-shrinkage data the shrinkage that took place during joining the large assemblies.

Study of Weld Cracking by Use of Restrained Specimens

Many tests in various configurations have been proposed and used for evaluating a sensitivity of a material for weld cracking.[7,18] Those tests which employ butt welds include:

1. U. S. Army Ordnance "H" test plate; U. S. Navy torture plate weld test; Lehigh restraint cracking test.
2. Tekken restraint test.
3. Houldcraft crack-susceptibility test.
4. RPI Varestraint test.
5. Circular-patch crack-susceptibility test.

In these tests a joint is held under considerable restraint during welding either by virtue of the size and assembly of the base metal pieces, or by welding the base metal pieces themselves to a massive backing plate.[18] Restrained-joint welding tests often are designed to provide information for a particular application by conducting a test under the degree of constraint which is equal to or exceeds that anticipated in actual structures.

A problem, here, is the lack of numerical studies. Only a few studies have been made to determine the degree of constraint of restrained-joint weld cracking specimens. Watanabe and Satoh[6,17] determined experimentally the degree of constraint of Lehigh restraint cracking specimens, as shown in Fig. 7, by driving a wedge into the slit. The degree of constraint was determined by measuring the force necessary to drive the wedge and the amount of opening of the slit. The results also are shown in Fig. 7.

Very recently, Fujita, et al.[19] conducted an analytical study to determine the degree of constraint of the Tekken restraint cracking test. The study, which was conducted independently of the MIT study, used the finite-element method for computing the degree of constraint.

Purpose of the Study

Discussions presented in the previous section cover the present state-of-the-art on the degree of constraint of butt welds which can be summarized as follows:

"The degree of constraint is an important parameter in studying shrinkage distortion and cracking. The degree of constraint of a simple joint has been calculated. However, no analysis has been developed of the degree of constraint of a practical joint and a restrained cracking test specimen which have rather complex configurations."

Consequently, information on cracking obtained on one type specimen cannot be transferred to another specimen, nor can it be applied directly to actual structures.

If there were an analytical method of determining the degree of constraint of various crack-susceptibility specimens and practical joints as well as simple joints, it would then be possible to combine scattered information presently available into one system of knowledge. The knowledge can further be developed into a system which can be applied to the prediction of distortion and cracking potential of actual structures. Such a system should be very useful for the fabrication of critical structures which may be built with high-strength materials which tend to be susceptible to cracking.

This study has been conducted to meet these needs to develop means to compute the degree of constraint of practical joints.

Values and Limitations of the Use of Elasticity Theory for Studying the Degree of Constraint

Definitions of the degree of constraint given in eqs (1), (2), and (3) are based upon the elasticity theory; so are the computations conducted in this study. However, as discussed in the beginning of Technical Background, plastic upsetting produced during welding plays an important role in residual stresses and distortion in weldments. Consequently, it is important to discuss the values and limitations of the elastic analysis of the degree of constraint.

The Role of This Study in the Analysis of Distortion. Figure 8 shows schematically the role of this study in the analysis of shrinkage and distortion in a free and a constrained welded joint. There are many factors that contribute to the shrinkage distortion of a weldment.

The material parameters include types and condition of base-metal and filler-metal materials.

Among the fabrication parameters are welding processes, including shielded metal-arc, submerged-arc, gas tungsten-arc, gas metal-arc, and others; procedure parameters, including current, arc voltage, arc travel speed, preheat and interpass temperatures, etc.; and assembly parameters, including welding sequence, among others.

The structural parameters include the geometry of the structure (whether it is a panel stiffened with frames, a cylinder, a spherical structure, etc.), plate thickness, and joint type. These factors may be classified into two groups:

1. Parameters which affect the heat pattern of a weldment, including arc current, arc voltage, arc travel speed, material parameters, among others.

2. Parameters which affect the degree of constraint of a weldment, including the geometry of structure, plate thickness, joint type, etc.

Shrinkage distortion in a free joint is affected only by parameters in the first group, while shrinkage distortion in a constrained joint is greatly affected by parameters in the second group.

As shown in Fig. 8, the analysis of distortion in a free joint involves the following steps:

1. Analysis of heat flow.

2. Analysis of thermal stresses and plastic upsetting produced during welding.

3. Determination of shrinkage and distortion in a free joint.

Mathematical analyses of the entire process which includes these three steps are extremely complex. The study of Step 2 is especially difficult, because it involves dynamic analysis of plastic strains at elevated temperatures. Consequently, in the study by Wanatabe and Satoh, an empirical formula, eq (5), was used to express the shrinkage of a free joint.

In analyzing the shrinkage of a constrained joint, it is first necessary to determine the degree of constraint which is discussed in this paper. Then an analysis needs to be made of thermal stresses and plastic upsetting under the influence of the degree of constraint. In the study by Watanabe and Satoh, an empirical approach was used again, as shown in eq (4).

In other words, the determination of the degree of constraint is an important part of the analysis of shrinkage distortion of a constrained joint. The degree of constraint, K, may be included in an empirical approach, as shown in eq (4), or it may be included in a completely analytical approach. No completely analytical study of the shrinkage of a constrained joint has been published.

The Nature of Transverse Shrinkage in a Butt Weld. The reason why the elastically determined degree of constraint is so useful in the study of transverse shrinkage may also be due to the nature of transverse shrinkage. Transverse shrinkage of a butt weld consists of two parts: shrinkage of the weld metal and shrinkage of the base metal. It has been found by a number of investigators that the shrinkage of the base plate is the majority, around 80 to 90%, of the transverse shrinkage of a butt weld.[20-22]

In the investigation by Kihara, et al.[1] on a slit-type specimen, as shown in Fig. 2, they found that about 85% of the shrinkage disappeared when the joint was cut after welding.

The results indicate the following mechanisms of transverse shrinkage in a constrained joint. During welding, base metal plates first expand due to the welding heat. Base plates are held together by the weld metal as it solidifies. As temperatures decrease, both the base plates and the weld metal contract. Since stresses in the transverse direction are relatively low, base plates after the completion of welding are primarily under elastic stresses. Consequently, the degree of constraint based on the elasticity theory can be used as a parameter for determining transverse shrinkage.

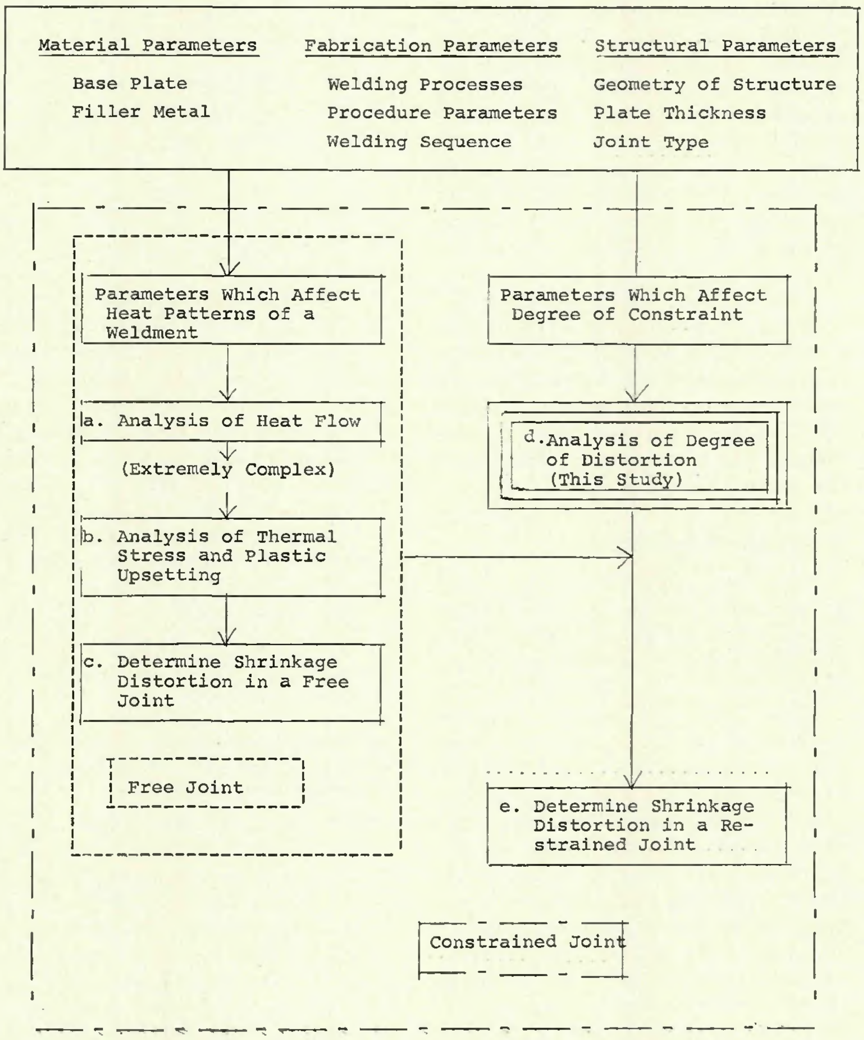


Fig. 8—Role of this study in the analysis of shrinkage and distortion in a constrained welded joint

Phase 1 Study—Selection of an Appropriate Model

Mathematical Approach

In this study a stress analysis technique called the finite element method was used. A general description of the finite element method is given in the literature.[11,12] The computer programs for the finite element methods in structure analysis are made available by the Civil Engineering Department of the

Massachusetts Institute of Technology, whose user's instructions have been published.[14]

The basis of the method is to divide the structure into a finite number of small elements, as shown in Fig. 9a, within each element, the displacements u and v at each point (x, y) in the x- and y-direction are assumed to be series functions of x and y of the form:

$$u(x,y) = \sum_i \sum_j a_{ij} x^i y^j$$

$$v(x,y) = \sum_i \sum_j b_{ij} x^i y^j$$

a_{ij} and b_{ij} are functions of the coordinates (x_K, y_K) of the nodes M_K limiting the elements. The functions $u(x,y)$ and $v(x,y)$ are such that displacement compatibility is satisfied along the boundaries of the elements to ensure the condition of convergence to the true solution. By use of the variational method, a system of linear equations between forces and displacements can be derived and can be solved for forces if displacements are given or vice versa.

The computer programs set up by the Civil Engineering Department for plates are of four types:

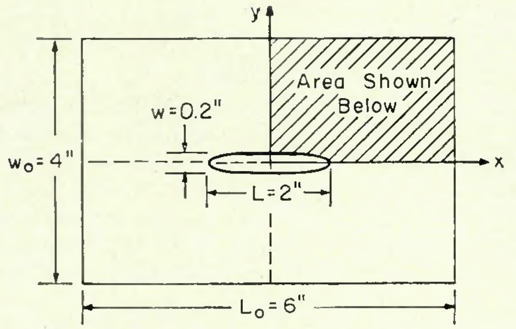
1. "CSTG": constant strain triangle, global formulation in which the elements are triangles, the strain within each element is assumed to be constant and one global referential system is used for the whole plate.
2. "CSTL": constant strain triangle, local formulation. The only difference with the "CSTG" is that each element has its own individual referential system.
3. "LST": linear strain triangle. The elements are triangles, the strain within each element is assumed to vary linearly.
4. "PSR": plane stress or plane strain rectangle. The elements are rectangles.

Selection of Appropriate Numerical Method

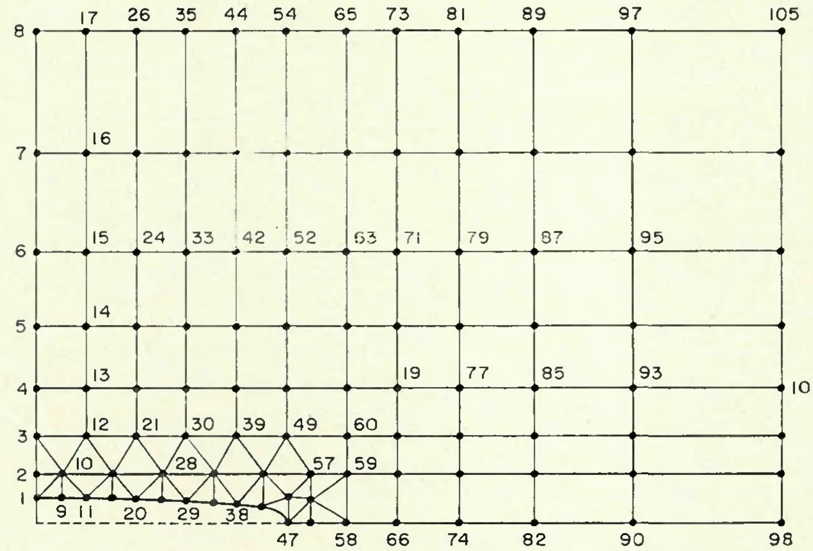
There are two alternatives in the use of the previously mentioned computer programs, namely the constant strain element—"CSTG", "CSTL", "PSR" or the linear strain element "LST."

During the first phase of this study, an effort was made to select an appropriate numerical method. Computations were made of the degree of constraint of a rectangular plate, 6 by 4 in., containing a slit of a very thin elliptic shape, 2 in. long, as shown in Fig. 9a. This specimen was used because an analytical solution for an infinite plate is already available in eq (3).

Computations were made using both the constant strain element and the linear strain element programs. In the case of constant strain, two types of grid system were used. The grid system used in computation 1-1 was made of a



a. General Shape of the Specimen



b. Grid System Used in Computation 1-1 ('CSTC' + 'PSR')

Fig. 9—Rectangular plate containing an elliptic hole, and the grid system used in the analysis

combination of triangles and rectangles, "CSTG" and "PSR" types, with 105 nodes in one quarter of the rectangular plate, as shown in Fig. 9b. In computation 1-2, a finer grid system of triangles, "CSTG" type with 170 nodes, was used. In computatuon 1-3, the linear strain element method, "LST," was employed using the same grid system as used in computation 1-1.

The procedure of computation can be explained using Fig. 2. Uniform stress of a given amount, σ_o, was applied along the central region of the slit. To simplify the analysis, computations were made only on a symmetric case where the central part is welded ($x_1 = -x_2$). The following values were computed:

[v]: transverse displacement of the slit edge.

Table 2—Values of the Degree of Constraint for a Rectangular Plate Containing a Thin Elliptic Hole[a]

	K, 10^6 psi/in.			$\bar{K} = K \cdot L/E$			
$R = \frac{l}{L}$	'CSTG' + 'PSR'	'CSTG' Finer Grid	'LST'	'CSTG' + 'PSR'	'CSTG' Finer Grid	'LST'	Infinite Plate $\bar{K}_\infty$
0.1	27.824	28.986	—	1.855	1.932	—	2.079
0.2	16.000	16.139	19.106	1.067	1.075	1.274	1.268
0.3	11.927	12.028	—	0.795	0.802	—	0.975
0.4	9.923	10.000	10.406	0.662	0.667	0.694	0.823
0.5	8.649	8.734	—	0.577	0.582	—	0.731
0.6	7.918	8.035	7.934	0.528	0.536	0.529	0.673
0.7	7.429	7.602	—	0.495	0.507	—	0.637
0.8	7.242	7.349	7.091	0.483	0.490	0.473	0.617
0.9	7.282	7.375	—	0.485	0.492	—	0.614
1.0	—	—	—	—	—	—	$\frac{2}{\pi} = 0.637$

[a] All calculations were made of plates in steel: $E = 30 \times 10^6$ psi or 21.1×10^3 kg/mm^2.

$[\bar{v}]_l$: average transverse displacement over the loading length, l.

Then the degree of constraint, K, was determined as follows:

$$K = \frac{\sigma_o}{[\bar{v}]_l} \qquad \text{Same as eq (2)}$$

Table 2 shows values of K determined by the three different methods for different values of $R = l/L$. Further details of the computations including values of $[v]$ and $[\bar{v}]_l$ are given in the literature.[10] Table 2 also contains the following values:

$\bar{K}$: non-dimensional degree of constraint, or specific degree of constraint

$$\bar{K} = \frac{K}{E/L} = \frac{KL}{E} \tag{6}$$

where, E = Young's modulus; L = length of slit.

$\bar{K}_\infty$: non-dimensional degree of constraint, or specific degree of constraint obtained analytically in the case of an infinite plate

$$\bar{K}_\infty = \frac{K_\infty L}{E} \tag{7}$$

where, K_∞ = degree of constraint for an infinite plate calculated from eq (3).

The use of non-dimensional values, $\bar{K}$ and $\bar{K}_\infty$, makes the results for different slit length comparable, and they may be called "the specific degree of constraint." Figure 10 shows relationships between $R = l/L$ and $\bar{K}_\infty$ and values of $\bar{K}$ determined by the three methods. All calculations were made of plates in steel.

The difference between $\bar{K}_\infty$ and $\bar{K}$ could not be considered as a measure of the accuracy of the numerical method, because $\bar{K}_\infty$ is the non-dimensional degree of constraint in an infinite plate and for which the only significant dimension is the slit length L of the weld and $\bar{K}$ is the non-dimensional degree of constraint in a finite plate and for which the significant dimensions include the slit length L, the plate length L_o, and the plate width W_o. But we can expect $\bar{K}$ to be smaller than $\bar{K}_\infty$ since the restraint due to a finite plate should be less than that due to an infinite plate.

From the results in Fig. 10, the following conclusions could be drawn:

1. The linear type gives an unusual swing; the curve of $\bar{K}$ is above that of $\bar{K}_\infty$ for small R and becomes smaller than that of $\bar{K}$ computed by the "CSTG" method for large values of R.

2. The constant strain types "CSTG", "PSR" in the two gridworks give more consistent results. The $\bar{K}$ curve obtained is almost parallel to the $\bar{K}_\infty$ curve. Also, there is not much difference between the two gridworks, one coarse and the other fine.

On the basis of these results, it was concluded that the constant strain element type with a relatively coarse grid system could be used in the computation of the degree of constraint of various joint configurations in plate structure.

Phase 2 Study—Determination of Degree of Constraint of Various Plate Specimens

Since it was found in Phase 1 Study that the finite element method using the constant strain element type provides satisfactory results, computations of the degree of constraint of various plate specimens were conducted in Phase 2 Study.

Joint Configurations

Figure 11 shows configurations of plate specimens on which computations were made. The metric system was used on some specimens, while the inch-pound system was used on others. The reason for this is to compare results of this study to other results obtained in Japan and the United States.

1. *Slit Type Joint.* Computations were made on a rectangular plate with a straight slit, as shown in Figure 11a. Results were compared with those obtained on a rectangular plate with an elliptic hole, as shown in Fig. 9a.

2. *Straight Slit with Circled Ends.* Computations were made on a rectangular plate with a straight slit having circular holes at both the ends, as shown in Fig. 11b. This type specimen was included from the following practi-

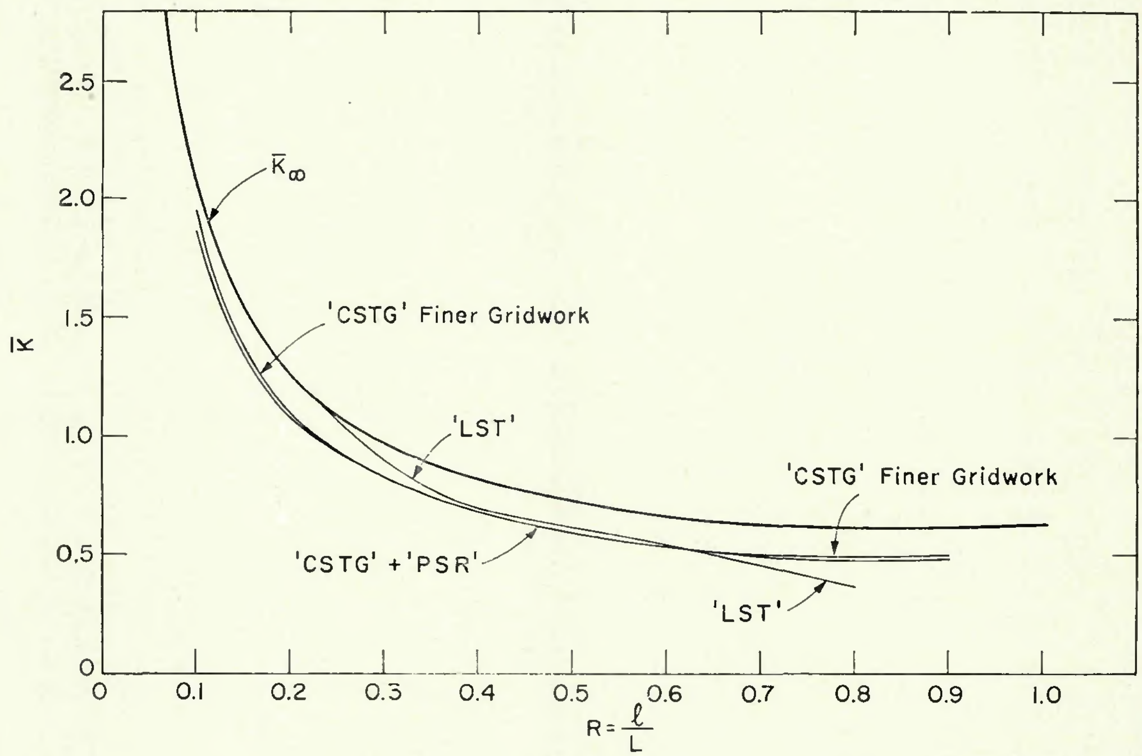


Fig. 10—Relationship between weld length to slit length, R, and nondimensional degrees of constraint, $\bar{K}$ for an elliptic hole in a finite plate. The gridworks used are made of: (a) a mixture of Constant Strain Triangles and Plane Stress Rectangles: 'CSTG' + 'PSR'; (b) Constant Strain Triangles 'CSTG'; (c) Linear Strain Triangles 'LST'

cal considerations.

In the previous study by Kihara, et al.[1] they used the slit type specimen, as shown in Fig. 2 and 11a, primarily because an analytical solution of the degree of constraint, eq (3), was available on the slit type specimen. However, a slit type specimen is rather expensive to make; and a specimen as shown in Fig. 11b is much less expensive to make.

Another reason for calculating the degree of constraint of a specimen as shown in Fig. 11b is to evaluate the effectiveness of a "crack-stopping hole" in reducing the degree of constraint. In a repair work of a structure containing a crack, it is a common practice to first drill holes at both the ends of the crack to prevent further extension of the crack. These holes are welded after the welding of the crack, or the slit, is completed.

3. *H-Type Slit.* Computations were conducted on an H-type slit specimen as shown in Fig. 11c, because specimens of this type have been used by several investigators (see Fig. 4c). Comparison between results obtained on specimens shown in Fig. 11b and 11c should indicate how effective the transverse cuts in the H-type slit are in reducing the degree of constraint.

4. *Lehigh-Type Specimen.* Computations were made on Lehigh-type specimens in two configurations: with sawcuts, as shown in Fig. 11d, and without sawcut. The specimen without sawcut looks more like the specimens in Fig. 11b, although dimensions are slightly different. Although dimensions of the original Lehigh restraint cracking test specimen are given in inches (12 in. long, 8 in. wide, and so on), the specimen sizes as shown in Fig. 11d were used because experimental values of the degree of constraint have been obtained by Watanabe and Satoh, as shown in Fig. 7.

5. *Large H-Type Slit.* Computations also were made on a large H-type slit specimen, as shown in Fig. 11e. Comparison of results obtained on specimens shown in Figs. 11c and 11e should show effects of (1) increasing slit length and (2) decreasing relative size of constraining areas. In this particular case, calculations were made for both steel and aluminum, because experiments were made in a different research program on shrinkage during welding of this type specimen in various materials including steel and aluminum.

Grid Systems

A grid system similar to that shown in Fig. 9b was employed on each of the joint configurations used in Phase 2 Study The number of nodes, however, varied depending upon the geometrical complexity of the joint. For example, the grid system employed on the Lehigh-type specimen with sawcut had 269 nodes.

Results

Results of computation are summarized in Table 3, showing the following values: L = slit length, in inches or millimeters; $R = l/L$: ratio of the length of loading zone, l, and slit length; K = degree of constraint in 10^6 psi/in. and kg/mm²/mm; $\bar{K} = K \cdot L/E$: Non-dimensional degree of constraint, or specific degree of constraint.

Rectangular Plates Containing Slits with Various Shapes. Figure 12 shows relationships between $R = l/L$ and values of the specific degree of constraint, $\bar{K}$, of rectangular plates containing slits with various shapes as follows:

1. Elliptic slit, as shown in Fig. 9a.
2. Straight slit, as shown in Fig. 11a.
3. Straight slit with circular holes at the ends, as shown in Fig. 11b.
4. H slit, as shown in Fig. 11c.

Also shown is the value of the specific degree of constraint for an infinite plate, $\bar{K}_\infty$.

As expected, the $\bar{K}$ value for a finite plate is smaller than that for an infinite plate. The effect of finiteness is also evidenced by the difference in the $\bar{K}$ value between the elliptic slit and the straight slit. In the first case, the ratio between the plate length L_o and the slit length L is (Fig. 9a):

$$\left(\frac{L_o}{L}\right) \text{elliptic hole} = \frac{6}{2} = 3$$

while the same ratio for the straight slit, shown in Fig. 11a, is

$$\left(\frac{L_o}{L}\right) \text{straight slit} = \frac{300}{120} = \frac{3}{1.2} < 3$$

Both plates have the same aspect ratio $L_o/W_o = 3/2$. It can therefore be concluded that for straight slit weld

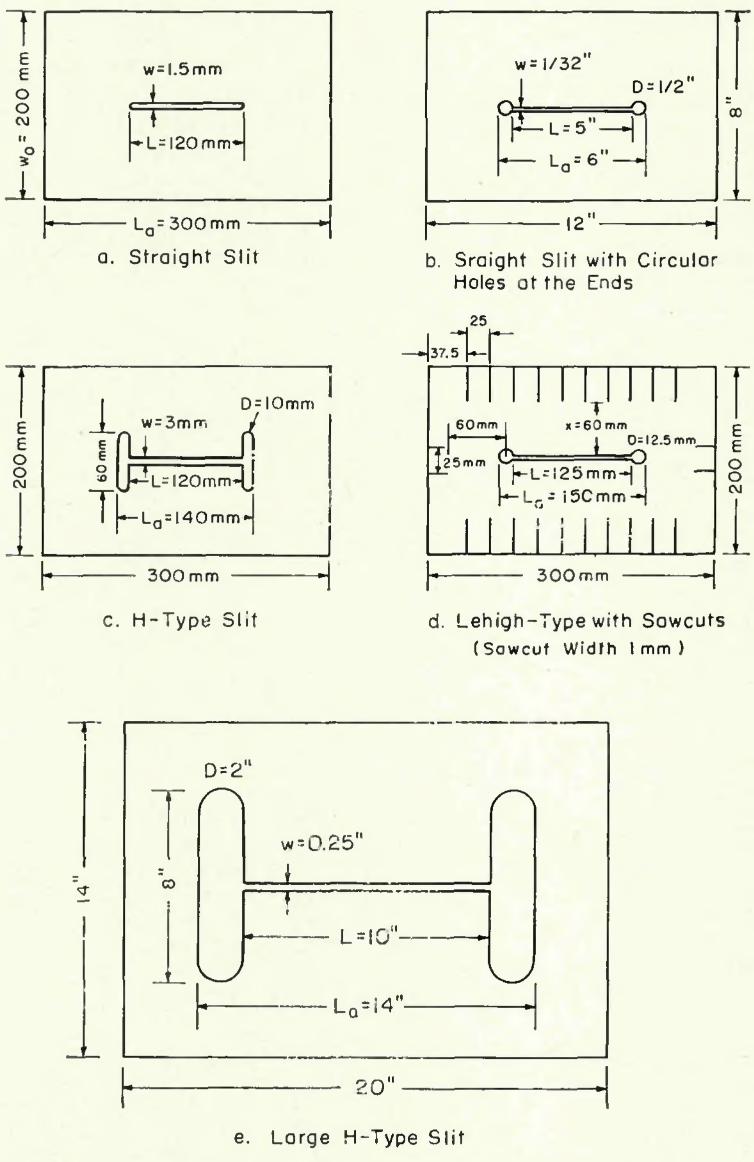


Fig. 11—Joint configurations used in Phase 2 Study

configuration, (there is not much difference between an ellipse with an axis ratio of $a/b = 10$ and a straight slit), the value of $\bar{K}$ depends on the ratio between plate length L_o and slit length L. $\bar{K}$ is larger for larger ratio L_o/L and for straight slits having the same length, the degree of constraint $\bar{K}$ is larger for the one with larger L_o/L.

In the case of a straight slit with circles at the ends, the diameter of the circle is larger than the slit width, the value of $\bar{K}$ is smaller than that for a straight slit without large-circled ends. The lowest value of $\bar{K}$ is obtained for an H-slit. It is noticed that "crack-stopping holes" have significant influence in reducing the degree of constraint; however, further reduction in the degree of constraint by having transverse H branches is rather minor.

All these results seem to conform to common sense and could be considered as an indication of the correctness of the computation method employed in this study.

Lehigh-Type Specimens. Values in kg/mm²/mm of the degree of constraint of the Lehigh-type specimens without sawcuts are as shown in Table 4.

The calculated values are somewhat higher than the measured values. At present, however, it is not known whether there were errors in the experiments or in the computation. Computations using a finer grid system should provide information on the accuracy of the computations.

Values shown in Table 3 indicate that the degree of constraint of the Lehigh-type specimen is considerably lower than that of a slit type specimen, especially when only a part of the slit is welded.

Curve ABC of Fig. 13 shows the relationship between the length of a slit, L, in an infinite plate and the weld

Table 3—Values of the Degree of Constraint

Slit Length L	$R = l/L$	K, 10^6 psi/in.[a]	K, kg/mm²/mm[a]	$\bar{K}=\frac{K}{(E/L)}$	Remarks
1. Rectangular plate with a straight slit (Fig. 11a):					
	$^1/_3$	4.60	127.36	0.724	
	$^1/_2$	3.55	98.21	0.559	
L=120 mm	$^2/_3$	3.05	84.49	0.481	
	$^5/_6$	2.86	79.30	0.451	
	1	2.94	81.53	0.464	
2. Straight slit with circular holes at the ends (Fig. 11b):					
	0.2	4.878	135.1	0.813	
	0.4	2.829	78.2	0.472	
L=5 in.	0.6	2.174	60.2	0.362	
	0.8	1.876	52.0	0.313	
	1.0	1.769	49.0	0.295	
3. H-slit (Fig. 11c):					
	$^1/_3$	3.13	86.78	0.493	
	$^1/_2$	2.30	63.78	0.363	
L=120 mm	$^2/_3$	1.89	52.35	0.298	
	$^5/_6$	1.65	45.83	0.258	
	1	1.46	40.45	0.230	$\bar{K}_s$=1.14[b]
4. Lehigh-type specimen without sawcut:					
L=125 mm	1	1.74	48.17	0.285	K measured =44 kg/mm²/mm
5. Lehigh-type specimen with sawcut (Fig. 11d):					
L=25 mm	1	1.11	30.69	0.182	K measured =27 kg/mm²/mm
6a. Large H-slit specimen in steel (Fig. 11e):					
L=10 in.	1	0.214	5.92	0.0714	$\bar{K}_s$=0.469[b]
6b. Large H-slit specimens in Aluminum (Fig. 11e):					
L=10 in.	1	0.0714	1.98	0.0714	$\bar{K}_s$=0.469[b]

[a] 10^6 psi/in. = 27.7 kg/mm²/mm.

[b] $\bar{K}_s = \frac{L_c}{B\left(1 + \frac{L_c}{2l_s}\right)}$, see Fig. 4c.

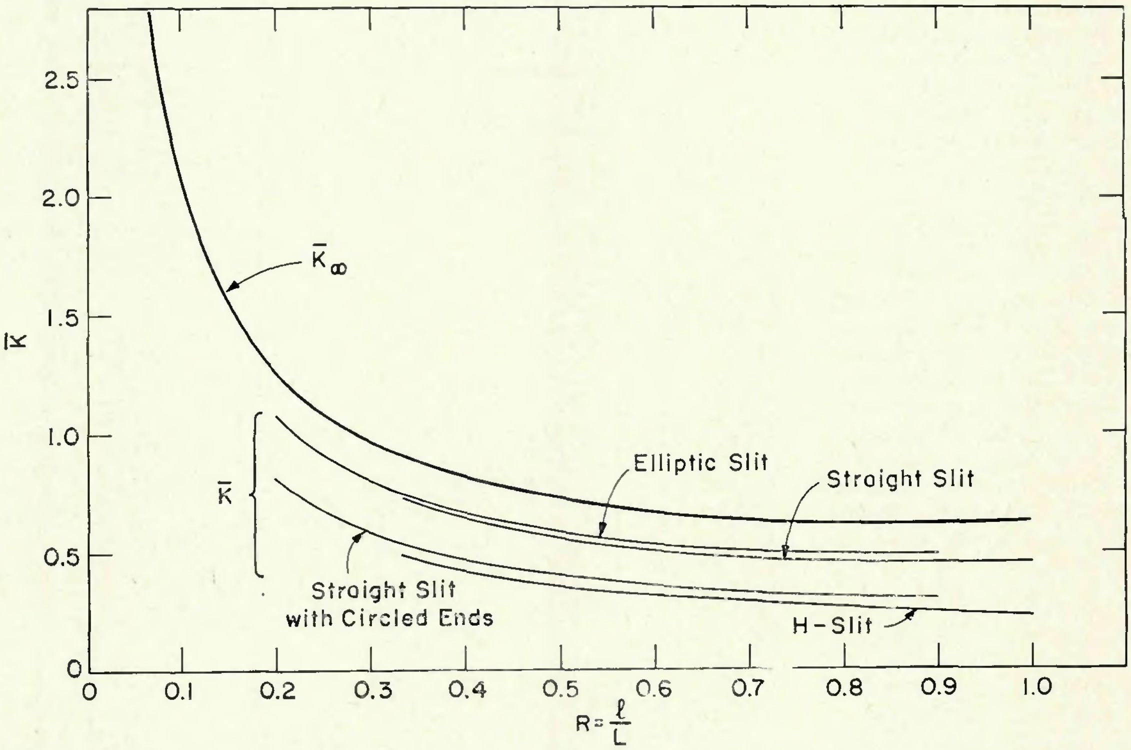


Fig. 12—Relationship between weld length to slit length ratio, R, and nondimensional degrees of constraint for different cases being studied, $\bar{K}_\infty$ for an infinite plate and $\bar{K}$ for finite plates

Table 4—Calculated vs. Measured Degrees of Constraint for Lehigh-Type Specimens

	Calculated, K_c	Measured, K_m	$\frac{K_c - K_m}{K_c}$
Without sawcut	48.17	44	7%
With sawcuts	30.69	27	12%

length, l, which gives the degree of constraint $K_\infty = 1.74 \times 10^6$ psi/in. The curve is obtained in the following way. When the slit length, L, is 20 in., for example, the value of $\bar{K}_\infty$ for $K_\infty = 1.74 \times 10^6$ psi/in. is:

$$\bar{K}_\infty = \frac{KL}{E} = \frac{1.74 \times 10^6 \times 20}{30 \times 10^6} = 1.16$$

From Fig. 10, the value of R of $\bar{K}_\infty = 1.16$ is obtained to be 0.22. Then:

$$l = 0.22 \times 20 = 4.4 \text{ in.}$$

Curve ABC is obtained by calculating for different values of L values of l which give $K_\infty = 1.74 \times 10^6$ psi/in. Point A is obtained from the fact that $\bar{K}_\infty$ (for $R = 1$) $= \frac{2}{\pi}$, as follows:

$$\bar{K}_\infty = \frac{KL}{E} = \frac{1.74 \times 10^6}{30 \times 10^6} \cdot L = \frac{2}{\pi}$$

$$L = \frac{2}{\pi} \cdot \frac{30}{1.74} = 11.0 \text{ in.}$$

By curve ABC and line OP (for $l = L$), the entire region is classified into the following three regions:

Region I: When the combination of L and l is in Region I, the degree of constraint is lower than 1.74×10^6 psi/in.

Region II: When the combination of L and l is in Region II, the degree of constraint is higher than 1.74×10^6 psi/in.

Region III: Region III is impractical because the weld length cannot be larger than the slit length.

Figure 13 can be used in the following way. Suppose that a cracking test was conducted on a Lehigh specimen with no sawcut using a set of base and filler metals and certain welding conditions, and no crack was observed; this could be interpreted that the critical degree of constraint for the materials and welding conditions are somewhat greater than 1.74×10^6 psi/in. If this result could be transferred to the case of welding of a butt joint in a large plate, it simply means that welds could be made without cracking if the combination of the slit length and the weld length falls in Region I.

If a weld shorter than 4.4 in. was made in a 20 in. long slit, cracking would occur unless tests were passed with some premium. When welds were made on a slit shorter than 11 in., they would also crack.

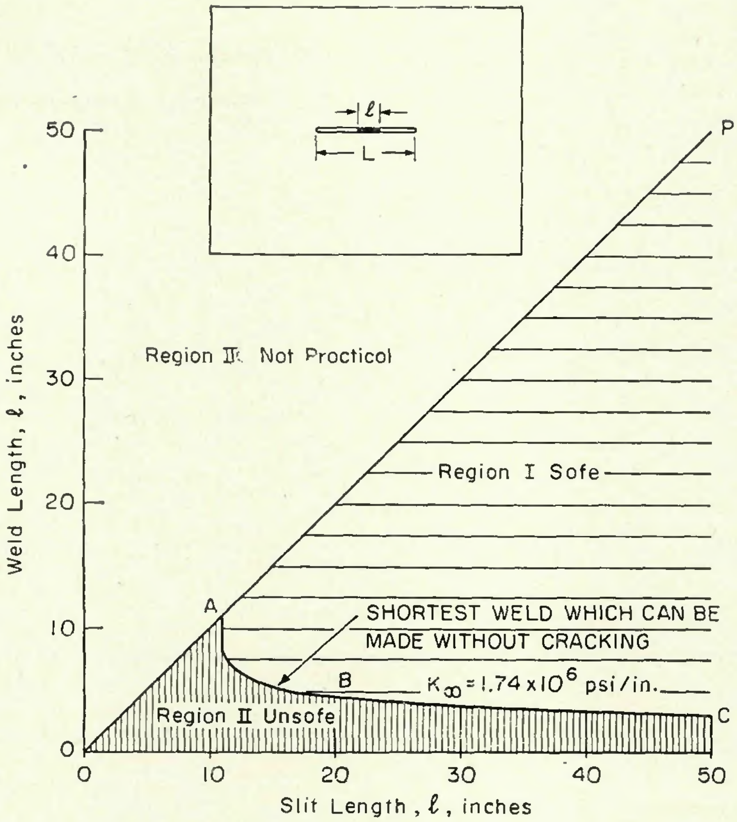


Fig. 13—Relationship between the length of slit in an infinite plate and weld length which gives the K_∞ value of 1.74×10^6 psi/in.

We recognize that discussions related to Fig. 13 are extrapolations of analytical studies and the validity of such discussions needs to be verified experimentally. The discussions are presented primarily to demonstrate one of the potential uses of this study.

Large H-Slit. Results obtained on large H-slit specimens in steel and aluminum show that values of the degree of constraint are very low, especially in aluminum.

For comparison, values of the degree of constraint calculated by the approximate solution, shown in Fig. 4c, $\bar{K}_s$, also are given in Table 3 for the H-slit (shown in Fig. 11c) and the large H-slit. In both cases, values obtained by the approximate solution are much higher than values calculated by the finite-element method. This indicated that the approximate solution cannot be used unless (1) the H-branch is very deep and (2) the width of the restraining member, L_s, is considerably small.

Conclusions

In this study, computations were made of the degree of constraint of several butt joints in different configurations.

Results obtained with a rectangular plate with a slit agreed very well with analytical solutions. Variations of results for joints with different configurations seem to conform to common sense. Computed values coincided fairly well with measured values on some specimens.

All of these results indicate that the finite-element method, with constant strain element, is very useful for computing the degree of constraint of a complex joint.

Results of this study also indicate a tremendous potential which the finite-element method could offer to the welding industry. It is quite feasible that the degree of constraint of various specimens used for weld cracking as well as various types of joints in complex welded structures can be computed. Such studies will eventually lead us to the establishment of a rational system for controlling weld distortion and eliminating weld cracking.

An example also is given to discuss how information obtained on a crack-susceptibility test could be used for the procedure control in an actual structure. Properly planned experimental programs will be most useful for further development of the knowledge.

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University Research Conference

The University Research Committee of the Welding Research Council has arranged for a Conference in conjunction with the 51st Annual Meeting of the AMERICAN WELDING SOCIETY in Cleveland on *Monday afternoon, April 20, 1970, from 4:30 to 6:00 P.M. in the Cleveland Room, on the Main Floor, Sheraton-Cleveland Hotel, Cleveland, Ohio.*

Subject: RESEARCH IN PROGRESS

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