Arc-Welding Temperatures in a Circular Disk Structure

Measured temperatures and a semi-theoretical simulation model based on the assumption of an almost quasistationary process of heat transfer are in good agreement

BY W. SOEDEL AND R. COHEN

ABSTRACT. Temperature measurements on a small disklike structure being arc welded on its periphery suggested the applicability of a quasistationary heat transfer model of a point source moving along the edge of a disk in order to simulate the temperature distribution in regions of interest removed from the center of the structure. An approximate solution to the problem was obtained by mapping the solution of a moving heat source on an infinite plate into the unit disk.

The influence of surface dissipation, temperature dependent heat transfer coefficients, one imperfect boundary condition and heat sinks was taken into account by introducing into the disk equation adjustment coefficients. These coefficients were found to be applicable to the total set of geometry dependent data once they were determined by fitting the solution to two particular measurements. Agreement between experimental values and the semi-theoretical model was found to be good.

The technique outlined in this paper reduces experimental labor considerably whenever an exploration of the influence of design parameters and welding machine parameters on the spatial and temporal welding temperature distributions of disk structures becomes necessary.

Introduction

This paper describes an investigation of arc welding temperatures in the diaphragm of a pressure-displacement converter. The diaphragm was a thin (0.004 in.) austenitic stainless steel (Type 302) disk. This disk was connected to a low carbon steel housing by means of arc welding as shown in Fig. 1. The center of the disk is in contact with a brass buffer which provides an unknown amount of heat sink effect.

The motivation for this study was threefold. A knowledge of the transient temperature distribution in the diaphragm, and preferably a mathematical model of it, was necessary in order to explain buckling behavior. Second, knowledge of the temperature distribution as a function of design parameters and welding machine parameters was necessary for control of metallurgical properties. Third, a knowledge of the maximum temperature profile over the disk was necessary for a proper strain gage installation.

These objectives were fulfilled, and results concerning transient temperature distribution and its mathematical modeling are given. Although this investigation was undertaken on a specific application, it is believed that the findings are of general interest and the same approach can be used on other applications.

Measurements

Two sets of temperature measurements were made on the diaphragm with thermocouples. Each set consisted of four measuring points, in one case grouped radially and in the other case grouped circumferentially as shown in Fig. 2. The thermocouples used were iron-constantan TG 36 ATP and ATN and were attached to the diaphragm surface with a spot-weld. The instrumented diaphragm disk and the housing parts were then preassembled and placed into the welding fixture.

Four Honeywell Electronik 19 single pen recorders were used as recording equipment. A switching circuit with a voltage divider was used to synchronize the four recorders. The signal from the voltage divider was also used for calibration purposes. A standard electric timer was used for timing turntable speed.

Identical measurements were made on several diaphragms and good repeatability was shown. Typical experimental results are shown as the solid curves in Figs. 3 and 4.

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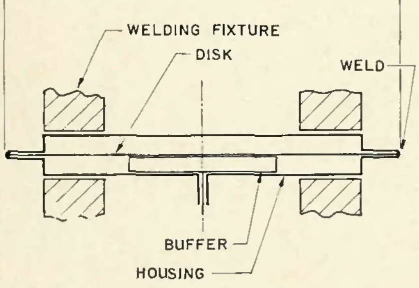

Fig. 1—Welding arrangement

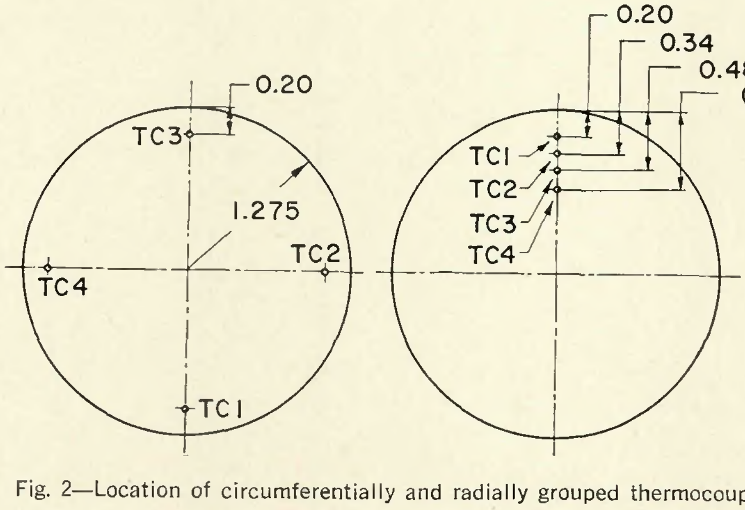

Fig. 2—Location of circumferentially and radially grouped thermocouples

Theoretical Model

An inspection of these measured temperature histories and comparison with results obtained by Grosh and Trabant[1] and Grosh and Hawkins[2] for large rectangular plates suggested the existence of a quasistationary state of temperature distribution. Such a quasistationary state exists if an observer, while not changing position relative to the heat source, would always experience a constant temperature as he moves along with the heat source. Obviously this will not be quite true in the present case since the diaphragm will generally warm up as the arc progresses. Also, the quasistationary state will not exist at the moment of initial arc formation but will need time to build up. However, it was felt that if such a mathematical model could be made to approximately fit the data, it would allow investigation of the influence of such parameters as current, arc speed, etc., without having to resort to an excessive amount of experimental labor.

Such an approximate mathematical model was obtained by adapting the solution given by Rosenthal[3] for the problem of a heat source moving along the edge of an infinite plate. This was done by mapping Rosenthal's solution to the unit disk. Rosenthal's solution is:

$$T - T_0 = \frac{q'}{2\pi k} e^{-\lambda v \dot{v}} K_0 (\lambda v \sqrt{u^2 + v^2}) \quad (1)$$

which satisfies the boundary conditions:

$$\frac{\partial(T - T_0)}{\partial u} \to 0 \text{ as } u \to \pm \infty \quad (2)$$

$$\frac{\partial(T - T_0)}{\partial v} \to 0 \text{ as } v \to \pm \infty \quad (3)$$

$$-\frac{\partial(T - T_0)}{\partial \rho'} 2\pi k \rho' \to q' \text{ as } \rho' \to 0 \quad (4)$$

where:

$2\pi\rho'$ = circle around heat source

$\rho' = \sqrt{u^2 + v^2}$

and, because of symmetry:

$$\frac{\partial(T - T_0)}{\partial u} = 0 \text{ at } u = 0 \quad (5)$$

To transform the unit circle in the z – plane (Fig. 5) into the infinite plane in the w – plane we use the transformation

$$w = \ln z \quad (6)$$

where

$$w = u + iv \quad (7)$$

$$z = x + iy \quad (8)$$

Thus, we have

$$u = \ln |z| = \ln \xi \quad (9)$$

$$v = \arg z = \theta \pm n\pi \qquad n = 2,4,\ldots. \quad (10)$$

Substituting eqs (9) and (10) in eq (1) and recognizing that the velocity $\dot{v}$ in the w – plane transforms directly into the angular velocity ω in the z – plane

$$\dot{v} = \frac{dv}{dt} \to \frac{d(\theta \pm n\pi)}{dt} = \omega \quad (11)$$

Thus for the disk:

$$T - T_0 = \frac{q'}{2\pi k} e^{-\lambda a^2 \omega (\theta \pm n\pi)} K_0 (\lambda a^2 \omega \sqrt{(\theta \pm n\pi)^2 + \ln^2 \xi}) \quad (12)$$

The w-plane boundary conditions will not directly carry over into the z-plane except condition (5) which becomes

$$\frac{\partial(T - T_0)}{\partial \xi} = 0 \text{ at } \xi = 1 \quad (13)$$

Examining the boundary conditions imposed by eq (12) gives:

$$(T - T_0) \to 0 \text{ as } \xi \to 0 \quad (14)$$

$$(T - T_0) \to 0 \text{ as } (\theta \pm n\pi) \to \pm \infty \quad (15)$$

$$\frac{\partial(T - T_0)}{\partial \theta} \to 0 \text{ as } (\theta \pm n\pi) \to \pm \infty \quad (16)$$

Condition (14) satisfies physical reality completely if the radius a approaches infinity. In cases of small disks, as in this investigation, eq (12) will generally have to be confined to a region sufficiently removed from the center. Conditions (15) and (16) are compatible with physical reality if we interpret the disk as a continuous spiral. Thus, at $\theta = 2\pi$ for instance, T_0 is increased by the value of T at $\theta = 2\pi$.

At the heat source itself eq (12) will be singular (as is eq (1)) since:

$$(T - T_0) \to \infty \text{ as } [(\theta \pm n\pi) \to 0 \text{ and } \xi \to 1] \quad (17)$$

Defining as substitute radius around the heat source

$$\rho'' = \sqrt{(\theta \pm n\pi)^2 + \ln^2 \xi} \quad (18)$$

in eq (12) and differentiating with respect to ρ'' will give a condition analogous to condition (4), namely:

$$-\frac{\partial(T - T_0)}{\partial \rho''} 2\pi k \rho'' \to q' \text{ as } \rho'' \to 0 \quad (19)$$

Thus, eq (12) should adequately describe the transient temperature distribution in the disk in regions sufficiently removed from the center. The measurements of this investigation indicate that this is the case at least for locations corresponding to $\xi > 0.5$.

It is a basic hypothesis of this work that the influence of surface dissipation, temperature dependent heat transfer coefficients and heat sinks remain sufficiently constant for designs and welding machine settings in the vicinity of nominal values. Thus they are assumed to be accountable by introducing adjustment coefficients into eq (12). Considering the welding process,

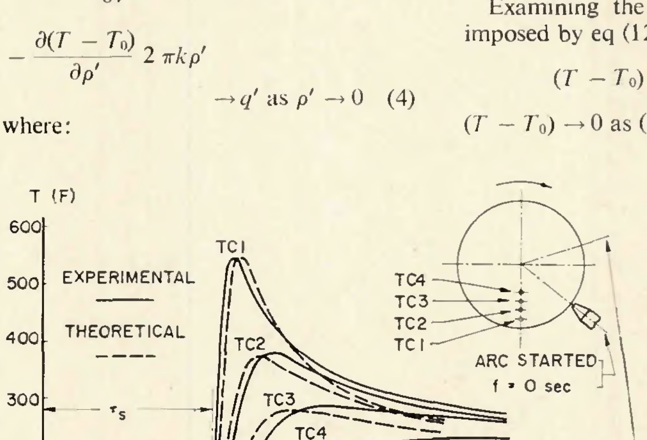

Fig. 3—Comparison of theory and experiment for radially grouped thermocouple locations

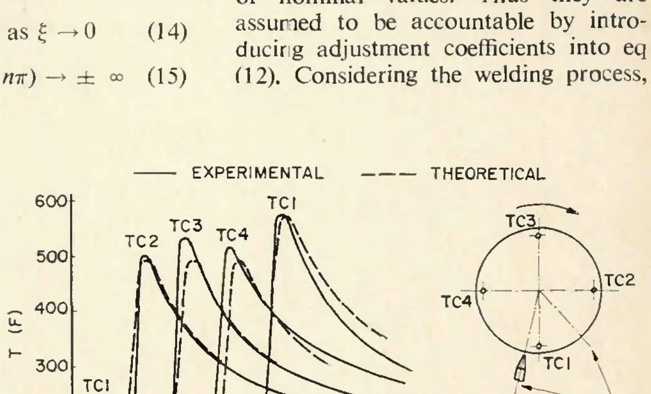

Fig. 4—Comparison of theory and experiment for circumferentially grouped thermocouple locations

$$q' = \frac{\mu q V I}{s} \tag{20}$$

eq (12) becomes:

$$T - T_0 = C_1 \frac{\mu q V I}{2\pi k s} e^{-C_2 \lambda a^2 \omega(\theta \pm n\pi)} K_0 \left(C_2 \lambda a^2 \omega \sqrt{(\theta \pm n\pi)^2 + C_3 \ln^2\left(\frac{r}{a}\right)} \right) \tag{21}$$

Equation (21) represents the quasistationary temperature field around the welding arc with the arc itself at $\xi = 1$ and $(\theta \pm n\pi) = 0$ of the moving coordinate system. To replace the angular coordinate by the time scale we use the relationship:

$$(\theta \pm n\pi) = -\omega\tau \tag{22}$$

where τ = time [sec]

= 0 at the moment when the arc passes the thermocouple location θ_r

The minus sign comes from the fact that the temperature field, which moves as a whole, will be inverted by a sensing thermocouple. Thus, eq (21) becomes:

$$T - T_0 = C_1 \frac{\mu q V I}{2\pi k s} e^{C_2 \lambda a^2 \omega^2 \tau} K_0 \left(C_2 \lambda a^2 \omega \sqrt{\omega^2 \tau^2 + C_3 \ln^2\left(\frac{r}{a}\right)} \right) \tag{23}$$

Thus we have an equation which allows calculation of the temperature rise at any location and time as a function of angular velocity ω of welding fixture, voltage V across arc, current I, efficiency μ of arc, conductivity k, thickness s of total melting zone, radius a of disk, density ρ and specific heat c, provided that reasonable values of c_1, c_2, and c_3 are known.

It is proposed that these values be estimated using the measurements of two thermocouples during the welding of a single nominal design prototype at a single nominal welding machine setting. This is done by fitting c_1 and c_2 to a data curve obtained by a thermocouple located close to the periphery of the disk structure and by determining c_3 from a data curve obtained by a second thermocouple located, radially inward from the first thermocouple, at approximately $\xi = 0.5$. The equation then will not only predict the temperature at other locations, but for other welding conditions and for other designs not far removed from the nominal one.

Comparison of Mathematical Model and Experiment

The following physical parameters were known: I = 70 amp, V = 20 v, s = 0.1 in., $\omega = 0.361 \frac{\text{rad}}{\text{sec}}$, a = 1.28 in.

The following parameters were estimated using cited references:[1-3] $\mu = 0.6$, $k = 0.6 \frac{\text{cal}}{\text{cm °Csec}}$, $\lambda = 5 \frac{\text{sec}}{\text{cm}^2}$.

Converting these values to compatible units and substituting in eq (23) gives:

$$T - T_0 = 630\, C_1 e^{19.1\, C_2 (0.361\tau)} K_0 \left(19.1\, C_2 \sqrt{(0.361\tau)^2 + C_3 \ln^2\left(\frac{r}{a}\right)} \right)$$

Next, the adjustment coefficients c_1 and c_2 were obtained by fitting the theoretical equation to one temperature curve measured by thermocouple TC1. This curve is shown in Fig. 3. The adjustment coefficient c_3 was obtained using one additional measurement: the curve generated by thermocouple TC4 in Fig. 3. The values of the adjustment coefficients were found to be: $c_1 = 1.47$; $c_2 = 0.11$; $c_3 = 4.0$.

It was then found that eq (24) would predict the transient temperature behavior of the disk at thermocouple locations TC2 and TC3 in good agreement with experimental data. Not only this, but measurements using circumferentially grouped thermocouples were predicted very well using the same set of adjustment coefficient values. Figure 4 shows such a comparison for a different arc current of 60 amp using the same, previously obtained, coefficient values. The total time, t, plotted on the abcissa is ($t = 0$ at moment when arc starts):

$$t = \tau_s + \tau \tag{25}$$

As it can be seen, the quasistationary state model gives results which agree with the data quite satisfactorily. The reason that the theoretical values tend to drop below the measured values as time becomes large is that the model does not incorporate the fact that the disk will retain heat which will keep the temperature from dropping off as predicted. In other words, condition (14) is more and more removed from physical reality. However, deviations become appreciable only after the time equivalent of a 2π rotation has elapsed—that is, after the welding process is completed and the almost quasistationary state has ceased to exist. Further, the maximum temperatures are often the ones of most concern.

The differences in the maximum temperature values measured by thermocouples TC3 and TC4 (Fig. 4) may be due to alignment errors, both in thermocouple location and eccentricity

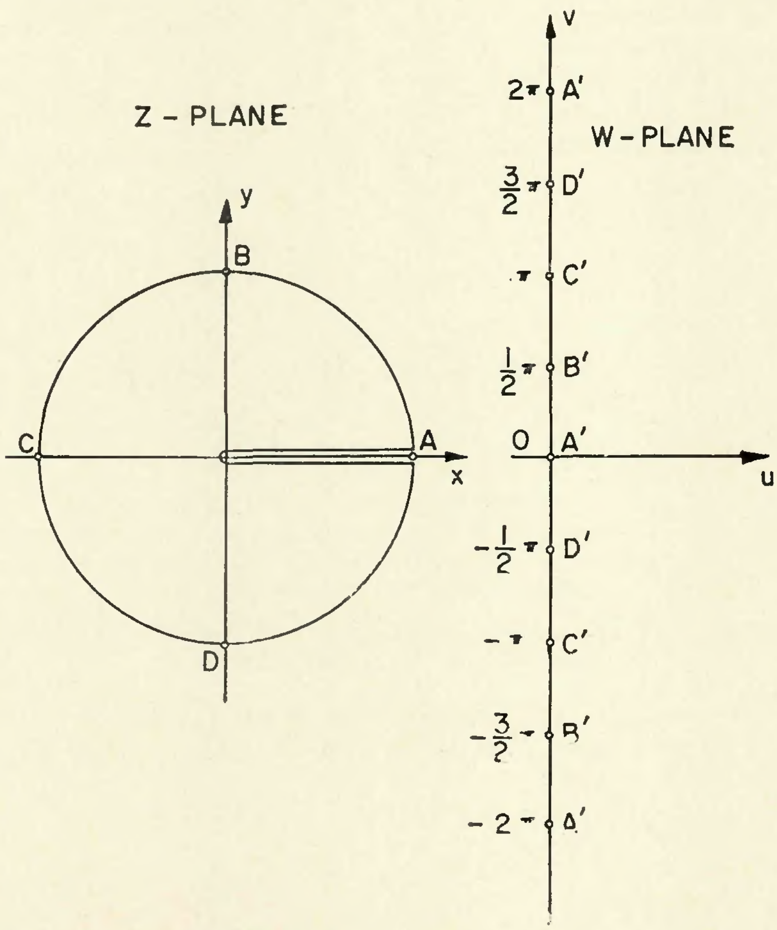

Fig. 5—Mapping planes

of the welding fixture. Agreement in interval length and location of peak temperatures as registered by the circumferentially spaced thermocouples is quite good.

Conclusions

The main conclusions are as follows:

1. The heat transfer process in the small disk structure investigated was almost quasistationary in regions removed from the disk center.

2. Rosenthal's solution mapped into the unit disk was modified by three adjustment coefficients to take into account heat sinks, surface dissipation and one imperfect boundary condition. These three coefficients can be obtained from temperature measurements at two points of a prototype subjected to one set of welding conditions. The semi-theoretical model describes then the spatial and temporal temperature distributions at all points of the disk structure except those very close to the axis of rotation with good accuracy.

3. The semi-theoretical model can be used to establish prototype design parameter and welding machine parameter influence trends. It can be used to investigate the influence of welding on buckling of the diaphragm disk. The adjustment coefficients have to be found only once.

4. The considerable amount of experimental labor necessary for an empirical exploration of design parameter and welding machine parameter influences on the temperature distribution during welding can be reduced drastically by way of the semi-theoretical model.

Acknowledgements

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Appendix: Nomenclature

q' = heat source intensity $\left[\frac{\text{cal}}{\text{in. sec}}\right]$

$\dot{v}$ = velocity of point source $\left[\frac{\text{in.}}{\text{sec}}\right]$

$K_0(.)$ = modified Bessel function of the second kind and of zero order

r, θ = coordinates of z-plane

u, v = coordinates of w-plane

$\xi = \frac{r}{a}$

a = radius of disk

k = conductivity $\left[\frac{\text{cal}}{\text{in}^\circ\text{F-sec}}\right]$

$\lambda = \frac{\rho c}{2k}\left[\frac{\text{sec}}{\text{in.}^2}\right]$

ρ = density $\left[\frac{\text{lb}}{\text{in.}^3}\right]$

c = specific heat $\left[\frac{\text{cal}}{\text{lb}^\circ\text{F}}\right]$

T = temperature [°F]

T_0 = initial temperature [°F]

V = voltage across arc [volt]

I = current [amp]

q = 0.239 $\left[\frac{\text{cal}}{\text{watt}}\right]$

μ = efficiency of arc

s = thickness of melting zone [in.]

C_1, C_2, C_3 = constants

$\tau_s = \frac{\rho_T}{\omega}$

θ_T = θ—coordinate of thermocouple position with respect to the start of the arc.

ω = angular velocity

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