

Motion in the Weld Pool in Arc Welding

Motion that occurs in weld pools is primarily caused by the action of the Lorentz force which decreases in intensity away from the constriction at the arc root

BY R. A. WOODS AND D. R. MILNER

ABSTRACT. The motion which occurs in pools of metal melted beneath a tungsten arc has been studied, by an examination of the mixing of dissimilar metal droplets in the pools, and by direct observation and cine photography of the underside of arc melted pools. The observed flow behavior of liquid metal within the pools has then been simulated in model experiments with pools of mercury.

The motion in the weld pool is caused primarily by the action of Lorentz forces. A subsidiary cause of motion is the action of the arc plasma jet on the surface of the weld pool.

The Lorentz force stirring results from the current constriction at the arc root which causes fluid flow away from this high current density region to other parts of the pool. If the current path is symmetric the form of the motion within the pool is a double circulation. With an asymmetric current path one half of the double circulation pattern becomes dominant, giving rise to a pure rotation. The intensity of motion increases both with the square of the arc current and with the melting point of the weld pool material. The latter effect is probably because there is a higher current density at the arc root of high melting point materials such as iron and nickel than with lower melting point materials such as aluminum.

The stirring forces are not always sufficiently intense to overcome basic incompatibilities in the physical and chemical properties of the weld pool and additive metals. Thus, when an addition is made to a weld pool, the extent to which mixing occurs is influenced by the relative melting points (higher melting point additives tends to solidify), the relative density (lower density additives tend to "float" on the surface and vice versa), and the relative miscibility of the materials involved.

R. A. WOODS is with Kaiser Aluminum and Chemical Corp. at the Center for Technology, Pleasanton, Calif., and D. R. MILNER is with the Dept. of Industrial Metallurgy, University of Birmingham, Edgbaston, England.

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Introduction

Motion in the weld pool in arc welding plays an important part in:

1. Determining the degree of homogeneity of the weld bead where filler and base metals differ in composition.

2. Heat transfer within the weld pool.

3. Gas-metal and slag-metal reactions in that it determines the distribution and interaction of reacting constituents.

Evidence for motion and its effect on weld homogeneity comes from Houldcroft et al.¹ and Pumphrey², who have shown that for a number of arc welding techniques, and over a range of welding conditions, aluminum alloys welded with a filler metal of different composition to the base metal give rise to substantially uniform weld beads. The well known Shaefller diagram³ implicitly assumes complete and perfect mixing of filler metals and base metal plates of different composition and is found to work out well in practice. Erokhin⁴ made a weld bead from a composite mild steel-stainless steel electrode and showed that the composition of the resulting bead conformed with the assumption of complete mixing. However, marked inhomogeneity has been found by D'Annessa and Willner⁵ in the gas tungsten-arc welding of thin magnesium alloy sheet with a-c around 35 amp, while examples of gross inhomogeneity have been described by Stout et al.^{6,7} and Muir⁸, associated with the "finger" type of penetration found at the base of the weld bead in gas metal-arc welding.

On the subject of heat transfer Apps and Milner⁹ determined the "effective thermal conductivity" of molten lead under an arc and found it to be considerably greater than that of static molten metal, showing the existence of forced convection and fluid flow. The same authors,¹⁰ and also

Christensen,¹¹ attributed the shape of the weld pool, at least in part, to the effect of heat transfer due to the motion in the molten metal. Conflicting general patterns for the fluid flow within the weld pool have been deduced by Ishizaki¹² and Bradstreet.¹³

In the field of gas-metal reactions, Salter, Howden, Pollard and Milner¹⁴⁻¹⁷ have shown that the rate of reaction and the distribution of the reacting constituents are determined by transport processes within the weld pool. Belton, Moore and Tankins¹⁸ have likewise attributed the distribution of silicon in the weld bead, arising from flux-metal interactions, to very rapid motion in the weld pool. There is also some evidence that motion in a molten metal affects the solidifying structure.¹⁹⁻²²

Despite the significance of motion in the weld pool very little is known about it and the object of the present work has been to investigate the extent to which it occurs, the cause of the motion, and the effect it has on the mixing action in the weld pool.

First, mixing experiments are described which show qualitatively the extent to which motion occurs under the arc. Then the mixing action is evaluated for a number of base metal/additive combinations. Next the motion is observed directly by cine photography, and then simulative experiments with a mercury pool as a model system are described which aim to show how the motion arises and the form which it takes.

Experimental

The investigation has been concerned with the mixing action and motion in a pool of molten metal under an arc in the nonconsumable tungsten electrode system. In the first place the effect of the arc on the degree of mixing that occurs has been established by comparing the behavior

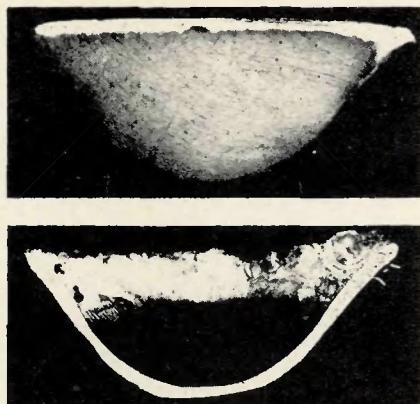


Fig. 1—Demonstrating the slow rate of mixing of dissimilar metals added to a pool of metal melted with an oxyacetylene flame. A (top)—indium on the surface of bismuth-tin eutectic; B (bottom)—lead rich zone at the base of a copper pool, both after 20 sec had been allowed for mixing, X6 (reduced 40% on reproduction)

of arc heated pools with that of pools heated by an oxyacetylene flame. The extent of mixing was determined by adding a drop of a different molten metal (comprising about 5% of the pool volume) to a molten pool of the base metal allowing a certain time for mixing to occur, then freezing the pool and examining for non-homogeneity. A survey is then made of the mixing behavior of a range of base metal/additive combinations.

For these experiments the base metal has been melted in thin crucibles of copper or steel, so that a water jet could be positioned underneath to give a rapid quench. Where the crucible would melt through or interact with the liquid metal, a molten pool was formed by melting part of a block of the metal being investigated (ideally a $1\frac{1}{2}$ in. diameter pool in a 2 in. x 2 in. x $1\frac{1}{2}$ in. block). In this case the water quench took longer to act. It was found that mixing times decreased rapidly with increasing current and that it was therefore difficult to make quantitative determinations on high melting point materials since these required a high current to keep them molten.

Thus for more detailed and quantitative evaluations a low melting point system has been used. Desirable characteristics of such a system were similar melting points and densities of the base metal and the additive, complete miscibility of the components and etching characteristics that clearly showed up the distribution of the additive. The most satisfactory combination found was the bismuth-tin eutectic (58% bismuth/42% tin), melting point 138°C (280°F), density 8.8 gm. cm^{-3} for the base metal, with indium, melting point 156°C (315°F), density 7.3 gm. cm^{-3} , as the additive.

Table 1—Mixing Behavior Under a 150 Amp D-C Argon Arc

Base metal	Mixed	Alloying addition		Unmixed (immiscible addition)
		Partially mixed	Low or compar. m.pt. addition	
Copper	Pb, (Sn, Sb) ^a	Ag, Au, Al, Mg	Mo, Ni, Fe	
Iron	Ni, Cu, Al, Cr		Mo	Ag
Titanium	Fe, Cu, Sn			
Nickel	Cu, Al, Fe		Mo	Ag
Aluminum		Mg, Sn, Zn	Fe	Pb
Tin		Bi, Pb, In, Al	Cu	

* A very slight trace of inhomogeneity.

F), density 7.3 gm. cm^{-3} , as the additive.

Metallographic examination of sections of partially mixed systems gave indications of the type of motion taking place in the molten metal. More direct observation of weld pool motion was made by cine photography of the circulation at the underside of pools in thin sheet, where the movement of small oxide particles showed the flow taking place. These experiments lead to the hypothesis that motion occurs primarily as a result of electro-magnetic forces within the pool and this concept was tested by setting up simulative systems using mercury pool models. For these tests the mercury was held in a suitable container with electrodes inserted to give the required current flow through the pool. Motion induced within the pool was then traced by following particles on the mercury surface.

Results

Qualitative Experiments on Mixing

Mixing in Non-Arc Pools of Molten Metal. These experiments were concerned with establishing the degree of mixing that occurs in a non-arc melted pool, for subsequent comparison with mixing in arc melted pools.

Initially, the bismuth-tin system with indium additions was investigated for conditions approximating to a stagnant pool. To achieve this the alloy was heated from below with an oxyacetylene torch to well above its melting point. The heat source was then removed and 20 sec allowed for

the system to stabilize before addition of the indium drop. It was found that after 20 sec mixing time the indium drop spread out on the surface of the molten pool to form an indium rich surface layer, with no dispersion of the indium into the body of the pool. When the bottom of the pool was heated throughout the 20 sec mixing period so that natural convection currents were operative, the same result was still obtained (Fig. 1A). Furthermore, when the pool was heated by the flame impinging on the surface, thereby determining any effect of fluid motion produced by the impinging gas stream, the same result was again obtained.

Mixing under oxyacetylene torch heating conditions was next extended to copper, a higher melting point base metal, with the more intense flame that this required. Tin, lead and silver additions were made to the molten copper heated from the bottom, and it was found that after 20 sec, the denser silver (10.5 gm cm^{-3} , compared to copper 8.9 gm cm^{-3}) and lead (11.3 gm cm^{-3}) had segregated to the base of the pool as silver and lead rich layers (Fig. 1B). The less dense tin (7.3 gm cm^{-3}) showed significantly more mixing, and there was no surface segregation as such, although the pool as a whole was still relatively inhomogeneous after 20 sec mixing time.

When the melt was heated from the top, so that natural convection was eliminated, but the force due to the flame impinging on the surface brought into play, the rate of mixing was considerably increased. Tin was uniformly dispersed throughout the pool within 5 sec, while although the silver still showed some segregation at the base of the pool after 20 sec, this was much less pronounced than with the base heated pool.

Mixing in Arc Melted Pools. The mixing action under arc heating was determined for six base metals at a fixed direct current of 150 amp and the effect of the use of a-c and a dissociable gas atmosphere also ascertained. The six base metals were iron,



Fig. 2—Example of incomplete mixing due to solidification of the higher melting point addition. Iron added to aluminum after 5 sec mixing time with a 150 amp arc. X5 (reduced 38% on reproduction)

nickel, titanium, copper, tin and aluminum to which drops of metal were added covering a range of melting points, densities and intersolubilities. It was found that mixing was considerably more rapid than under oxyacetylene heating so that mixing times were shortened to 5 sec before the pool was quenched.

The results of this survey are summarized in Table 1, where the behavior of each additive is classified as mixed, partially mixed or completely unmixed, in the base metal. As would be expected, where the additive and base metal were completely immiscible there was no mixing. Thus with silver in iron and in nickel the silver drop was pushed to one side of the pool and remained there, while with lead in aluminum some of the drop sank to the base of the pool, with a smaller quantity remaining on the surface. The addition of a drop of high melting point metal to a pool of lower melting point resulted in only partial mixing as the higher melting point material solidified—Fig. 2. This effect was apparent in the iron plus molybdenum system, where the melting point difference is 1100°C and also in nickel/molybdenum (1150°C , 2102°F), copper/molybdenum (1500°C , 2732°F), copper/nickel (370°C , 698°F), copper/iron (450°C , 842°F), aluminum/iron (840°C , 1544°F) and tin/copper (850°C , 1562°F). The remaining combinations show marked differences in the rate of mixing depending on the base metal.

In one group are the rapid mixing metals, iron, titanium and nickel

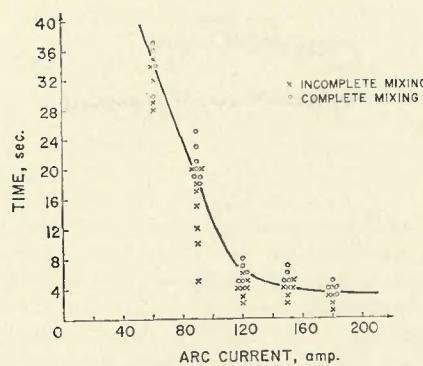


Fig. 3—The time required for complete mixing of a molten indium droplet into bismuth-tin eutectic as a function of arc current; 1.2 cm arc

which showed complete mixing in 5 sec. In a second group are aluminum and tin in which mixing took place much more slowly. Copper formed somewhat of an intermediate case since with most additions it behaved as a slow mixer, yet the low melting point additions, tin and lead, dispersed rapidly. Since tin had also mixed rapidly into copper under oxyacetylene heating, a further experiment was carried out in which tin was first added to molten copper followed by silver, which otherwise was a poor mixer. With this prior addition of tin the silver dispersed completely in the 5 sec period, suggesting that the addition of tin effects a basic improvement in the mixing action of the copper.

Under otherwise similar experimental conditions the mixing behavior using a-c arcs was determined for a selection of combinations. The results were generally similar to those ob-

tained with d-c arcs. The most significant difference was an improvement in the degree of mixing of additions to aluminum pools. Here more intermixing occurred, but there was still not complete dispersion of additions within the 5 sec period. There was also some evidence for slightly reduced mixing in the iron pools. The improved mixing with aluminum appeared to be associated with the clearing of surface oxide when using a-c.

To determine whether such an oxide would affect mixing in the weld pool, some experiments were carried out with d-c in which an aluminum drop was first added to a copper pool, and then a further addition of the rapid mixing tin or lead was made. The aluminum spread out on the surface of the copper and formed an oxide film and inhibited the mixing action, since a lead rich layer segregated at the bottom of the weld pool, while the tin segregated as a tin rich layer between the copper and aluminum. Without the aluminum addition both tin and lead had completely dispersed in copper under otherwise similar conditions.

Two systems in dissociable gases were investigated. First, the mixing of silver added to copper melted under a nitrogen arc, and secondly copper in 4% silicon-iron under a CO_2 arc. The silver drops mixed much more effectively into the copper under a nitrogen arc than under an argon arc, complete homogeneity being obtained in the 5 sec period. With the CO_2 arc, however, the situation was reversed; whereas under argon complete mixing occurred, under a CO_2 arc there was

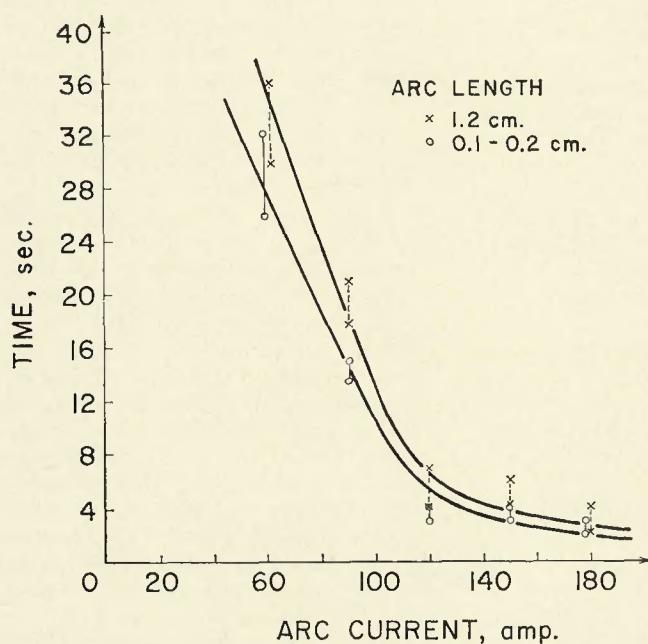
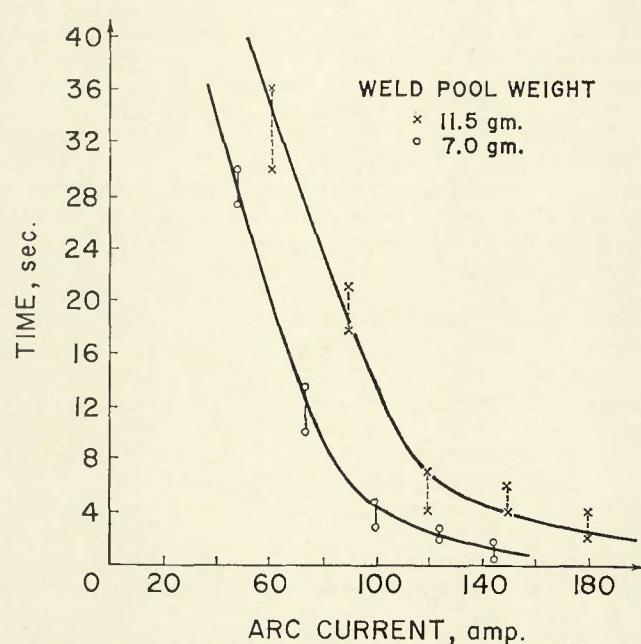


Fig. 4—The effect on the time required for complete mixing of an indium droplet into a pool of molten bismuth-tin eutectic: A (left)—reducing the arc length from 1.2 to 0.28 cm; B (right)—reducing the pool size from 11.5 to 7 gm



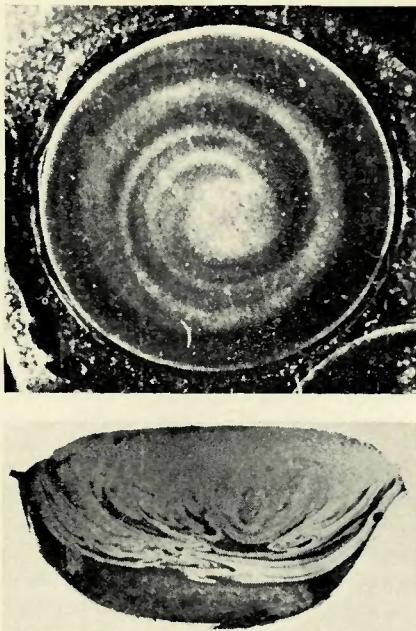


Fig. 5—The mixing of an indium droplet into bismuth-tin eutectic after 1 second under a 120 amp arc (compare with Fig. 1A for mixing under an oxyacetylene flame). A (top)—the rotational flow pattern observed on a horizontal section, X6 (reduced 17% on reproduction); B (bottom)—the sweeping of the indium into the depths of the pool, X8 (reduced 45% on reproduction)

gross heterogeneity.

Quantitative Experiments on Mixing in a Low Melting Point System

The aim of this part of the investigation was to quantitatively evaluate the mixing process. The low melting point bismuth-tin system, with indium additions, was used as this allowed experiments to be made with low, 50 amp, as well as moderate (200 amp) currents and the lower pool temperatures meant that the system could be more readily controlled and manipulated.

The Rate of Mixing. The effects of changes in arc current, arc length and size of molten pool on mixing were determined by adding drops of indium to many specimens, allowing various times for mixing, then quenching and examining metallographically to determine whether the indium was uniformly dispersed throughout the solidified pool. The type of result obtained with variation in current is shown for a 1.2 cm arc with tungsten electrode negative in Fig. 3.

In Fig. 3, it can be seen that the time for complete mixing at currents below about 100 amp is very long, compared with the life of a weld pool, and that the mixing action increases very rapidly with increasing current until at about 120 amp only a few seconds are required for complete dispersion. Reducing the arc length to

0.2 cm resulted in slightly more rapid mixing; complete dispersion occurred in appreciably shorter times with smaller pools—Fig. 4.

The Mixing Pattern. Observation of the surface of pools and of sections of tin-bismuth pools in which the indium drop had not completely dispersed yielded some information on the pattern of fluid flow that was taking place. The motion appeared to be of a three dimensional rotational nature—Fig. 5, circulating the lighter indium and at the same time carrying it progressively into the depths of the pool.

The Cause of Motion. There appeared to be two possible causes of the motion in the weld pool. First, it is known that high velocity plasma jets exist in welding arcs, and the initial experiments had shown that the oxyacetylene flame used for melting copper caused motion in the pool, presumably due to the momentum imparted by the impinging gas stream. Second, the current flowing through the pool could induce motion by electromagnetic forces. Both of these possibilities were investigated.

The contribution due to momentum imparted by the gas stream was evaluated by measuring the gas flow pressure created at the anode by the arc jet and then imparting a similar degree of momentum by a nitrogen jet acting on a super-heated pool. The character of the arc jet was determined by a technique due to Wilkinson,²³ in which a water-cooled copper anode containing a small hole connected to a manometer is traversed beneath the arc. The variation in pressure across the anode at currents of 100 and 145 amp, and the maximum pressure over the current range 40 and 200 amp were determined. Then, by a process of trial and error, it was found that nitrogen at suitable velocity passed through a 1 mm. diameter capillary tube would simulate the pressure distribution of the arc jet—Fig. 6. When the nitrogen jet was directed perpendicularly onto the molten metal it produced mixing currents, but complete dispersion of the indium drop took very much longer than under the action of the arc—Fig. 7. When the jet was angled to the surface it was much more effective in creating mixing currents and produced complete dispersion in times comparable with those of the vertical arc—Fig. 7.

The effect of current flow through the melt was simulated by an electrode touching the surface of the molten pool.²⁴ Nestor has shown that for the arc length used for determining mixing times in Fig. 3, i.e., 1.2 cm, the largest proportion of the current enters the anode over an area approx-

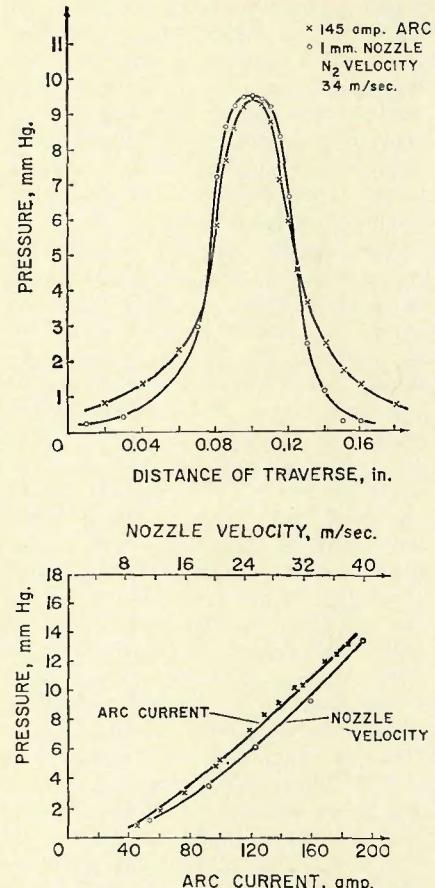


Fig. 6—The pressure distribution under an arc as measured by traversing a water cooled copper anode containing a 0.015 in. hole connected to a manometer. A (top)—for a 145 amp arc; B (bottom)—as a function of current, with similar data for a comparable nitrogen jet

imately 1 cm in diameter. Therefore, a 1 cm diam tungsten electrode was used to carry the current into the pool. It was found that rapid mixing resulted, more rapid in fact than had been obtained under arc conditions—Fig. 8, and that the mixing pattern, as might be expected from more rapid motion, was more well defined.

It therefore appeared that, while the plasma jet can cause motion, particularly if inclined at an angle to the surface, the primary source of motion arose from the passage of the current through the molten pool. It had not been possible to determine the effect of changing the electrode polarity using an arc, as considerable fuming resulted from the low melting point bismuth-tin base metal when the tungsten electrode was positive, or when a-c was used. However, with the short circuit conditions these experiments could be carried out and it was found that the mixing times for complete dispersion were the same whether the molten pool was the anode or the cathode or when a-c was used.

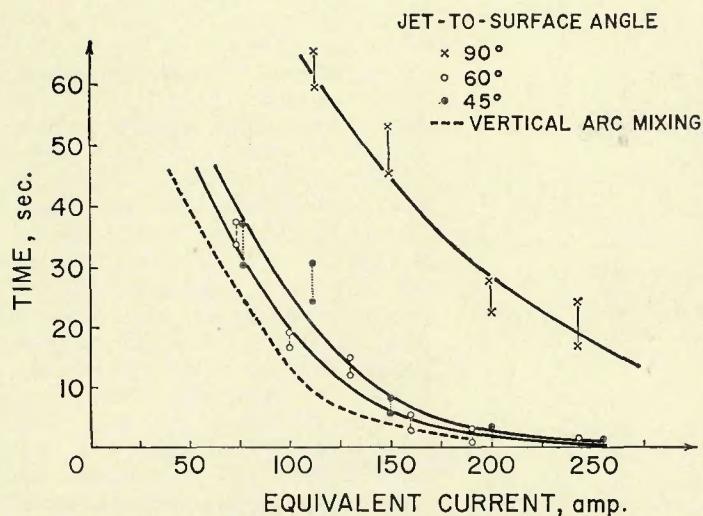


Fig. 7—The time required for the complete mixing of an indium droplet into bismuth-tin eutectic with a nitrogen jet, simulating an arc plasma jet, directed into the surface of the pool. Vertical and angled nitrogen jets are compared with the time required for mixing under an arc

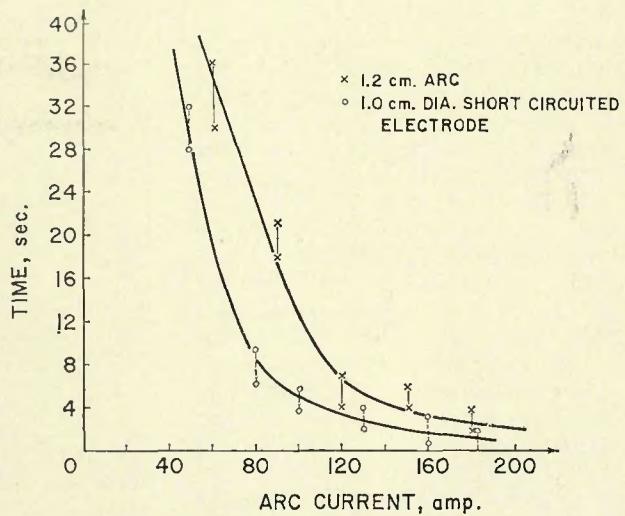


Fig. 8—The time required for complete mixing of an indium droplet into bismuth-tin eutectic with a tungsten electrode touching the molten pool. This simulates the effect of the arc current flowing through the pool axis, and is compared with the time required for mixing under an arc

Motion in Various Metals

The experiments described so far have provided general information on the mixing behavior of various systems and detailed data for the low melting point bismuth-tin indium system. To obtain more specific information for high melting point materials, the pattern of fluid flow was studied by cine photography of the underside of static molten pools formed in 0.075 in. thick sheet by a conventional argon arc welding torch. Specimens were mounted in a water-cooled jig so that a stable pool could be established and the rear of the specimen was protected by a flow of argon. Under these conditions sufficient oxide particles were usually present on the underside of the pool to make the flow visible. In addition dissimilar metal drops were added to the pool immediately

prior to a rapid quench so that mixing patterns could be correlated with the observed motion.

The behavior of nine base metals was studied together with the effect of changing the polarity, using a-c, angling the electrode and using a dissociable gas instead of argon for the arc atmosphere. In addition to observing the motion in the base of a static pool, the motion on the underside of a moving weld pool was observed and found to yield an important clue to factors determining the pattern of fluid flow.

Motion with Tungsten Electrode Negative and an Argon Atmosphere. All the metals conformed to a general pattern of behavior. At low currents, of the order of 30 amp, a double circulation was observed on the bottom of the molten pool—Fig. 9; the pattern often rotated as a whole or

moved erratically around the pool. As the current was increased to the order of 100 amp the double circulation gave way to a pure rotation—Fig. 10, which became more rapid at higher currents.

There was a considerable difference in the intensity of the motion with different metals. Those which had shown rapid mixing, i.e., iron, titanium and nickel, and also mild steel, stainless steel and cobalt, which were not included in the mixing tests, showed very rapid motion and more erratic changes in flow pattern. The motion was particularly intense with iron and mild steel. With these metals there also appeared to be an area of super-heated metal, or "hot spot" which changed in shape and position very rapidly—Fig. 11.

The remaining metals, copper and aluminum (which had shown slow

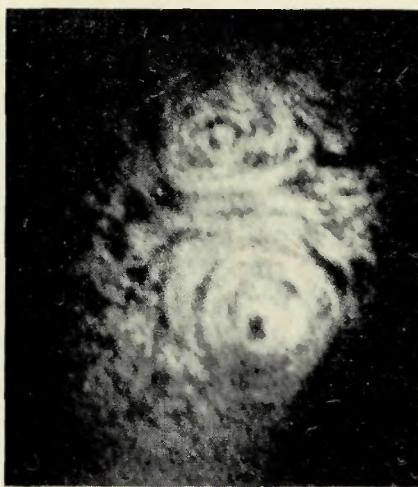


Fig. 9—A print from a 16 mm cine film showing double circulation on the underside of a silver pool melted under a 90 amp arc

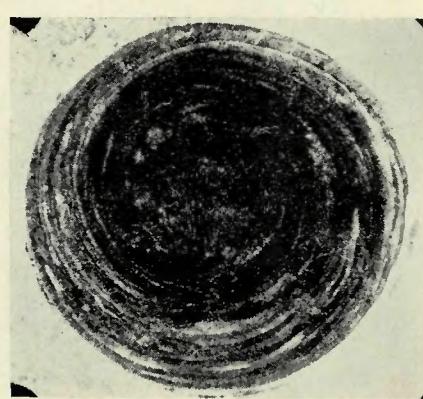


Fig. 10—Section showing a rotational flow pattern in a 150 amp arc melted copper pool to which a droplet of silver was added immediately prior to solidification. X4

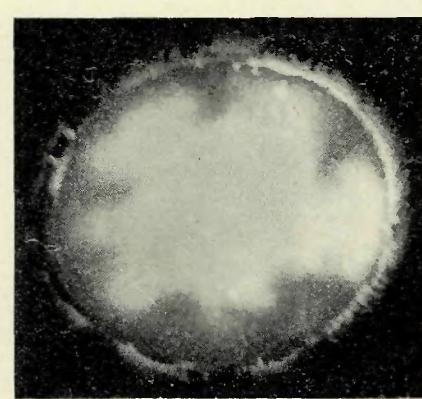


Fig. 11—The appearance of the rapidly moving "hot spot" on the underside of an Armco iron specimen melted under a 100 amp arc. X4 (reduced 32% on reproduction)

mixing) and silver exhibited much slower and more well defined, although not reproducible, motion and it was apparent that the rotation arose by the one half of the double circulation growing at the expense of the other half. Both clockwise and counter-clockwise rotation were observed on the bottom of the pool. In some cases when there was rotation on the bottom of the pool, double circulation still remained on the top despite the pool being only 0.75 in. thick. When the torch was angled, the dual motion effect became more frequent with rapid double circulation in the direction in which the welding electrode pointed on the upper surface. When the direction of the torch was reversed the direction of the double circulation also reversed, but the rotation on the underside did not change direction.

The Effect of Reverse Polarity, A-C and a Dissociable Gas Atmosphere. With the tungsten electrode positive the arc was very unstable; some specimens were, however, successfully obtained at high currents in Armco iron and those showed that the basic motion consisted of a very fast rotation similar to that observed with the tungsten electrode negative.

There were also difficulties in obtaining results with a-c. With Armco iron at currents below 100 amp the behavior on the underside of the pools was very similar to that observed with the d-c arc. At higher currents the metal tended to vibrate fiercely and the pool frequently dropped through, so that only a few results were obtained; these indicated a more diffuse rotation than occurred with d-c. Some results were obtained with copper; these again indicated slower, but otherwise the same flow pattern as the motion observed with d-c.

The effect of using a nitrogen atmosphere with copper was that an area of apparently very hot metal was formed, similar to that observed with iron in an argon atmosphere, but there was no obvious change in the form and intensity of the motion on the underside of the pool.

Motion in a Moving Weld Pool. It will be apparent that the pattern of fluid flow is a variable phenomenon and that under the conditions in which a well defined flow pattern was observed this could be a double circulation, or a pure rotation in either the clockwise or counter-clockwise direction. A major variable influencing this situation was observed during experiments in which the motion on the underside of molten pools in thin strip was examined for a fused zone run. A length of copper strip was traversed under a 120 amp arc with the current off-take point perpendicular to the

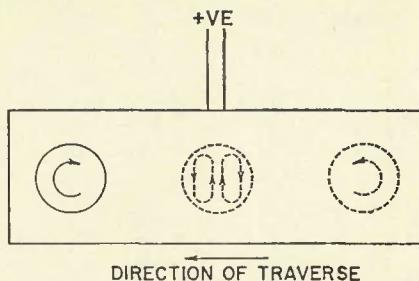


Fig. 12—The relation between the motion in a copper pool and its position relative to the positive contact

center of the strip, as shown in Fig. 12. At the commencement of traverse there was clear counter-clockwise rotation. As the pool moved past the current off-take this changed to double circulation, and then when the pool moved further away from this point the fluid motion changed to clockwise rotation.

Thus, as the pool moved past the position of the off-take contact the motion changed from rotation in one direction through a double circulation, to rotation in the other direction, showing that the type of motion was dependent upon the geometry of the welding current path.

Simulation Experiments To Determine the Mechanism of Motion Using a Mercury Pool Model

The work described in previous sections has shown that the motion within arc melted pools of metal varies in intensity and form, depending on the metal, the current, the angle of the welding torch and the position of the current off-take contact. It has also been demonstrated that the passage of the current through the pool is mainly responsible for the mixing action, and that this is largely unaffected by the polarity of the current, and by

decreasing the arc length.

It is well known in the welding field that the passage of an electric current through a fluid can cause fluid flow due to the action of the Lorentz force.^{23, 25} When a current flows through a fluid conductor, a magnetic field is set up around the current and the current and magnetic field interact to produce a self compressive force on the conductor. In a fluid this leads to a pressure build-up with the increase in pressure greatest where the current density is highest. A pressure gradient is therefore set up which leads to a flow of the conducting fluid from regions of high to regions of low current density. In the tungsten arc, for example, the highest current density is at the narrowest part of the conducting column formed by the cathode spot. Away from the cathode spot the arc gradually becomes broader so that the current density, and hence pressure, decreases and gas flows along the arc to form a plasma jet.

A similar flow would be expected to occur in the molten pool, as there is a region of high current density at the pool surface bounded by the limits of the conducting area of the arc root, after which the current flow path broadens out in the conducting metal pool. Simulative experiments were therefore carried out with mercury pools, first to determine the extent of Lorentz force streaming in a liquid metal when current flows from a high to a low current density region, and second, to determine whether the Lorentz force could also be responsible for the various types of motion which had been observed in weld pools.

Lorentz Force Streaming in Mercury

The apparatus for this aspect of the investigation comprised a bath of mercury into which a 3 mm diam pointed copper electrode insulated up to its tip projected horizontally. Opposite to this was set a larger, 1.5 cm diam, flat ended copper electrode. When a current was passed through the bath, turbulence could be seen in the mercury; but in order to show up any movement it was found necessary to clean the surface of the pool and cover it with a layer of 50% hydrochloric acid. Particles of silicon carbide powder, which floated on the surface of the mercury, were very effective in showing up the flow pattern clearly.

Under these conditions the motion caused by the passage of current through the bath could be clearly seen and was of the form shown in Fig. 13. This consisted of a fast streaming of mercury away from the smaller elec-



Fig. 13—The flow pattern in a horizontal mercury bath with a current of 100 amp flowing from a small (3 mm diam) pointed copper electrode to a larger (1.5 cm diam) electrode

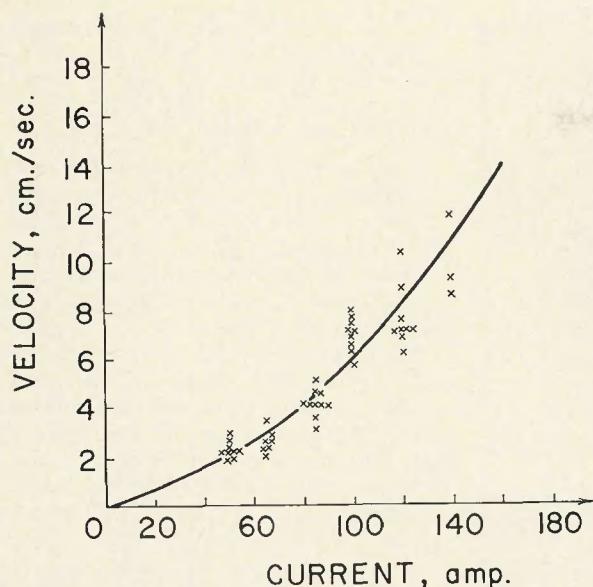


Fig. 14—The variation with current of the velocity of the central stream of mercury show in Fig. 13

trode tip towards the larger electrode, with continuity of flow maintained by a more general flow of roughly double circulation around the bath. The velocity of particles traveling down the central zone of this main stream was measured by filming the motion at 64 frames per sec. This showed that the metal accelerated at the electrode tip and that the velocity thereafter remained practically constant down the length of the flow stream while the velocity increased with current at a greater than linear rate—Fig. 14.

Changing to reverse polarity or a-c did not affect the flow pattern and the variation in maximum velocity with current was substantially the same as with the small electrode negative—Fig. 15.

When the smaller, 0.3 cm diam, electrode was replaced by a larger one of 0.6 cm diam, the maximum velocity of flow at any given current was about halved—Fig. 15. The central stream was wider and the general circulation throughout the bath more

diffuse.

When changes were made in the direction in which the small, 0.3 cm diam electrode pointed in the bath it was found that the streaming always occurred in a direction away from the electrode tip—Fig. 16.

Experiments on the Pattern of Fluid Flow

In the previous section Lorentz force streaming was examined with two electrodes in line with each other and lying in the same plane. In welding, however, the arc impinges vertically onto the weld pool and the current then flows through the weld pool and the base plate to the current take-off contact which would usually be lying at right angles to the arc. To simulate this type of geometry a cylindrical crucible was constructed from a non-conducting material and the current passed into a pool of mercury held in the crucible, through a long vertical length of 1 cm diam copper bar. Provisions were made so that the second contact could be made in one of three ways; first, through another length of 1 cm diam copper bar fitting into the base of the crucible; second, through a length of 5 cm diam bar which formed the complete base of the crucible; and third, through a 2 cm wide copper strip which was bent over the side of the crucible to dip into the mercury.

When the current was taken from the smaller, 1 cm base connection, it was found that there was turbulence in the pool, but no clearly defined flow pattern, with currents up to 200 amp. On substituting the larger 5 cm electrode for the smaller one at the base the mercury flow took on a rotational form at around 100 amp. More in-

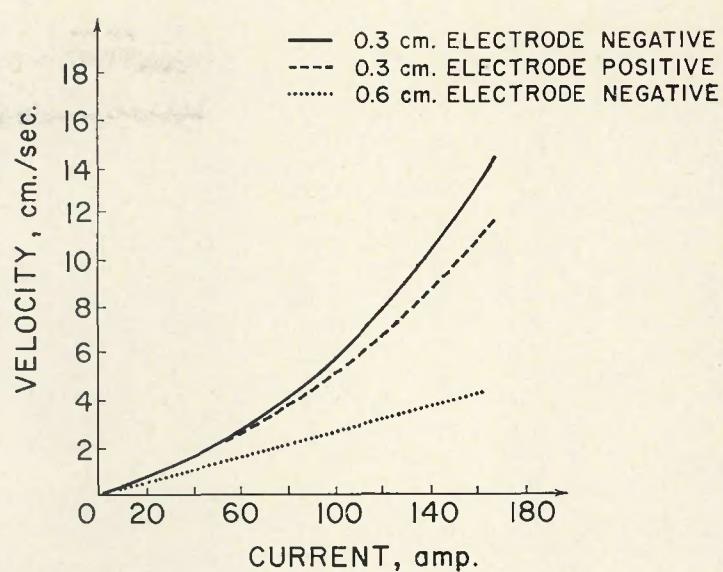


Fig. 15—The effect of electrode size and polarity upon mercury streaming velocity

tense rotation occurred, however, when the current was taken out through the 2 cm side contact strip. The direction of this rotation was determined by the path of current flow leaving the pool. Thus when the connecting bar was in position AB—(Fig. 17), very fast clockwise rotation occurred at currents of the order of 70 amp. As the connecting bar was rotated towards AC the clockwise rotation became gradually less intense until, at position AC, there was turbulence in the pool, but no evidence of rotation up to currents of 200 amp. As the bar was moved further towards position AD, counter-clockwise rotation was set up in the pool, which became gradually more intense as position AD was approached. Similarly if the current off-take were maintained in the central position AC and the smaller electrode angled to the pool from either side then rotational flow occurred, the direction of which was determined by the side from which the smaller electrode was inserted—Fig. 18.

Double circulation and a transition

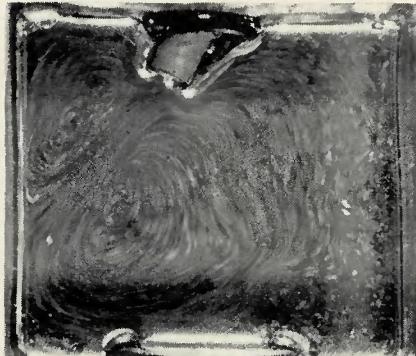


Fig. 16—The dependence of streaming on the direction in which the smaller electrode is pointing, current 100 amp

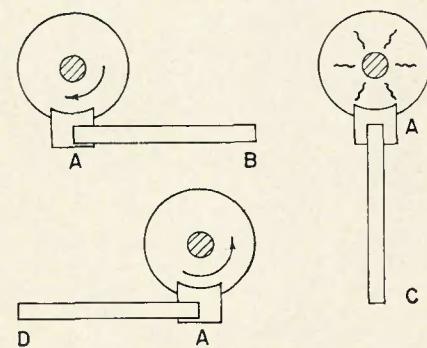


Fig. 17—The relation between the direction of the current off-take connection and the motion in a mercury pool

from double circulation to rotation was observed when the smaller electrode was angled in line with the current off-take from the mercury pool, with the connection outside the bath made at right angles. (In this case the current was taken out of the mercury pool by a flat ended 2.5 cm diam bar cemented into the side of the crucible, instead of the strip bent over the side.) When the smaller electrode was lowered, i.e., at about 20 deg from the horizontal—Fig. 19, there was a well defined double circulation. As the electrode was raised, one half of the pattern grew at the expense of the other half until pure rotation was obtained—Fig. 19.

A Note on the Effect of Motion on the Form of the Weld Pool

It was mentioned in the introduction that the fluid flow within the weld pool determines the heat transfer in the liquid metal. While the technique of melting stationary pools in thin metal specimens was being used, a correlation was observed between the type of motion within the pool and the pool shape. To correlate motion with pool shape specimens were produced in iron, nickel and copper with 130 amp vertical and inclined arcs.

In iron the weld pools were circular under both the vertical and inclined arcs, corresponding to a rotational flow pattern. The pools in nickel were circular under a vertical arc with rotational motion observed in the pool. Under the inclined arc the pools were pear shaped during the early stages of their development, but on becoming larger, they gradually became circular. To begin with, an area of double circulation existed on the upper surface at the front of the pool, which caused the formation of the nose to the pool. This area of double circulation gradually became absorbed by

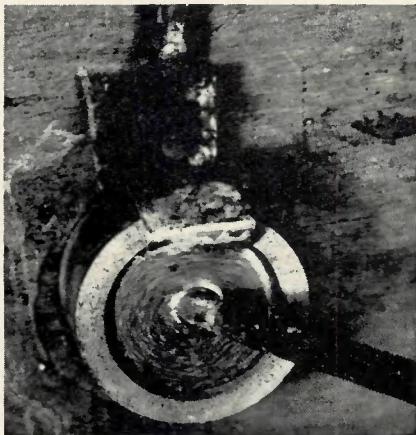


Fig. 18—The effect of angling the electrode dipping into a mercury pool on the motion within the pool. A (top)—clockwise rotation; B (bottom)—counter-clockwise rotation

the rotational movement and, although some double circulation still existed, it was not strong enough to affect the pool shape. With copper the pools were circular under a vertical arc, with clockwise rotation observed on the underside. When the arc was inclined the pools became ovoid—Fig. 20, with double circulation apparent

on the upper surface in the direction of the major pool axis away from the electrode tip.

Discussion

It has been demonstrated that motion in a molten pool arises primarily due to the current flowing through the molten metal, although a significant degree of motion can be produced by the plasma jet acting on the surface of the weld pool if the torch is angled. The motion is largely independent of current polarity, occurs to the same extent with a-c and is more intense with a greater degree of constriction at the current entry point. These features point to the motion being induced by the Lorentz force. Maecker has shown how fluid motion arises from the action of the Lorentz force.²⁶ Consider in the first place a current following a cylindrical path through a fluid—Fig. 21A. Then the flow of current creates a magnetic field and the magnetic field and current interact to produce an inwards acting force so that an excess pressure develops in the fluid which is equal and opposite to the Lorentz force. The force, and hence also the pressure increase, is proportional to the product of the current density and field strength, i.e.,

$$p \propto j \cdot H \quad (1)$$

or, since the field strength is proportional to the current:

$$p \propto j \cdot I \quad (2)$$

When, instead of following a cylindrical path, the current broadens out, then the current density decreases and hence a pressure gradient is set up away from the region of highest current density so that fluid flows away from this region—Fig. 21B. Since for laminar fluid flow the velocity is proportional to the pressure:

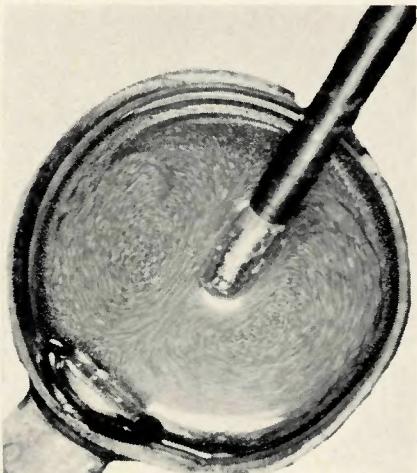


Fig. 19—The effect of the angle of the electrode on the motion in a mercury pool: A (left)—electrode at 20 deg to the horizontal; B (center)—45 deg; C (right)—nearly vertical. Current 120 amp

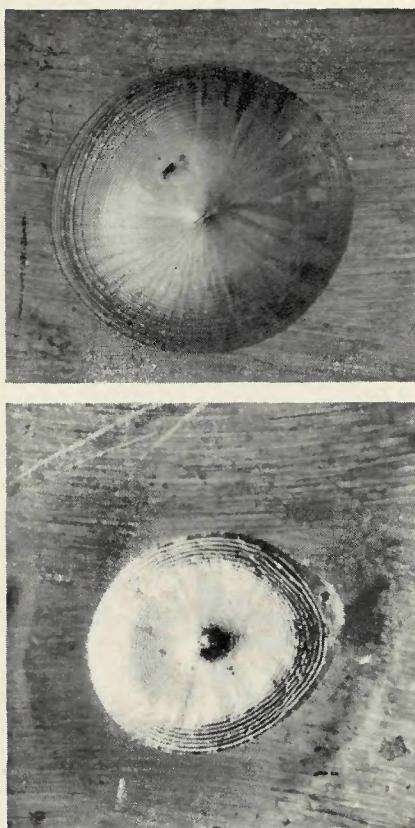


Fig. 20—The effect of inclining the arc and thereby getting a double circulation flow pattern, on the shape of a molten pool in copper strip: A (top)—vertical arc; B (bottom)—inclined arc. Current 130 amp. X1

$$v \propto Ij$$

or, for a fixed area of constriction, i.e., fixed diameter of electrode in a mercury pool, or fixed size of arc root with a weld pool, then:

$$v \propto I^2 \quad (3)$$

The velocity of the central stream of mercury for the configuration shown in Fig. 13 (and plotted in Fig.

14) is re-plotted as a function of the square of the current (Fig. 22), and conforms well with this law.

Where mixing of dissimilar materials is involved, it would be expected that the mixing rate would be related to the force/velocity relationships outlined above. And, in fact, when the mixing rate (i.e., mixing time), for both arc mixing and short circuit electrode mixing, in the indium-tin-bismuth system is plotted as a function of the square of the current, linear proportionality is found—Fig. 23.

While the motion would thus appear to be Lorentz force induced it remains to explain how the different forms of double circulation and pure rotation arise. The classical form of motion is that shown in Fig. 13 where there is a streaming away from a smaller, constricting, electrode towards a larger diameter electrode, and this gives rise to a double circulation. The experimental data showed that rotation occurred when there was asymmetry in the current flow. This was particularly clear with the moving weld pool, depicted in Fig. 12, where rotation occurred in either a clockwise or a counter-clockwise direction depending on the direction of current off-take. Additionally, in many instances it was observed that rotation arose by one half of a basically double circulation flow pattern growing at the expense of the other half. When the current flow is asymmetric the magnetic field is stronger on one side (Fig. 24), so that, from eq (1), the Lorentz force is greater on the side of high field strength and asymmetry is introduced into the fluid flow.

A further feature of the experimental results was that more rapid motion occurred with some metals, and that the metals which showed the most intense motion also showed more erratic behavior. The metals showing the most intense and erratic motion were iron and steel; stainless steel, nickel, titanium and cobalt showed similar but less intense characteristics, while the motion was comparatively slow and regular with copper, silver and aluminum. The intensity of the motion is related to the degree of constriction of current at the anode spot.

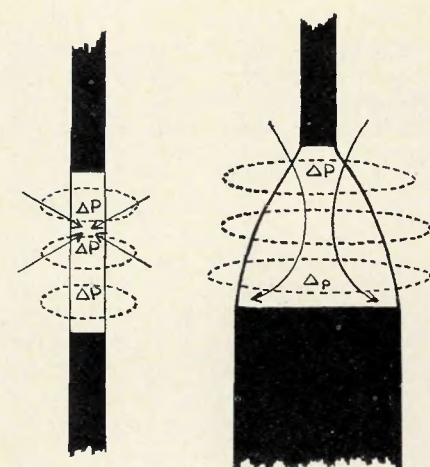


Fig. 21—The action of the Lorentz force to produce fluid flow: A (left)—the current follows a cylindrical path and the Lorentz force creates a cylindrical region of excess pressure P ; B (right)—the current path broadens out and the pressure in front of the smaller electrode diminishes with the current density thereby giving rise to fluid flow

The essential feature of the anode region is that current has to be carried from the arc column, where charge carriers are readily available as a result of ionization at the high temperatures involved, to the much lower temperature metal electrode. The mechanisms by which this is accomplished are not entirely clear, but it is

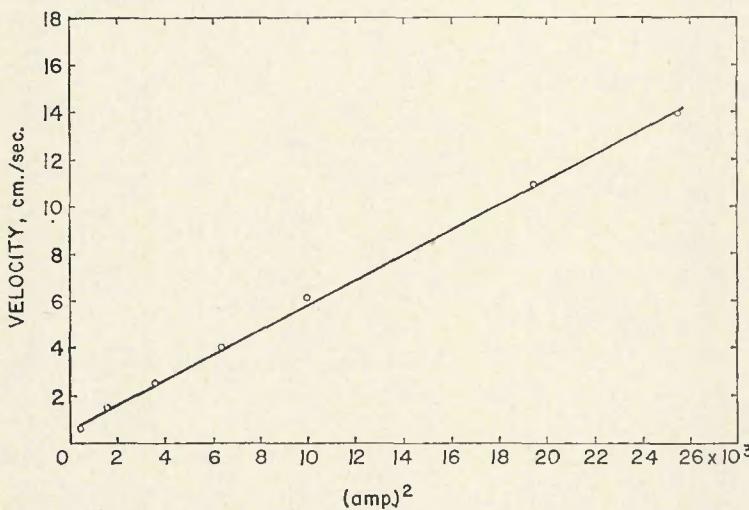


Fig. 22—The velocity of the central stream of mercury (Figs. 13 and 14) replotted against the square of the current

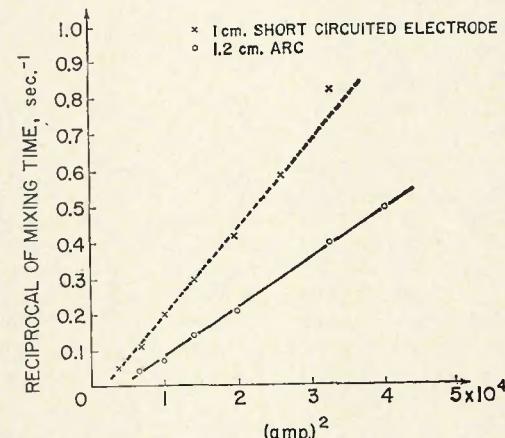


Fig. 23—The mixing rate for an indium droplet added to a pool of bismuth-tin eutectic as a function of the square of the current (from Figs. 3 and 8) under a 1.2 cm. arc and with a short circuited electrode

known that it is facilitated by high temperatures and the associated thermal ionization.^{27,28} It may be significant that the metals showing the most rapid motion, indicative of the greatest degree of constriction, have the highest melting point and could thus favor a high current density. Similarly, the current density at the arc root would be expected to be different when the weld pool was made the cathode or when dissociable gases were used and could account for the experimental differences observed.

The existence of double circulation on the top surface, and pure rotation on the bottom surface, of a pool only 0.075 in. thick shows that marked changes in the flow pattern can occur over very short distances. It was observed that one half of a double circulation pattern can dominate the other half, so that one half only may penetrate through to the bottom of the pool to give rotation. When the electrode was angled and the double circulation flow on the top surface changed according to the direction in which the electrode pointed, the rotation on the bottom was unaffected. It would appear that the rotation was caused by current flow through the pool and the double circulation by the plasma jet acting on the top. Where the current enters the weld pool, resistance heating occurs; if the current path is restricted by a small anode spot, the resistance heating in the immediately adjacent molten metal is greater. This could be the origin of the "hot spot" observed in the molten pools in thin sheet. The resistance heating creates a local increase in temperature of the molten metal. Since the resistance of molten metal increases with temperature it becomes increasingly more difficult for the current to follow this path. Thus the current is continually seeking out fresh paths, the "hot spot" moves around, and the motion associated with the concentration of current becomes erratic.

This research has been concerned with attempting to obtain greater understanding of the extent and cause of motion in weld pools. It has not been possible to look into the practical implications of the results, so that only a few unsubstantiated comments can be made about such possibilities. As pointed out in the Introduction, it is generally accepted that rapid mixing occurs where electrode and base metal differ in composition. The rate of mixing has been shown to increase with the square of the current, so that, as D'Annessa and Willner found, inhomogeneity is more likely to occur at low currents.⁵ In addition, D'Annessa and Willner were working with

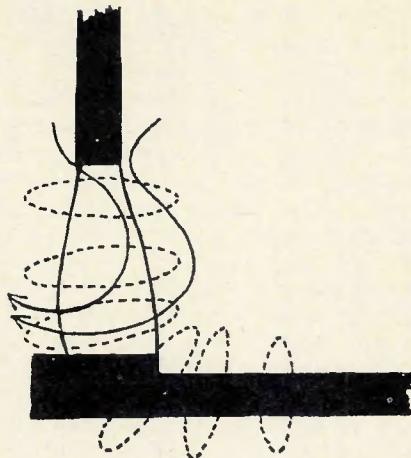


Fig. 24—The effect of asymmetry of the current path to produce rotational fluid flow

magnesium which is a low melting point material and thus probably exhibits slow mixing anyway.

The present authors examined an electroslag weld made with dissimilar compositions of base and filler metals and found gross inhomogeneity; this is very likely to occur because the motion arises from expansion of current flow away from a constriction. While this occurs in weld pools in arc welding, it takes place in the flux in electroslag melting with the current entering the molten metal surface over a much larger area. Grain refinement is known to result from motion in the solidifying metal¹⁹⁻²² and electroslag welds tend to exhibit large grain sizes. Additionally arc welding is notorious for the lack of reproducibility under apparently rigorously controlled conditions. It may be that fluid motion, which in itself is affected by factors such as the position of the current off-take connection, and which in turn affects the heat transfer and thus the penetration, is one of the unresolved parameters involved.

Conclusions

1. The motion that occurs in weld pools is primarily caused by the action of the Lorentz force which decreases in intensity away from the constriction at the arc root thus giving rise to fluid flow away from this region. If the current path is symmetrical the flow pattern is a double circulation, but if the current path is markedly asymmetric one half of the double circulation dominates the fluid flow to give a pure rotation.

2. The motion is more rapid with the higher melting point materials, such as iron, steel, stainless steel, etc., than with lower melting point materials such as aluminum, silver and copper. This is probably due to the arc root being smaller with the former

group giving rise to a greater degree of constriction at the arc root.

3. An additional, secondary cause of motion is the action of the arc plasma jet on the surface of the weld pool.

4. As a result of the rapid motion any dissimilar metal additives are rapidly mixed into the weld pool. The rate of motion, and hence the rate of mixing, increases with the square of the current, so that inhomogeneity is more likely to arise at low currents.

5. The extent to which mixing occurs when an additive is made to a weld pool depends on the relative melting points (higher melting point additives solidify), the relative density (lower density materials "float" on the surface and vice versa), and the relative miscibility.

Acknowledgements

The authors wish to thank Professor E. C. Rollason, Head of the Dept. of Industrial Metallurgy at the University of Birmingham, for his interest and encouragement, and the Superintendent, The Naval Construction Research Establishment, Dunfermline, for financial support and sponsorship of this research project.

References

- Houldcroft, P. T., *British Welding Journal*, Vol. 1, p. 468, 1954.
- Pumphrey, W. I., *British Welding Journal*, Vol. 2, p. 93, 1955.
- Shaeffer, A. L., "Selection of Austenitic Electrodes for Welding Dissimilar Metals," *WELDING JOURNAL*, 26 (7), Research Suppl., 601-s to 620-s (1947).
- Erokhin, A. A., *Welding Production*, No. 7, p. 1, 1963.
- D'Annessa, A. T., and Willner, E., "Welding Characteristics of LA14IXA Magnesium-Lithium Alloy Sheet," *WELDING JOURNAL*, 43 (1), Research Suppl. 1-s to 9-s, (1964).
- Warren, D., and Stout, R. D., "Porosity in Mild Steel Weld Metal," *Ibid.*, 31 (9), Research Suppl., 406-s to 420-s (1952).
- Oyler, C. S., and Stout, R. D., "Porosity in the Welding of Carbon Steel," *Ibid.*, 32 (9), Research Suppl., 454-s to 460-s (1953).
- Muir, A. R., *British Welding Journal*, Vol. 4, p. 323, 1953.
- Apps, R. L., and Milner, D. R., *British Welding Journal*, Vol. 10, p. 348, 1963.
- Apps, R. L. and Milner, D. R., *British Welding Journal*, Vol. 2, p. 475, 1955.
- Christensen, N., Davies, V. de L., and Gjermundsen, K., *British Welding Journal*, Vol. 12, p. 54, 1965.
- Ishizaki, K., "Physics of Welding Arc Symposium," The Institute of Welding, p. 195, 1966.
- Bradstreet, B. J., "Effect of Surface Tension and Metal Flow on Weld Bead Formation," *WELDING JOURNAL*, 47 (7), Research Suppl., 314-s to 322-s (1968).
- Salter, G. R., and Milner, D. R., *British Welding Journal*, Vol. 7, p. 89, 1960.
- Salter, G. R., and Milner, D. R., *British Welding Journal*, Vol. 12, p. 222, 1965.
- Howden, D., and Milner, D. R., *British Welding Journal*, Vol. 10, p. 304, 1963.
- Pollard, B., and Milner, D. R., to be published.
- Belton, G. R., Moore, J. J., and Tankins, E. S., "Slag-Metal Reactions in Submerged-Arc Welding," *WELDING JOURNAL*, 42 (7), Research Suppl., 289-s to 297-s (1963).

19. Brown, D. C., Crossley, F. A., Rudy, J. F., and Schwartzbart, H., "The Effect of Electromagnetic Stirring and Mechanical Vibration on Arc Welds," *Ibid.*, 41 (6), Research Suppl., p. 241-s to 250-s (1962).
 20. Crossley, F. A., *Iron Age*, Vol. 186, p. 102, 1960.
 21. Dudko, D. A., and Rublevskii, I. N., *Automatic Welding*, No. 9, p. 10, 1960.
 22. Trochum, I. P., and Chemysh, V. P., *Welding Production*, No. 11, p. 6, 1965.
 23. Wilkinson, J. B., and Milner, D. R., *British Welding Journal*, Vol. 7, p. 115, 1960.
 24. Nestor, O. H., *Journal of Applied Physics*, Vol. 33, p. 1638, 1962.
 25. Milner, D. R., Salter, G. R., and Wilkinson, J. B., *British Welding Journal*, Vol. 7, p. 73, 1960.
 26. Maecker, H., *Z. Physik*, Vol. 141, p. 198, 1955.
 27. Finkelnburg, W., and Maecker, H., "Handbuch der Physik," Vol. 22, p. 254, Springer, Berlin, 1956.
 28. Buxz-Peuchert, W., and Finkelnburg, W., *Z. Physik*, Vol. 144, p. 244, 1956.

Errata for WRC Bulletin 107

"Local Stresses in Spherical and Cylindrical Shells Due to External Loadings"

By K. R. Wichman, A. G. Hopper and J. L. Mershon

It will be recalled that this report summarizes, modifies and extends the work carried out by Prof. P. P. Bijlaard and his associates at Cornell University for the Pressure Vessel Research Committee. The information is presented in a "cook-book" form to facilitate its use by design engineers.

A number of minor errors in the report were found by the authors and others since it was originally published in August 1965. Because of continued interest, the report was reprinted with corrections in December 1968 and again in July 1970. A list of the corrections is presented below:

p. i, right-hand column, para. 2, line 11: ... the *test* data indicate that the *calculated* data ...

p. 6, Table 2, row 6 under column BL: minus sign inserted.

p. 6, Table 2, row 9 under column Du: plus sign changed to minus sign.

p. 6, Table 2: change formula for Combined Stress Intensity as follows—When σ_x & σ_y have like signs, $S =$ larger of $\frac{1}{2}[\sigma_x + \sigma_y + \sqrt{(\sigma_x - \sigma_y)^2 + 4r^2}]$ or $\sqrt{(\sigma_x - \sigma_y)^2 + 4r^2}$ (*this was not in the first revised printing*).

p. 7, Table 3, row 2 under column Bu: plus sign changed to minus sign.

p. 7, Table 3: change formula for Combined Stress Intensity as follows—When σ_x and σ_y have like signs, $S =$ larger of $\frac{1}{2}[\sigma_x + \sigma_y + \sqrt{(\sigma_x - \sigma_y)^2 + 4r^2}]$ or $\sqrt{(\sigma_x - \sigma_y)^2 + 4r^2}$ (*this was not in the first revised printing*).

not in the first revised printing).

p. 10, Table 5: change formula for Combined Stress Intensity as follows—When σ_ϕ & σ_x have like signs, $S =$ larger of $\frac{1}{2}[\sigma_\phi + \sigma_x + \sqrt{(\sigma_\phi - \sigma_x)^2 + 4r^2}]$ or $\sqrt{(\sigma_\phi - \sigma_x)^2 + 4r^2}$ (*this was not in the first revised printing*).

p. 11, Table 7: right-hand column should read C_c for N_x instead of C_e for M_x (*this was not in the first revised printing*).

p. 12, par. 4.3.3.1, eqn. for $6M_\phi/T^2:N_\phi$ in first bracket should be M_ϕ (*this was not in the first revised printing*).

p. 25: Fig. SP-8 revised.

pp. 28 and 29: formula for Υ changed

to $\frac{r_m}{l} = 5$

p. 32: $\Upsilon = 5$ changed to $\Upsilon = 15$.

p. 54, Fig. A-1: $D_m/T = 4 - 16U$ changed to $4 + 16U$.

p. 57, Table A-3, last column: superscript *a* in rows 2 and 3 changed to *d*; superscript *b* in rows 10 and 11 changed to *a*; superscript *b* in row 15 changed to *c*; footnotes added—^e Based on S.C.F. of 1.20; ^d Based on S.C.F. of 1.15.

p. 68, Fig. B-2: ratio $\frac{r}{h}$ in legend on

abscissa changed to $\frac{2r}{h}$

p. 69, right-hand column, line 5: ... point B, ... changed to ... point C, ...

It should be noted that the plus

and minus signs in Table 4 on page 9

were very light and barely legible. In the interest of clarity, the signs for Table 4 are also shown below:

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