Estimation of Fatigue-Crack Propagation Life in Butt Welds

Analytical model assesses the influence of joint geometry, weld reinforcement and other factors relative to the propagation of fatigue cracks

BY F. V. LAWRENCE

ABSTRACT. A method for calculating the fatigue crack propagation portion of the fatigue life of welds initiating fatigue failure at the toe of the weld is presented. The method can be applied to arbitrarily shaped and loaded members having external fatigue cracks of an assumed initial size. The results indicate that estimates using this procedure can provide reasonable estimates of the fatigue life of mild constructional steels. In the higher strength quenched and tempered steels, however, the initiation period which is neglected in this analysis may constitute the major portion of the fatigue life. The analytical model allows the relative influence of weld geometry on the crack propagation portion of the fatigue life to be assessed.

Introduction

Crack Initiation and Propagation

The fatigue resistance of a weld is usually less than that of the metal

F. V. LAWRENCE is Assistant Professor of Civil and Metallurgical Engineering, University of Illinois, Urbana, Illinois, 61801.

List of Symbols Used

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>da/dN</td>
<td>crack advance per cycle (in/cycle)</td>
</tr>
<tr>
<td>K</td>
<td>stress intensity factor</td>
</tr>
<tr>
<td>Kic</td>
<td>critical stress intensity factor</td>
</tr>
<tr>
<td>ΔK</td>
<td>range in stress intensity factor (ksi/√in.)</td>
</tr>
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<td>C, n</td>
<td>crack propagation material properties constants (kip, inch units)</td>
</tr>
<tr>
<td>Np</td>
<td>crack propagation fatigue life (cycles)</td>
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<tr>
<td>a</td>
<td>crack length</td>
</tr>
<tr>
<td>a0</td>
<td>initial crack length</td>
</tr>
<tr>
<td>a1</td>
<td>final crack length</td>
</tr>
<tr>
<td>Δa</td>
<td>finite advance of crack</td>
</tr>
<tr>
<td>h</td>
<td>height of weld reinforcement</td>
</tr>
<tr>
<td>w</td>
<td>width of weld reinforcement</td>
</tr>
<tr>
<td>t</td>
<td>plate thickness</td>
</tr>
<tr>
<td>θ</td>
<td>flank angle of the weld</td>
</tr>
<tr>
<td>φ</td>
<td>edge preparation angle of the weld</td>
</tr>
<tr>
<td>Kf</td>
<td>stress concentration factor</td>
</tr>
<tr>
<td>S</td>
<td>maximum applied tensile stress (stress cycle O-S)</td>
</tr>
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<td>σ</td>
<td>stress along crack interface</td>
</tr>
<tr>
<td>b0, b1, b2, b3, b4</td>
<td>constants in stress polynomial</td>
</tr>
<tr>
<td>x</td>
<td>coordinate from toe of weld</td>
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</tbody>
</table>
which it joins. This fact can be attributed to several types of discontinu-
ities — geometrical discontinuities, internal flaws, or metallurgical discontinu-
ities — which serve as stress concentrators and accelerate fatigue damage in their locality.\(^1\)

The fatigue life of a weldment may be considered as consisting of two periods: (1) the number of cycles required to initiate a fatigue crack or to initiate a fatigue crack from a pre-existing flaw (initiation), and (2) the number of cycles required to propa-
gate the crack until failure occurs (propagation). The initiation period\(^2\) is difficult to measure or predict. The length of this period de-
pends upon the cyclic stress-strain behavior of the material, stress his-
tory, residual stresses, and the geometry of the defect.

In welded joints which often contain sizable discontinuities unavoid-
ably created during fabrication, it is often assumed that the initiation period is short relative to the crack propagation period. The fatigue crack propagation portion of the fatigue life is a predictable quantity if the rate of fatigue crack growth per cycle (\(da/dN\)) as a function of the range in stress intensity factor (\(\Delta K\)) has been empirically established for the environ-
ment, materials, and loading history in question. Under most con-
ditions of testing there is a threshold level of \(\Delta K\), \(\Delta K_{th}\), which is just suffi-
cient to propagate the crack. At very high values of \(\Delta K\), the crack growth rate becomes infinite, i.e., fracture occurs when the peak value of \(K\) during a cycle equals \(K_I\). Between these two extremes and particularly for low \(\Delta K\) values, the measured rates of crack growth can be expressed as a power of the range in stress intensity.

\[
da/dN = C(\Delta K)^n\tag{1}
\]

From Eq. 1, the crack propagation period of the fatigue life can be es-
imated.

\[
N_p = \int_{a_c}^{a_f} \frac{1}{C(\Delta K)^n} \, da \tag{2}
\]

If Eq. 2 proves impossible to obtain in a closed form, the integral may be evaluated by finite difference tech-
niques.

\[
N_p = \sum_{a_0}^{a_f} \frac{1}{C(\Delta K)^n} \Delta a \tag{3}
\]

In previous studies\(^2\), the author has used Eq. 3 to estimate the fatigue crack propagation life of welds contain-
ing internal discontinuities. In the present study, the fatigue crack propagation life has been estimated for butt weldments containing external flaws (toe cracks) using an elastic superposition procedure and Eq. 3.
Fatigue in Butt Welds with Reinforcement Intact

Butt weldments tested in fatigue with their reinforcement intact exhibit less fatigue resistance than plain plate or butt welds with their reinforcement removed due to the notch associated with the toe of the weld. Fatigue cracks invariably initiate at these locations (shown schematically in Fig. 1) and propagate through the heat-affected zone and base metal in a direction perpendicular to the applied stress.

The height of the reinforcement (h), the width of the reinforcement (w), and the thickness of the weld (t) influence the severity of this notch. If one considers the weld reinforcement to be a segment of a circle, the ratios h/w and w/t determine the geometry, or, alternatively, the geometry may be specified by the flank angle (θ) and the edge preparation angle (ϕ): see Fig. 1. The ratios h/w and w/t are functions of θ and ϕ respectively.

\[
\frac{h}{w} = \frac{1}{2} \tan \frac{\theta}{2} \tag{4}
\]

\[
\frac{w}{t} = \tan \frac{\phi}{2} \tag{5}
\]

The geometry of the weld will influence the state of stress along the direction of fatigue crack propagation and hence will influence the rate of crack growth and the fatigue crack propagation.
propagation life of a weldment.
To calculate the fatigue crack propagation lives of double-Vee butt welds of varying geometry, the following procedure was used.

**Procedures**

**Elastic Superposition**

With reference to Fig. 2, one wishes to know the stress intensity factor for an edge crack in a body which is arbitrarily loaded. This condition is represented by the first body in Fig. 2. Using elastic superposition, the state of stress of the first body can be considered to be the superposition of the stresses in bodies two and three (Fig. 2). In the second body, the crack is held closed by the tractions necessary to do so, so that the body may be considered to be unflawed. The stresses in the second body and in particular the tractions necessary to hold the crack closed may be found using an appropriate elastic solution, or by using approximate finite element methods.

The stresses in the third body which is not loaded externally are due to loading the internal surfaces of the crack with the negative of the tractions found in second body. By superposing the stress fields of the bodies two and three, one obtains the stresses in first body. More importantly for present purposes, the stress intensity factor associated with the crack in third body is identical to that in the first body.

In terms of the present problem, the calculation of the stress intensity factor for an edge notch in a particular geometry double-Vee butt weld consists of the following steps:

1. Find the tractions in an uncracked weldment along the line to be traversed by the crack, using finite element methods.
2. Fit these stresses with a polynomial function.
3. Calculate the stress intensity factor for any particular length of crack.
4. Use Eq. 3 to obtain the fatigue crack propagation life, \( N_p \).

**Stress Analysis Using Finite Element Techniques**

Examples of the geometries studied are shown in Fig. 3. Because of symmetry only one quadrant of the weld need be considered. Geometries having flank angles (\( \phi \)) of 0, 10, 20, 30, 45, and 60 degrees and edge preparation angles (\( \gamma \)) of 30, 45, 60, and 90 degrees were analyzed using finite element methods. The geometry of a particular weld was modeled by subdividing it into a mesh of interconnected triangles (Fig. 4). Linear strain triangles were used and plane strain conditions were assumed. The applied (tensile) stresses were replaced by equivalent nodal forces. The boundary conditions allowed freedom of displacement along the respective axes, and restraints perpendicular to the axes. The stresses at each node were determined by averaging the stresses of all triangles joined at that node. A finer mesh was necessitated at the toe of the weld where the stress level changes rapidly.

The calculated stress levels for different flank angles are shown in Fig. 5 and listed in Table 1. It can be seen that the maximum value of stress (\( \sigma_{max} \)) occurs at the toe of the weld and is between 1.2 to 1.8 times larger than the applied stress (\( S \)). The
stress level decreases rapidly with distance away from the toe of the weld, and after a distance of approximately 0.1t, the stress level is approximately that of the applied stress (S). At greater distances, the stress is slightly below the applied stress.

The calculated stress concentration factor ($K_t = \sigma_{\text{max}} / S$) is plotted as a function of the geometries studied in Fig. 6. The stress concentration factor increases rapidly with increasing flank angle $\theta$ (or h/w ratio) but does not increase much after $\theta = 45$ deg (h/w $\sim 0.2$). For $\theta = 60$ deg, $\phi = 90$ deg a maximum $K_t$ of 1.8 was found. Reducing the angle of the edge preparation, $\phi$, reduces the stress concentration markedly: when $\theta = 60$ deg, $\phi = 30$ deg, $K_t$ is reduced to 1.27.

The calculated stresses shown in Fig. 5 were fitted with a fourth order polynomial using a least squares fit.

\[ \sigma = b_0 + b_1(x_1) + b_2(x_1)^2 + b_3(x_1)^3 + b_4(x_1)^4 \]  

where $\sigma$ = stress at any point $x$, $S$ = applied stress, $x$ = distance from toe of weld, $b_0$, $b_1$, $b_2$, $b_3$, $b_4$ = constants.

This analytical expression can be conveniently used in stress intensity factor calculations (Eq. 8).

### Stress Intensity Factor for Edge Cracks

The stress intensity factor for an edge crack in a semi-infinite solid uniformly loaded at a distance from the crack is given approximately by:  

\[ K = 1.1\sigma\sqrt{\pi a} \]  

The stress intensity factor for an edge crack in a semi-infinite solid loaded internally by an arbitrary system of stresses has been given by Emery $^{10,11}$ (see Fig. 7).

\[ K = \sqrt{\pi a} \left\{ 1.1\sigma_a + \int_0^a f\left(\frac{x}{a}\right) \frac{d\sigma}{dx} \, dx \right\} \]  

where $a$ = crack length, $\sigma_a$ = stress at crack.

When the stress does not vary along the internal surface of the crack, i.e., $\frac{d\sigma}{dx} = 0$, Eq. 8 reduces to Eq. 7.

Substituting Eq. 8 into Eq. 2 one obtains:  

\[ N_p = \int \frac{d\sigma}{\sigma} \left\{ 1.1\sigma - \int_0^a f\left(\frac{x}{a}\right) \frac{d\sigma}{dx} \, dx \right\} \]  

or in terms of finite differences and Eq. 3, $N_p = \int \frac{d\sigma}{\sigma} \left\{ 1.1\sigma - \sum_{i=0}^{n} f\left(\frac{x}{a}\right) \frac{d\sigma}{dx} \, dx \right\}$

Estimation of Fatigue Crack Propagation Life, $N_p$

Since the variation of stress along the interface to be traversed by the toe crack is a known function of distance (Eq. 6), $\Delta K$, the range in stress intensity can be calculated for any crack length (a) and the fatigue crack propagation life calculated by Eq. 10.
The exact computation procedure is diagrammed in Fig. 8. Both of the summations are performed using an open-ended form of Simpson’s Rule for numerical integration. The use of Simpson’s Rule was found to give results within one percent of closed form solutions (found by substituting Eq. 7 into Eq. 2)* when the increment in crack size was 0.01 in.

In the calculations performed, the plate size of the weld (t) was assumed to be 1 in. for the purpose of comparing the calculations with available test results. The final flaw size was chosen to be 0.2 in. or 0.2t. This choice avoids the necessity of correcting for the opposite free surface of the plate. The fatigue life spent in propagating the remaining plate thickness is small compared with the life spent in propagating to a length of 0.2 in. Further, for most practical purposes a flaw size of 0.2t can be considered as constituting failure.

Results and Discussion

Effect of Weld Reinforcement Geometry on \(N_p\)

The calculated growth of a 0.01 in. toe crack in a one-inch thick double-Vee butt weldment subjected to zero-tension load cycling is shown in Fig. 9. The material is assumed to be a ferrite-pearlite steel having \(C\) and \(n\) values of \(0.36 \times 10^{-9}\) and 3.0, respectively.

Increasing the flank angle of the weld (\(\phi\)) from zero to 20 degrees greatly accelerates the rate of crack propagation to a length of 0.2 in.

Table 2 — Results of Stress Analysis for Various Weld Profiles

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(h/w)</th>
<th>(\phi)</th>
<th>(w/t)</th>
<th>(b_3)</th>
<th>(b_4)</th>
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<td>0</td>
<td>0</td>
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</table>

*This comparison is valid when \(\phi = 0\).
equivalent to very large reductions in cp are those of ferrite-pearlite steels for the values for martensitic steels. Average C and n values calculated fatigue life N is compared for ferrite-pearlite and martensitic steels. The sensitivity of the results is demonstrated in Fig. 13. The proper assumption of initial flaw size is critical for accurate predictions of Np, and at present one can only guess at the appropriate value. Most calculations in this work have been carried out with an assumed crack size of 0.01 in. This is a reasonably sized “small” crack which will most certainly exist at some (early) stage in the fatigue life of a weld. Assuming smaller cracks (0.001 in.) poses no problems for the analytical procedures described, but the behavior of such small cracks may not be properly described by the power law (Eq. 1). Nonetheless, crack propagation lives Np have been plotted for flaw sizes of 0.1, 0.01, and 0.001 in.

Comparison With Fatigue Test Data

The predicted fatigue crack propagation lives are compared with the total fatigue lives of constructional grade ferritic-pearlitic steels (A36 and A441) in Fig. 14 and constructional grade low-alloy martensitic steels (HY-130, HY-100 and T1) in Fig. 15. Naturally, this comparison is strained because any period spent in crack initiation is ignored. On the other hand, several current investigators have asserted that the initiation period in welds failing at the toe of the weld is negligibly short.

The data for the A36 and A441 are in quite good agreement with the predicted fatigue crack propagation life. The spread in the test data is due to controlled variations and weld reinforcement geometry. The predicted fatigue crack propagation life should be compared with test specimens giving the longest lives since these are of equivalent geometries. On this basis, it seems that initial flaw size assumptions of 0.01 in. and 0.001 in. bound the test data. For the A36 and A441 steel data, the majority of the fatigue life can be explained on the basis of fatigue crack propagation alone, with the assumption of a reasonable initial flaw size between 0.01 and 0.001 in. This, however, is not the case with the data for the martensitic steels. The large difference between the predicted crack propagation life and measured total fatigue life implies that a major portion of the fatigue life is spent in initiation. Assuming an initial crack size smaller than 0.001 in. would seem unreasonable and beyond the range of applicability of the power law (Eq. 1). Although the crack propagation life of the martensitic steels is shorter (for the condition at hand), the longer (apparent) initiation period contributes to the life of these specimens so that there is little difference between the fatigue life of the martensitic steel weldments and the ferritic-pearlitic weldments which exhibit longer crack propagation lives but lesser initiation periods.

Fig. 12 — S-Np plot for martensitic and ferrite-pearlite steels

Fig. 13 — Effect of initial flaw size, a0, on fatigue crack propagation life, Np

growth. Increases in flank angle beyond 30 degrees have little further effect. The major period of a crack's fatigue life is spent at very small crack lengths.

The calculated influence of flank angle Ө, and edge preparation angle φ upon Np is shown in Fig. 10. Decreasing φ or decreasing the width of the weld (w) for a given thickness lengthens the fatigue crack propagation life. Small reductions in φ are equivalent to very large reductions in Ө when Ө is larger than 15 degrees.

Effect of Material Properties on Np

The effect of varying material properties is shown in Fig. 11 in which the calculated fatigue life Np is compared for ferrite-pearlite and martensitic steels. Average C and n values for martensitic steels are 0.66 × 10⁻⁸ and 2.25 respectively. These values yield Np values for martensitic steels which are approximately one-third those of ferrite-pearlite steels for the initial flaw size chosen (a0 = 0.01 in.). The reduction in Np with increasing Ө is less pronounced in this material. These effects can also be seen in the S-N plots of Fig. 12. The slope of the S-N curve is equal to the reciprocal of the exponent (n) in Eq. 1. An initial flaw size of 0.01 in. is assumed in this plot. Ferrite-pearlite steels exhibit longer lives and more sensitivity to the weld reinforcement geometry than the martensitic steels. The difference between the predicted lives for the two steels diminishes with increasing stress level.

Effect of Assumed Initial Flaw Size on Np

In addition to the values of C, n, stress level, and geometry, the computed fatigue crack propagation life is extremely sensitive, if not the most sensitive, to the assumed initial flaw size. The sensitivity of the results is demonstrated in Fig. 13. The proper assumption of initial flaw size is critical for accurate predictions of Np, and at present one can only guess at the appropriate value. Most calculations in this work have been carried out with an assumed crack size of 0.01 in. This is a reasonably sized "small" crack which will most certainly exist at some (early) stage in the fatigue life of a weld. Assuming smaller cracks (0.001 in.) poses no problems for the analytical procedures described, but the behavior of such small cracks may not be properly described by the power law (Eq. 1). Nonetheless, crack propagation lives Np have been plotted for flaw sizes of 0.1, 0.01, and 0.001 in.
The effects of weld geometry upon the fatigue lives of the A36 and A441 fatigue specimens is shown for two different stress levels in Figs. 16 and 17, respectively. Again, the data are bounded by the calculated fatigue crack propagation lives for assumed initial crack sizes of 0.01 and 0.001 in. As the flank angle, $\theta$, or height to width ratio, $h/w$, is reduced, the total fatigue lives of the tested specimen increase more rapidly than expected at the higher stress levels. Possibly, the initiation period becomes proportionally larger as the severity of the stress at the toe of the weld is reduced.

Conclusions

An analytical model has been developed to calculate the fatigue crack propagation life of arbitrarily shaped and loaded weldments containing an external crack of assumed initial size.

With the model, the effects of weld geometry, material properties, stress level, and initial flaw size were considered. Differences in weld geometry were found to influence the fatigue crack propagation life by as much as a factor of three while material properties and initial flaw size can have a much larger effect.

Comparisons with fatigue results for A36 and A441 steel weldments reveal a good agreement between the calculated fatigue crack propagation lives and the total lives of tested specimens implying that in these materials the crack initiation period is relatively short. In such cases the calculation methods discussed can provide a reasonable lower bound for the fatigue life of such materials. There is a large discrepancy between the test results for low-alloy martensitic steels and the calculated fatigue crack propagation lives using similar assumptions of initial crack sizes, which implies that a larger proportion of the fatigue life is spent in crack initiation in these materials.
Although the uncertainty as to the proper choice of initial flaw size prohibits the exact calculation of the fatigue crack propagation life, comparisons of calculated lives for differing geometries with identical initial crack sizes does allow the influence of geometry on the fatigue crack propagation life to be studied.

Acknowledgements
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References

This compact manual presents information on how to gas and arc weld zinc-coated steel, including galvanized steel, thermal sprayed steel and steel painted with zinc-rich primers.

Welding Zinc-Coated Steel covers most of the commercially used welding processes, and includes numerous tables listing actual welding conditions and even the soundness of the resulting welds.

The excellent long-term protection of steel by galvanizing or thermal spraying, together with the attendant low maintenance cost, have led to the widespread application of zinc coatings to large structures such as highway bridges, power and television transmission towers, etc.

The use of zinc-rich paints in the form of welding primers for the temporary protection of shot-blasted steel during fabrication and prior to the application of the final paint coating is also increasing each year, typical applications being ship hulls and plating and all forms of structural steelwork.

To exploit the exceptional advantages of zinc coatings, both for permanent and temporary protection, it is essential to be able to weld zinc-coated steel and to produce joints having qualities equal to those of joints in uncoated steel.