Bending and Buckling Strength of Tapered Structural Members

Effects of oxygen cut vs. shear cut plates, single fillet welded web-to-flange joints and certain design limitations are determined

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Introduction

Although tapered steel members have been used for some time in a large variety of structural applications, their design has for the most part, been conservative. This is a natural result when data upon which to base a design is not available. Recognizing the need for such data, a joint task committee of the Column Research Council and the Welding Research Council was formed in 1966 to study and formulate design information relating to tapered steel members.

This report summarizes the results of an experimental program in which fifteen tapered steel members were tested to destruction. These tests were conducted at the State University of New York at Buffalo under the technical guidance of the Joint CRC-WRC task committee.

The primary objective of the testing program was to determine the bending and buckling strength of several linearly tapered members whose cross-sectional dimensions were similar to the dimensions of tapered members that are commonly used in construction. The member lengths and conditions of support were chosen so that failure of the member would occur within the inelastic range. The testing program therefore represents a continuation of the work carried out by Butler (Refs. 1-3) in which the elastic strength of tapered members was studied.

All of the tapered members tested had an I-shaped cross-section. The section was fabricated from plates by using a continuous fillet weld on only one side of the web plate. This is consistent with some present-day fabrication practice. Because of this welding procedure, residual stresses that are unsymmetrically distributed with respect to the weak axis of the cross-section were developed. The effect of this factor on the inelastic lateral buckling characteristics of a tapered member is discussed in more detail later.

Two sets of tapered members that were geometric duplicates of one another were used. One set of members was fabricated from plates that were cut to size by a shearing process while the other set was fabricated from oxygen cut plates. In every other way the two sets were the same. The reason for this was so that information relating to the effect of the fabrication process could be obtained.

While the main thrust of the testing program was directed toward finding the buckling strength of the tapered members, data relating to the in-plane bending behavior of each of the specimens was also taken. This data was to provide experimental verification for a recently developed analytical procedure to predict the in-plane response of a tapered member. It is to be noted that these procedures deal with in-plane bending and do not involve buckling in any way. Any comparisons made are thus limited to loads below the first buckling load. An analytical procedure to define the buckling load is currently being developed and will be described elsewhere. The results of this experimental study were referred to and used in the formulation of design recommendations of tapered members as reported in Ref. 4.

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Test Specimens and Program

Two series of tests involving a total of fifteen tapered members were conducted. In the first, each of the beams had its flanges laterally supported at several points as shown in Fig. 1. The second series involved members that were cantilevered as illustrated in Fig. 2. In this second set, the only lateral support provided was at the free end.

The individual member sizes used were based upon test results reported by Butler (Ref. 1). In each case the tapered beams were specially fabricated from ASTM A242 steel having a nominal yield of 42 ksi. Coupon tests of this material indicated a yield of 52 ksi and this actual value was used in all subsequent analytical work. The actual dimensions of all of the members tested are given in Table 1.

Test Specimens

Tapered or wedge beams are usually fabricated in one of two ways. In the first method, a rolled section is split longitudinally and some of the web material removed. The two parts thus formed are then reassembled into a wedge shape. The second method involves the fabrication of the member from precut rectangular and trapezoidal plates. Because of the wide variation in cross-section along the length of the member that is possible with the second method, it is usually preferred. All of the tapered members used in these tests were fabricated in this second way. In each case, the taper was limited to one dimension with the change in overall depth of the member resulting from a trapezoidal web plate. The width and thickness of the flanges was maintained constant over the entire length of the beam.

Two different procedures were used to cut the plate from which the test specimens were fabricated. In one case, all of the edges were shear cut while in the other, the plates were oxygen cut. This was done so that a comparison of the effect of the plate shaping process on the residual stresses could be made. The web plate and the flanges were then joined by a continuous 1/8 in. fillet weld placed on only one side of the web as illustrated in Fig. 3.

Because of the great importance of residual stress in a study of this type, two identical sets of test specimens were fabricated. One set was to be tested to destruction while the other set was sectioned to determine the actual residual stress levels present. In addition, so that the effect of the cutting process could be more clearly defined, a complete set of web plates having the same geometry as those in the beams tested, were sectioned for residual stress. A compilation of the results of this residual stress investigation is contained in Ref. 5. Similar residual stress patterns were obtained by McFalls and Tall (Ref. 6) in their study of built-up prismatic columns.

Typical residual stress patterns for the members as fabricated are shown in Fig. 3. From this figure it can be seen that there is a basic difference in the patterns of residual stress in sections fabricated from oxygen cut and sheared plates. Oxygen cutting results in tension at the flange tip while shearing the plate causes compression at that point. Because of this difference, one would expect the inelastic lateral buckling load to be higher for members having oxygen cut plates. In both cases, the residual stress pattern is seen to be unsymmetrical about the vertical axis of the cross section. This is caused by welding on only one side of the web and should also have an effect on the buckling characteristics of the member.

All of the test specimens were designed with all of the taper on the bottom of the member. Thus, at any point along the member, a cross section would be considered to be perpendicular to the axis of the top flange. It was felt that this geometry would be more consistent with practice. Prior to testing, stiffeners were welded to the specimens at all points of load application and to the ends. Since the members to be tested as cantilever beam columns were to be mounted at various angles with the horizontal, their ends had to be cut to the proper angle before the end stiffeners could be attached. Table 2 lists the final dimensions of each of the test specimens.

Because of the unsymmetrical welding procedure used in their fabrication all of the test specimens were bowed to some extent in the lateral direction (i.e. flange bent about a vertical axis through the plane of the web). While the magnitude of this initial lateral deflection varied in a random fashion, in all cases the flange welded to the web last was bowed more than the other.

The Testing Program

As pointed out earlier, the testing program was conducted as two separate series of tests. The first consisted of three tests in which the tapered members were simply supported for strong axis bending while laterally supported at the ends and quarter points. This arrangement is shown in Fig. 1. As shown, the beams were supported so that the top flange was horizontal. Two vertical loads Q_1 and Q_2 were applied through a loading beam at the quarter and three quarter points respectively. The magnitude of these loads was such that one of the following two conditions was realized.

1. In the elastic range, equal bending stress existed over the entire center portion of the beam.
2. A moment gradient case in which Q_1 was zero.

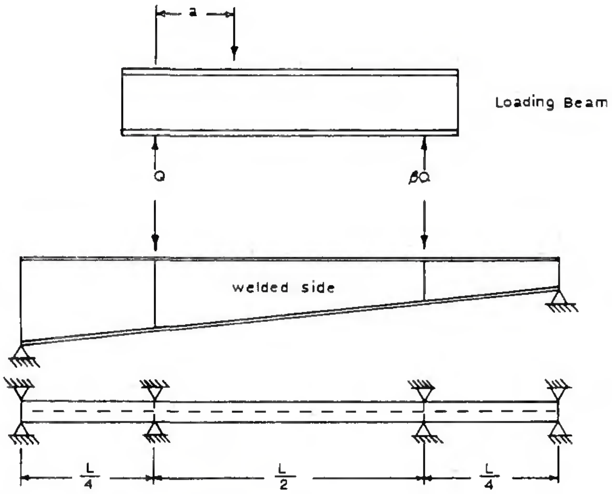


Fig. 1 — Continuous beam test setup

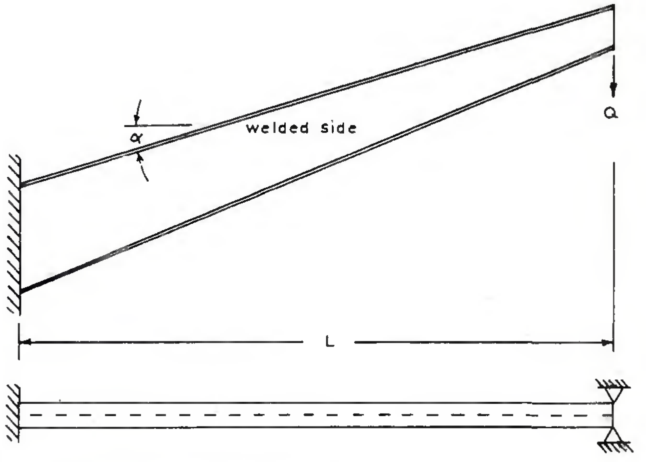
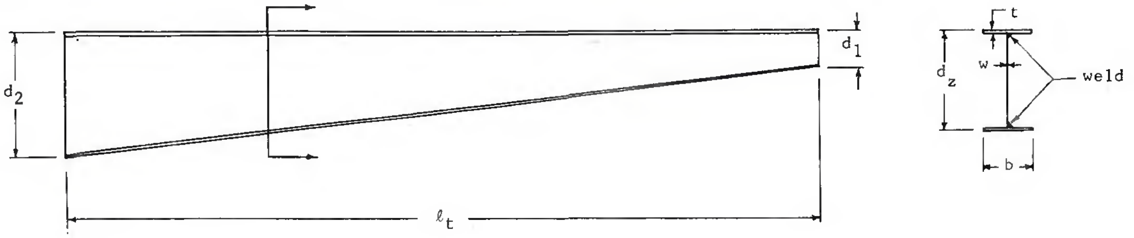


Fig. 2 — Beam column test setup

Table 1 — Dimensions of Test Specimens



Beam number	d_1, in.	d_2, in.	γ	w, in.	b, in.	t, in.	l_t, in.	$\frac{d^{*(a)}}{w}$	$\frac{b}{t}$	$\frac{l}{r_{y^1}}$	$\frac{l}{r_{y^2}}$	Edge preparation
LB-3	6.0	16.0	1.67	0.105	4.0	0.25	144.0	129	16	74	80	Shear cut
LB-5	6.0	16.0	1.67	0.105	4.0	0.25	96.0	129	16	49	53	Shear cut
LB-6	6.0	16.0	1.67	0.105	4.0	0.25	96.0	129	16	74	80	Shear cut
LB-C-1	6.0	12.0	1.0	0.105	6.0	0.25	120.0	114	24	76	88	Shear cut
LB-C-2	6.0	6.0	0.0	0.105	6.0	0.25	120.0	57	24	74	85	Oxygen cut
LB-C-3	6.0	6.0	0.0	0.105	6.0	0.25	120.0	57	24	74	85	Shear cut
LB-C-4	6.0	12.0	1.0	0.105	6.0	0.25	120.0	114	24	73	84	Oxygen cut
LB-C-5	6.0	18.0	2.0	0.105	6.0	0.25	120.0	171	24	71	82	Shear cut
LB-C-6	6.0	18.0	2.0	0.105	6.0	0.25	120.0	171	24	71	82	Oxygen cut
LB-C-7	6.0	6.0	0.0	0.105	6.0	0.25	120.0	57	24	74	85	Shear cut
LB-C-8	6.0	6.0	0.0	0.105	6.0	0.25	120.0	57	24	74	85	Oxygen cut
LB-C-9	6.0	12.0	1.0	0.105	6.0	0.25	120.0	114	24	73	84	Shear cut
LB-C-10	6.0	18,0	2.0	0.105	6.0	0.25	120.0	171	24	72	83	Shear cut
LB-C-11	6.0	18,0	2.0	0.105	6.0	0.25	120.0	171	24	72	83	Oxygen cut
LB-C-12	6.0	12.0	1.0	0.105	6.0	0.25	120.0	114	24	73	84	Oxygen cut

(a) d* is the depth at 3/4 L for the continuous beams tests and d_2 for the beam-column tests.

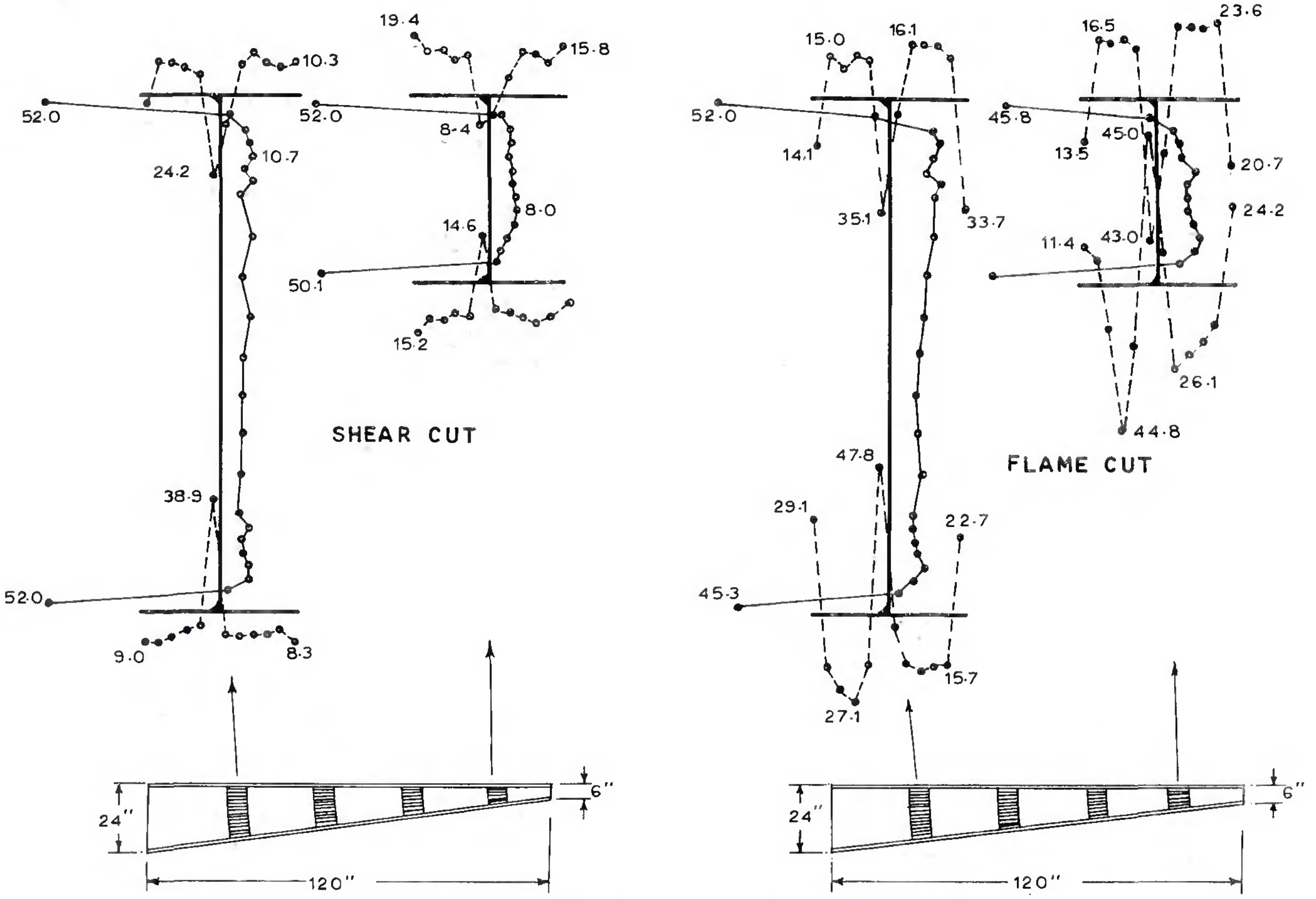


Fig. 3 — Residual stresses (ksi) in one side welded builtup I-sections with shear and oxygen cut edges

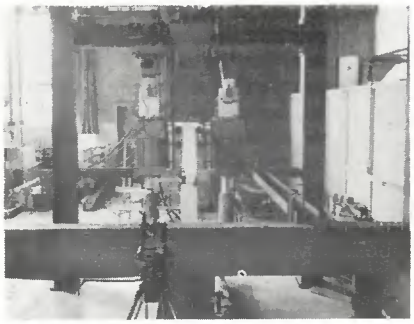


Fig. 4 — A general view of the test setup and the supporting frame

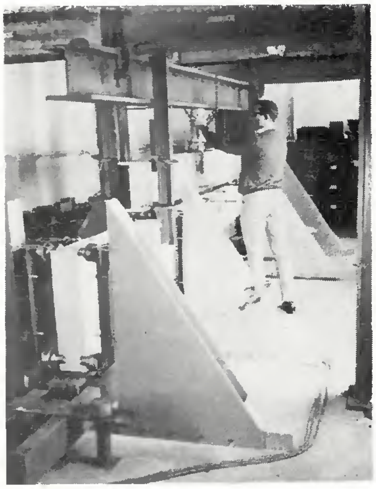


Fig. 5 — The lateral support and loading systems

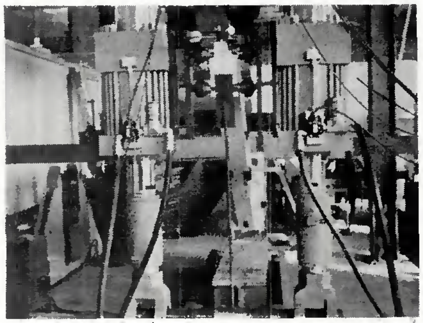


Fig. 6 — Loading end of beam column showing ball-bearing supports and loading link

The second series of tests involved a total of twelve beam columns supported at their deeper end and cantilevered at various angles from the horizontal. In each case the free or loaded end was restrained against lateral motion and twisting. This configuration, shown in Fig. 2, was used to simulate the roof member in a gable frame more closely. Since the inelastic response of such a member is load path dependent, it was felt that an axial force value that changed as the member was loaded was more realistic. Because of this the results obtained here are not directly comparable to the results of previous investigations where the axial force was held constant as the transverse force was increased.

Test Apparatus, Instrumentation and Procedure

The testing facility used is shown in Fig. 4. As shown, two H-shaped frames are rigidly attached to a reinforced concrete test floor and have a single horizontal loading beam spanning between them. For all except the first two tests, the loading was delivered by MTS controlled actuators in the stroke control mode (constant strain rate). The entire system was braced by a system of channels and wire cables.

The Loading System

The loading system employed for the first series of beam tests was as shown in Fig. 5. As shown, a single actuator was positioned between the heavy reaction beam and a shorter loading member. The ram was placed so as to achieve the proper moment conditions in the test specimen, as previously described. In all cases the lower loading beam was braced laterally in such a way as to ensure as close to frictionless vertical motion as possible.

Figures 6 and 7 show the arrangement that was used to test the cantilevered beam columns. The pair of actuators that were used were identical to one another and positioned so that they were equidistant from each side of the free end of test specimen. In this way they served as the basis for the lateral support of the free end of the beam column. The actuators were themselves braced horizontally to insure vertical motions of their pistons. The load was delivered from the actuator to the free end of the specimen by an adjustable 3/4 in. diam bar. This bar connected a special loading eye welded to the beam column and a 3½ in. diam bar that passed through the movable yokes of the actuators (Figs. 4, 6 and 7).

The Lateral Supporting System

As noted previously the first series of tests was conducted with the tapered members supported laterally at their ends and quarter points. The system used to provide this lateral support is shown in Fig. 5 and 8. As shown, a specially made 2 in. diam heavy duty ball bearing was placed at each of the four flange tips of the cross-section at the point where lateral bracing was required. The bearing surface for the ball bearing was a machined and hardened 1/8 in. tool steel plate backed up by a 1 in. thick steel plate. The thicker plate was mounted in such a way that it could be accurately positioned in the proper vertical plane. The entire bearing assembly was supported by steel reinforced timber and plywood frames which were attached to the test floor. Subsequent measurements indicated that the lateral deformation of these frames was negligible under load. Vertically adjustable rollers were used as supports at the ends of the beams to ensure that the top flange of the test specimen was horizontal.

The tapered specimens tested in the second series were all cantilevered from a specially designed "L" frame that was fastened to the test floor. This arrangement is shown in Fig. 7. Lateral support for the free end of these members was provided for in a manner similar to that employed in the first series of tests. The only difference was that for this series of tests the bearing plates were mounted on the actuators rather than on timber frames as in the first series. This is shown in Fig. 6.

Instrumentation

All vertical deformations were measured using standard dial gages. These measurements were made at center and at each load point for the first series of tests and at the free end only for the second series. Horizontal deformations were also measured at the free end for each member in the second series of tests. Also, measurements were taken to indicate any deformation in the supporting assemblies.

Lateral deformation of the test specimen was measured using a finely divided rule placed as a horizontal projection of the flange and read with an engineers transit. The lateral deformation of just the compression flange was measured in the first series of tests. In the second series of tests however, lateral deflections of both flanges were recorded.

Strains at several points were recorded automatically by electrical resistance strain gages and a digital read-out device. In each test gages were placed at the midpoint of the member near the flange tips, on the flange over the weld both top and bottom and at the mid-depth of the section. Strain gages were also placed over the weld 3 in. from the fixed support in all beam column tests (series 2).

The first beam tested in the first series was loaded using a standard hydraulic jack. Hydraulic pressure measurements were taken and then converted into loads. In each of the remaining tests, the load was delivered by one or more MTS stroke controlled actuators. The load magnitude was read from a digital display connected to the actuator load transducer. Actuator stroke measurements

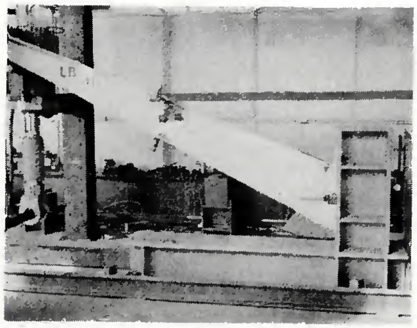


Fig. 7 — "L" shaped supporting frame for the beam-column tests

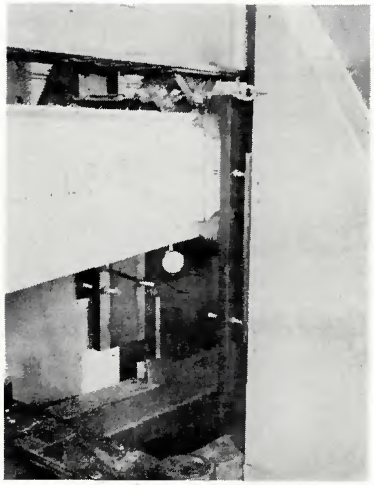


Fig. 8 — Close-up of lateral support and knife edge

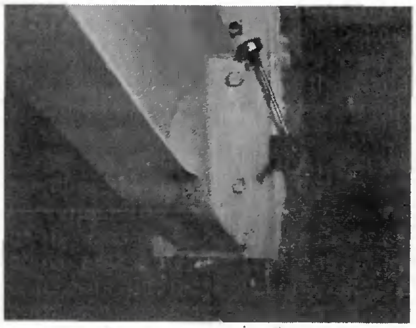


Fig. 9 — Local flange and web buckling near the fixed end of a beam column

were also taken and used to regulate the rate of strain. These readings could not be used for strain measurements, however, because they involved the deformation of both the test specimen and the test facility.

The Testing Procedure

Prior to positioning the member to be tested in the test frame, all strain gages were applied, it was given a coat of whitewash and all of the lateral restraint ball bearings installed. For the beam tests the lateral bracing frames were then positioned and the laterally adjustable bearing plates on one side accurately aligned with a transit. For the beam column tests, one of the single pair of bearing plates was aligned. The test specimen complete with its ball bearings was then placed in position and set to proper vertical alignment. At this point, the adjustable bearing plates on the other side of the member were tightened up against the projecting ball bearings. A continuing check was made on the alignment and plumbness of the plate. Points along the center of the top flange were also checked for alignment. The ball bearing assembly was designed to provide a small amount of lateral adjustment. Using this fine adjustment, the points of support on both sides of the beam were set parallel to the web plate and the center of the flange at each of these points was set to within 1/16 in. from line. Finally, the loading device was put in place and adjusted and a check made on all of the bracing and mechanical instrumentation. A check was also made to ensure that all of the strain gages were functioning properly.

With all parts of the test set up properly adjusted, a load of approximately 10% of the theoretical yield load (neglecting residual stress effects) was slowly applied and then removed. The purpose of this loading was to "seat" the system and to check for any movement in the loading and bracing system and to check for friction. The loading and unloading curves were compared and any necessary adjustments made.

Initially, there was some concern about frictional losses that could develop in the lateral bracing system, particularly for the beams tested in the first series. In addition to requiring a total of sixteen ball bearings for the lateral support of the test member, lateral support was also required for the loading system. The beam column tests on the other hand, required only four bearings. Upon investigation it was found that these losses did not merit consideration.

The actual testing was conducted at a controlled rate of strain. This was necessary because of the large deformation corresponding to a small increment in load when the load is near to the ultimate for the member. With this type of control in effect, deflections were increased in increments calculated to produce approximately 10% of the theoretical yield load for the particular member being tested.

This loading sequence was continued until noticeable yielding occurred. From that point on, the load corresponded to that produced by 1/2 in. increments of deflection. Measurements of load level versus time were taken for each 1/2 in. increment of deflection. The next increment was introduced only after the load for the previous increment showed no change with time. This procedure was continued until a clear ultimate was obtained. The appearance and progression of any yield lines were noted as were any conditions of local instability.

Experimental Results

In general, all of the experimental results obtained for the members tested followed a similar trend. Because of their method of fabrication, all of the specimens were, to some degree, warped and their flanges curved. This initial lateral deflection of the flange resulted in a more or less monotonically increasing lateral and twisting motion of the test specimen as the load was increased. Because of this steadily increasing lateral deformation, no sharply defined lateral buckling load could be detected using the transit alone. In a few tests, however, the strain gages that were mounted at the flange tips did indicate a significant change in strain at a particular load even though the transit did not.

The results of previous studies involving the lateral buckling of rolled, prismatic beams (Ref. 7) indicated that the onset of lateral buckling did not in itself, greatly influence the load carrying capacity of the member. Large angles of twist were necessary before there was any significant loss in strength. To produce these large twist angles, it was necessary that the member possess a large capacity for rotation about the strong bending axis. The main effect of the lateral motion and twist was to cause local buckling to occur prematurely. In all cases, local buckling in the flange was the limiting factor in the carrying capacity of the member.

The response of the tapered members tested in this program was very much the same as the response of the prismatic members described above and in Ref. 7. The main difference was that local buckling occurred with considerably less strong axis rotation. In most cases lateral buckling occurred shortly after noticeable lateral deflections were observed. Figures 9 and 10 show typical buckling failures for a beam column (series 2) and a beam test. It should be noted that the values of the geometric parameters b/L and d/w (see table 1) are higher than those permitted by the AISC Plastic Design Code for prismatic members. This accounts for the very noticeable lack of rotation capacity shown by these tapered members.

The results of all of the tests are summarized in Table 2 and typical test curves are shown in Figs. 11 and 12. Figure 11 illustrates the results for one of the beam tests from the first series. As indicated by the graph

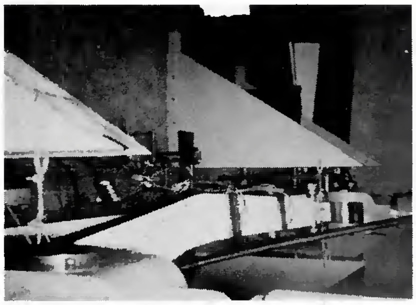


Fig. 10 — Lateral buckling of a continuous beam

of load versus strain gage reading at the flange tip, the lateral buckling load is approximately 32 kips. Also shown in this figure are curves relating both vertical and lateral deflections to load. Note that the ultimate load as defined by the local buckling is only slightly higher than the lateral buckling load. While the small strong axis rotation capacity shown does not detract from the members suitability from a working stress point of view, it does make it unsuitable for plastic design.

Figure 12 shows typical results obtained for one of the cantilever or beam-column tests. These results are seen to be very similar to those obtained for the beam tests of the first series. The dashed line in the vertical deflection curve represents an analytically defined solution for the same member and will be discussed in a later section.

One of the stated goals of the test program was to determine the effect of the method of cutting the plates on their response to load. To this end

Table 2 — Summary of Test Results

(a) Continuous beams

Beam no.	Interior bracing and loading points	β	Critical span length, in.	Test values at ultimate: Total load, kips	Vertical deflection, in.	Lateral deflection, in.	Remarks
LB-3	¼ L and ¾ L	0.28	72.0	39.0	1.04	0.10	lateral buckle
LB-5	¼ L and ¾ L	0.28	48.0	50	0.83	0.08	local flange buckle
LB-6	¾ L	0.0	72.0	46	0.64	0.61	local flange buckle

(b) Beam columns

Beam no.	Centroidal dimensions: $(d_o)_c$, in.	$(d_L)_c$, in.	l_c, in.	Pitch α deg	Theoretical ultimate load, kips	Test values at ultimate: Load, kips	Vertical deflection, in.	Lateral[(a)] deflection, in.	Remarks
LB-C-1	6.0	11.8	120.0	0	11.2	8.7	3.3	0.12 R	local flange buckle
LB-C-2	6.0	6.0	116.5	30	5.5	4.9	4.7	0.25 R	lateral, local flange buckle
LB-C-3	6.0	6.0	116.5	30	5.5	4.3	4.8	0.15 R	local flange buckle
LB-C-4	6.1	11.8	114.7	30	11.8	10.0	2.8	0.30 L	local flange buckle
LB-C-5	6.2	17.5	112.8	30	26.0	15.0	2.0	0.45 R	local flange buckle
LB-C-6	6.2	17.5	112.8	30	26.0	16.4	2.2	0.50 R	local flange buckle
LB-C-7	6.0	6.0	116.5	20	5.1	4.1	7.2	0.15 R	local flange buckle
LB-C-8	6.0	6.0	116.5	20	5.1	3.9	7-10	0.05 R	local flange buckle
LB-C-9	6.1	11.8	115.4	20	11.8	9.3	3.5	0.45 R	lateral, local flange buckle
LB-C-10	6.1	17.5	114.0	20	26.5	14.3	2.3	0.55 R	lateral, local flange buckle
LB-C-11	6.1	17.5	114.0	20	26.5	14.0	2.3	0.35 L	lateral, local flange buckle
L8-C-12	6.0	11.8	115.9	10	10.4	8.3	3.5	0.55 R	local flange buckle

(a) R = the direction away from the unwelded side of the web.
L = the direction away from the welded side of the web.

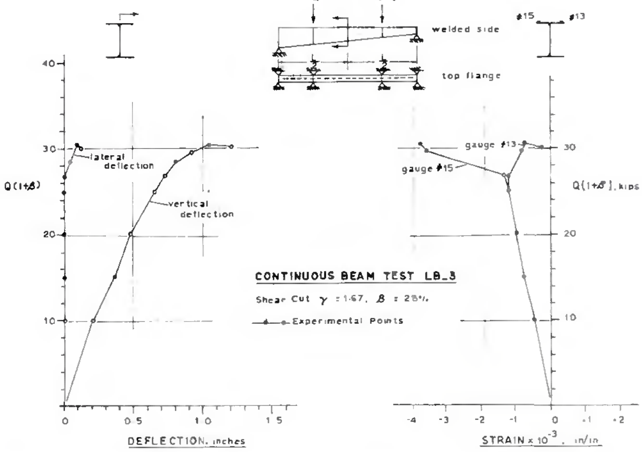


Fig. 11 — Test results of a continuous beam: LB-3

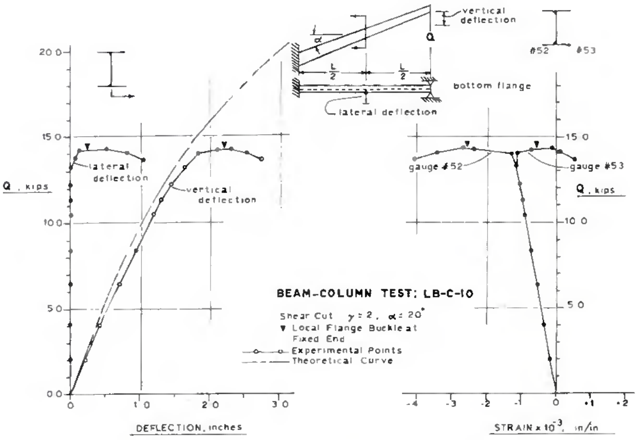


Fig. 12 — Results of beam column test: LB-C-10

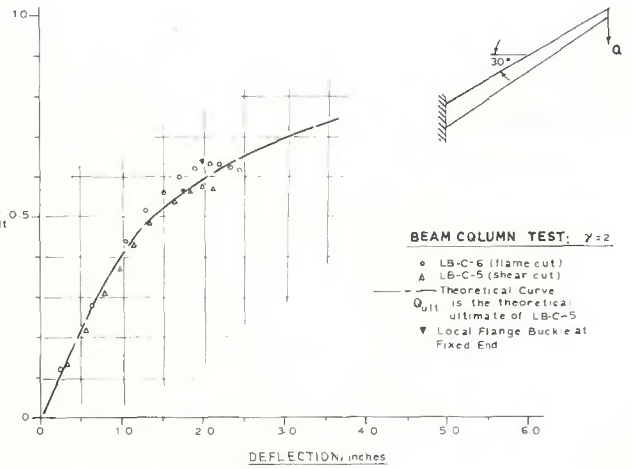


Fig. 13 — Beam column test result comparing shear and oxygen cut edge for two identical beams (LB-C-5 and LB-C-6)

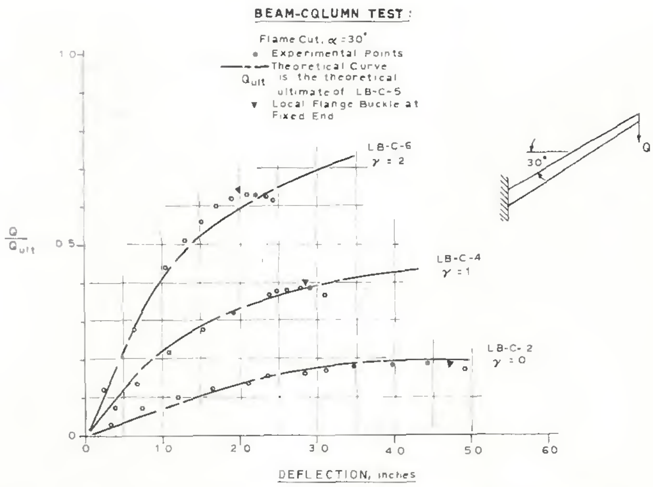


Fig. 14 — Beam column test results comparing three different tapers (LB-C-2, LB-C-4 and LB-C-6)

two sets of specimens that were geometrically identical were tested in exactly the same manner. The members were different in that the plates from which one set was fabricated were oxygen cut while those for the second set were shear cut. In each case the plates were cut from the same location on a larger plate. The points shown in Fig. 13 are typical of the results obtained for this study. As shown, the circles represent results for oxygen cut edges while the triangles are for shear cut edges. It can be seen that the strength of the shear cut specimen is lower than that of the oxygen cut specimen. This is due to the different residual stress patterns resulting from the alternate fabrication procedures. Since the oxygen cut specimen has a more favorable residual stress distribution (see Fig. 3), it would be expected to have a higher inelastic buckling strength. The same type of response was noted for each of the pairs of specimens tested. As in Fig. 12, the dashed line describes the analytical prediction of the inplane response for this member.

Figure 14 shows the results obtained for three different shear cut specimens, each tested at an angle of inclination of 30 deg and each having a different angle of taper. The load indicated in each case is normalized with respect to the computed ultimate load of the largest member. The lower curve represents the response of a welded prismatic member, the middle curve is that for a member whose large end depth is twice that of the small end while the top curve describes the response of a member having a depth ratio of d_L/d_s of three. In each case, the small end depth was the same. As expected, as the depth ratio increases, the load carrying capacity also increases. The onset of local buckling however, is seen to occur in reverse order.

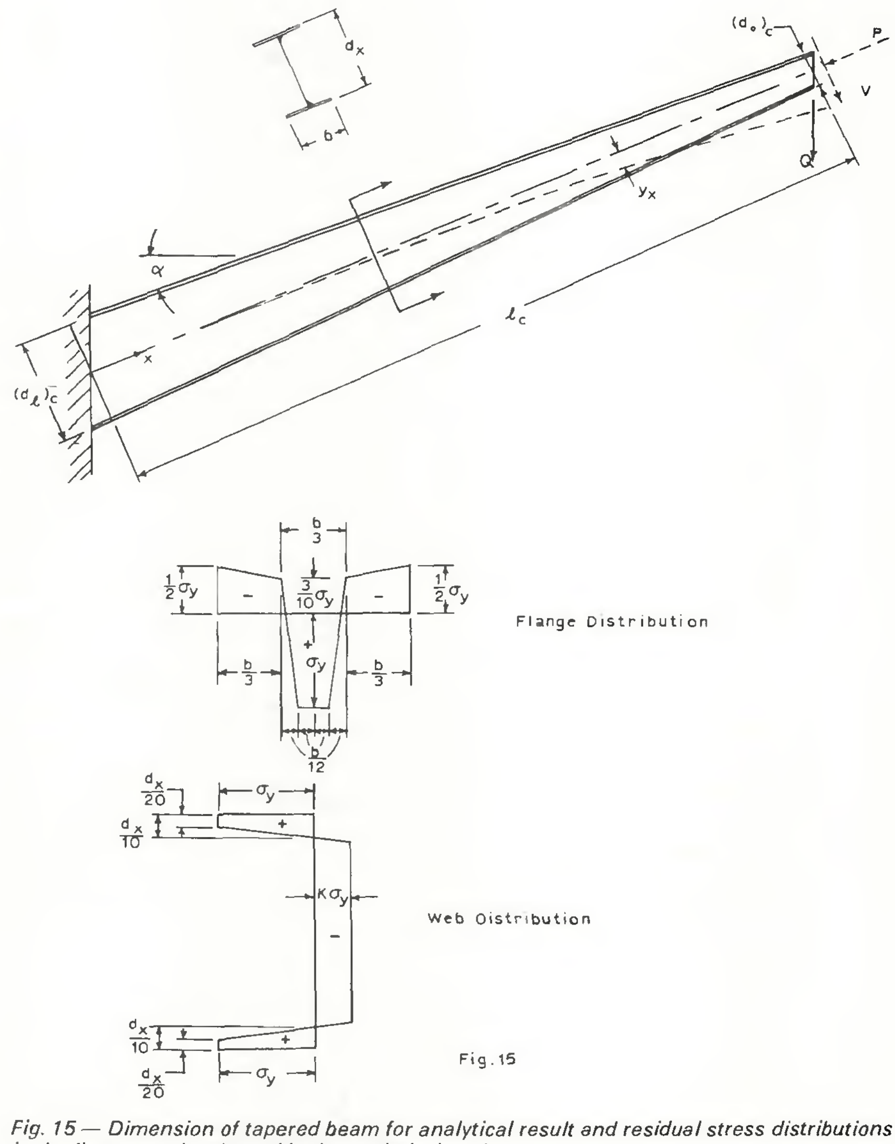


Fig. 15 — Dimension of tapered beam for analytical result and residual stress distributions in the flanges and web used in the analytical study

Analytical Considerations

At the time the testing program was begun, no adequate analytical procedure to predict the in-plane behavior of a tapered beam column was available. Since such solution would be very useful in interpreting test results, at least up to the point of first instability, the development of a suitable analytical procedure was undertaken as an integral part of the project.

The Moment Curvature Relationships

The key to analytically defining the in-plane bending behavior of a tapered beam column lies in the availability of suitable moment-curvature relationships. With these relationships available, the bending deformation could be obtained by a simple double integration process. The major effort of this part of the study was therefore devoted to the development of these families of curves.

The procedure used was an adaptation of that used by Ketter, Kamsinsky, and Beedle (Ref. 8) and later extended by Fukumoto (Ref. 9). The cross-section is first broken up into a large number of sub-areas and a stress resultant, based on an assumed stress strain law and residual stress pattern, is calculated for each. The algebraic sum of these stress resultants gives the value of the axial load while their first moments about the centroidal axis yields the bending moment. Using this procedure, families of dimensionless moment-curvature-axial load (M-Ø-P) curves were prepared for several points along the length of the tapered member.

The stress strain law that was used was of the Ramberg-Osgood form. That is,

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \frac{3}{7}\left(\frac{\sigma}{\sigma_y}\right)^N$$

where N was assumed to have a value of 100. The residual stress used was an idealized average of measured residual patterns for members with shear cut edges and is shown in Fig. 15.

A comparison of the computed M-Ø-P curves for each of the members considered indicated that the family of curves shown in Fig. 16 was representative of the entire set of tapered members. These curves were therefore used in all subsequent computations. To facilitate the use of these curves in the computer program written to define the deformation of the tapered member, an expression of the form

$$\frac{\phi}{\phi_y} = \frac{M}{M_y} + B\left(\frac{M}{M_y}\right)^T$$

was used. The quantities of B and T in this expression are functions of both the axial load and the depth of the member. By fitting this equation to each part of the series of curves in Fig. 16, the B and T surfaces shown in Fig. 17 were developed. Points on these surfaces defined the required values of ϕ/ϕ_y.

The computer program that was developed to compute the beam column deformation was nothing more than a step by step integration of the moment-curvature relationship. The structure considered and its corresponding model are shown in Fig. 15. The assumptions involved are:

1. Simple beam column action
2. The difference in length between the top and bottom flanges was not considered
3. The displacements computed are considered to be normal to the centroidal axis of the member.

The actual solution procedure used was iterative. The process was begun by computing the bending moment of the first node point from the deeper end of the member. The ratio P/P_y was also determined and from these the curvature calculated. Assuming that this curvature was constant over the small distance to the first node point the deformation at the first node was found. The process was repeated for all nodes up to and including the free end of the member.

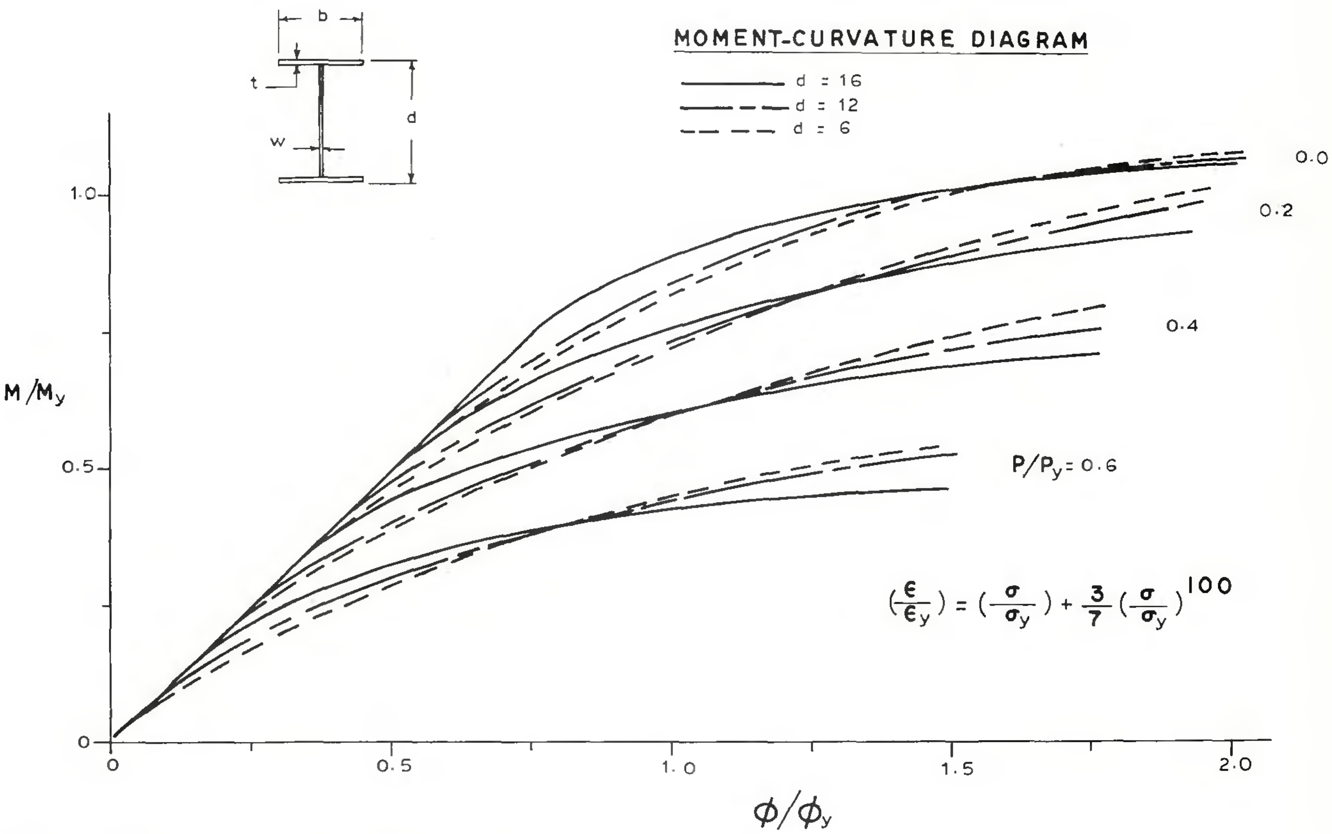


Fig. 16 — Typical moment-curvature diagram of tapered members at various depths

A second cycle was next initiated in which the entire process was repeated starting again at the deep end. This time, however, the value of the bending moment at each node point was modified for the effect of the axial load and the previously computed deformation. The new deformations so defined were then compared with those from the previous cycle. If the comparison was favorable, the process was terminated. If they were not sufficiently alike another cycle was started. Upon convergence, the displacements and the loadings were recorded.

The results of several of these solutions that correspond to experimental test results are shown in Fig. 12, 13, and 14. In each of these cases the in-plane deformational behavior predicted analytically agrees quite closely with that defined experimentally. Since the computational procedure contains no provision for buckling, this agreement terminates when buckling effects predominate over bending effects. The ultimate load defined by the analytical solution is therefore meaningless, and the true ultimate for the member is defined by post buckling behavior of one kind or another.

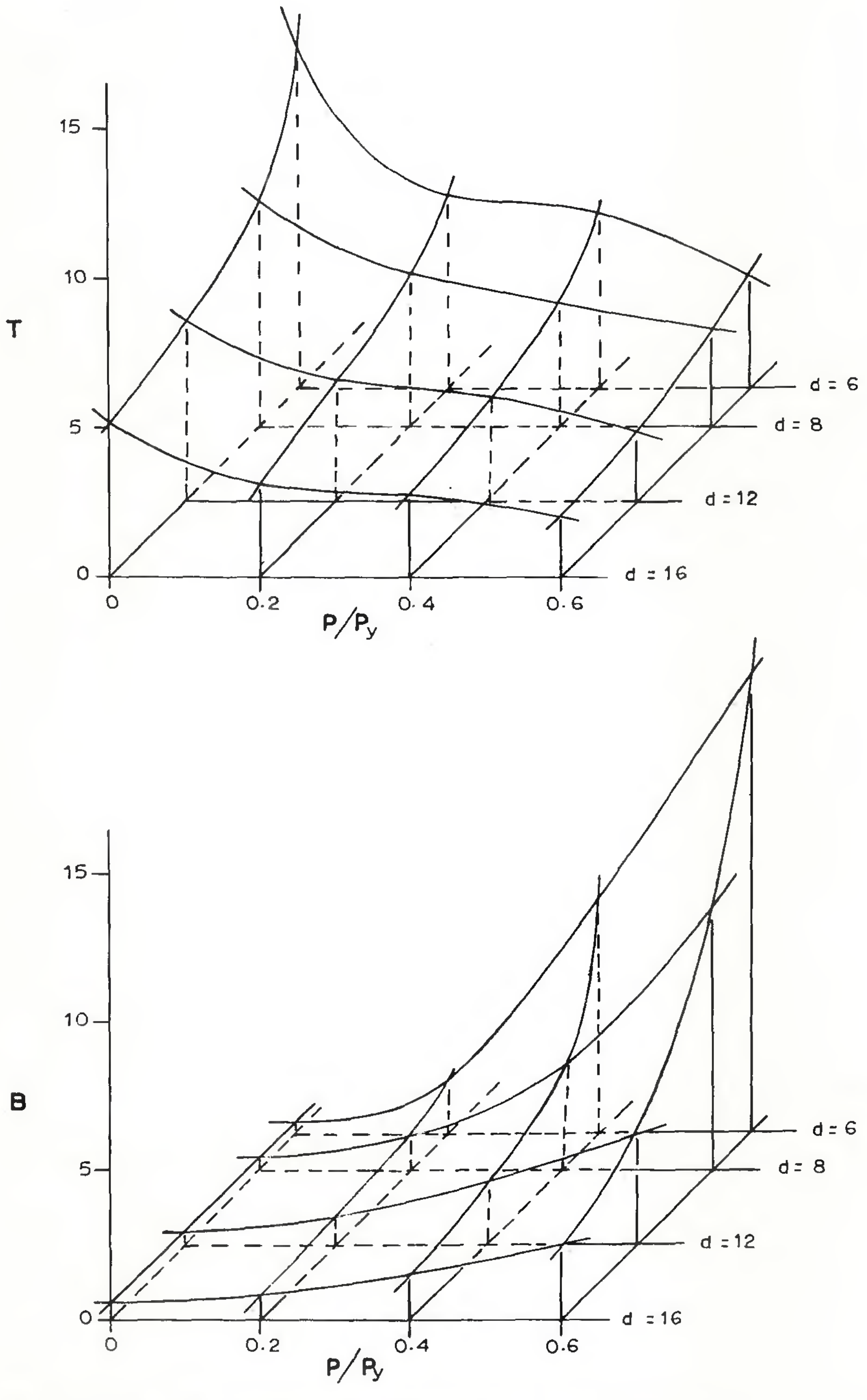


Fig. 17 — "T" and "B" surfaces used in the moment-curvature relationship

Concluding Remarks

The following may be concluded from the results of this study:

1. The procedure used to cut the plate during the fabrication of the tapered members has a decided effect upon the inelastic response of the member. This is because of the different residual stress patterns that are developed. Oxygen cut members appear to have a higher inelastic bending stiffness and correspondingly a higher inelastic lateral buckling strength.

2. Attaching the flanges to the web by welding on only one side of the web produces initial lateral deflections by forcing the flange to bow away from the side of the weld. The flange attached last had a larger initial lateral deformation. No very sharply defined lateral buckling load could be established because lateral bending was present at all load levels.

3. In all of the beam column tests local buckling in the compression flange near the deep end of the member led directly to failure. The rotation capacity needed for lateral buckling to have a significant effect on the load carrying capacity of the member was not delivered and it was very much less than that which would be expected from a rolled shape. The larger the angle of taper, the more pronounced was the local buckling effect.

4. Fabrication by one side welding does not seem to influence the static strength of laterally supported members if they are proportioned to satisfy local buckling requirements.

5. The analytical procedure developed for the program satisfactorily predicted the in-plane bending behavior of the tapered member up to the point at which buckling occurs.

Acknowledgment

The study reported herein was a part of a research program on tapered structural members that was conducted at the State University of New York at Buffalo. It was jointly sponsored by the Navy Facilities Engineering Command, American Institute of Steel Construction, American Iron and Steel Institute, and the Metal Building Manufacturing Association. Technical guidance was provided by the joint task committee on tapered members of the Column Research Council and the Welding Research Council. The committee members are: A. Amirikian (Chairman), D. J. Butler, T. R. Higgins, R. L. Ketter, K. H. Koopman, C. F. Larson, G. C. Lee, L. W. Lu, W. A. Milek, N. W. Rimmer and I. Viest.

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