Assessing the Significance of Flaws in Welds Subject to Fatigue

Use of fracture mechanics analysis for fatigue is considered and examples are given of applications to some practical types of weld joints and flaws

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Summary. It is now widely recognized that flaws will inevitably exist in welded structures and the old idea of removing all detectable defects must be replaced by the 'fitness for purpose' design philosophy. This makes it necessary to define reliable methods of assessing the significance of flaws, particularly in the context of fatigue, a fracture mechanism critically affected by flaws. The most promising approach to this problem lies in the use of the fracture mechanics based description of fatigue crack propagation. This paper is concerned with the practical application of fracture mechanics to the problem of fatigue of welded joints containing cracks or defects.

Introduction

The attitude of fabricators and designers to flaws in welded structures has changed over the past few years. The automatic reaction in the past of repairing all regions found to contain flaws is being replaced by the 'fitness for purpose' design philosophy, whereby flaws which would not bring about premature failure may be ignored. There are several reasons for this change in attitude but the principal ones are widespread recognition of the facts that all welds contain flaws and that the chances of detecting them depend on the sensitivity of the nondestructive inspection method; that repair is expensive and may be unnecessary; that repair may introduce more harmful flaws, such as cracks arising from repair welds made under difficult conditions.

This paper is concerned with flaw assessment in the context of fatigue, a fracture mechanism critically affected by the presence of flaws. The problems of presenting information about the fatigue behavior of flaws of varying sizes in the form of simple data is discussed. The main part of the paper deals with the application of fracture mechanics to this problem, with particular reference to both surface and buried planar flaws.

From the general point of view, the principles discussed should be applicable to any material. However, data presented all refer to structural C-Mn steels.

Problem of Flaw Assessment

The fatigue properties of many welded joints can be represented on the conventional S-N diagram where, for a given material and joint type, applied stress is plotted against endurance. Data presented in this way and analyzed statistically have been used to provide design S-N curves (Ref. 1). The value of the S-N diagram for estimating fatigue life depends on the fact that, in many cases, all relevant variables apart from applied stress, such as material and geometric stress concentration, remain approximately constant. Variations in these 'constants' contribute to scatter in S-N data and reduce the confidence with which lives can be predicted.

In most welded joints, certainly the lower strength joints which fail from the weld toe or root, fatigue failure initiates at flaws (Refs. 2,3). In many cases, the severity of the relevant flaw does not vary enough from joint to joint to limit the value of the S-N diagram. However, the severity of some flaws can vary considerably. In such cases, data represented on an S-N diagram would be of value only if the severity of the flaw could be expressed simply or specified. For example, if both the joint type and size of flaw were specified. This approach has proved to be possible in the cases of butt welds with uniform porosity (Ref. 4), where severity can be characterized simply in terms of its volume, and slag inclusions in arc welded joints, where the severity of the flaw depends on both its depth and length but may be characterized by the length since the depth does not vary much (Ref. 5). Thus, comparisons between the expected fatigue life of the flaw, the fatigue lives of other welded details present and the required service life can be made easily. Flaw accep-

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tance criteria based on such an approach are being prepared currently for inclusion in a British Standard.

In general, planar flaws cannot be treated in the simple way used to consider porosity and slag inclusions. The severity of the flaw depends not only on its size but also its shape and location, all of which may vary from one weld to the next. In order to provide a simple means of comparing the fatigue behavior of such flaws with that of other weld details, some means of relating the relevant variables is required.

An additional problem which may invalidate the use of an S-N curve is the use, or introduction by welding, of low toughness materials. Most fatigue data are obtained from tests of relatively small specimens in which failure is defined as the time when the net section stress reaches the ultimate strength of the material. The resulting life is usually approximately the same as the number of cycles required to produce a through-section crack. In practice, if the material toughness is not sufficient to tolerate a through-thickness crack, the critical crack size being much less than the section thickness, premature fracture would occur.

In order to use the 'fitness for purpose' approach to planar flaw acceptance, a reliable method of assessing the fatigue behavior of flaws is required. Ideally, this method should be capable of taking into account variations in stress concentration, initial flaw dimensions and critical crack size. Clearly, it is the propagation of flaws as fatigue cracks which must be considered and the fracture mechanics based description of fatigue crack propagation offers a promising method of analysis.

Fatigue Crack Propagation Law

The fracture mechanics parameter ΔK, the stress intensity factor range, provides the means of representing fatigue crack propagation (da/dN) in relatively simple mathematical terms. In its simplest form, the relationship is

$$da/dN = C(\Delta K)^m \quad (1)$$

where C and m are material constants and ΔK is a function of applied stress range, crack size and geometry. Since a given value of the stress intensity factor is proportional to the stress at the tip of any crack, the relationship between da/dN and ΔK may be regarded as a law of crack propagation relevant to any geometry of cracked body, provided factors not related to ΔK, such as material or environment, remain constant. Such a law provides the means of solving the problems discussed above.

Later, it will be seen that a crack propagation law of the form of Eq. 1 has wide practical application for analyzing cracks in both welded and unwelded structures. However, several factors can affect the constants in the equation and even the form of the relationship, which may be more complex than Eq. 1. Before attempting to use the law, it is important to appreciate how and why it may change.

Method of Obtaining Basic Data

Although many attempts have been made to deduce a law of fatigue crack propagation theoretically (Refs. 6-8) none have been found that agree with observed crack propagation behavior. Therefore, the crack propagation relationship is deduced from test data. Since the relationship between ΔK and loading incorporates corrections for geometry, theoretically such data may be obtained using any convenient specimen design. However, in practice, the data obtained can be affected by geometry, as illustrated for center-cracked plate specimens in Fig. 1 (Ref. 9). In that case it turns out that only region 2, which obeys Eq. 1, is of real practical value in the context of fatigue cracks associated with welded joints, because the plane strain fracture mode is appropriate. Furthermore, at the extremes of very low ΔK values, close to the threshold value at which da/dN is zero (Refs. 10,11), and high K_{max} values, close to K_c when da/dN accelerates, Eq. 1 no longer holds (Ref. 12).

Effect of Material Properties and Microstructure

Clearly, rate of fatigue crack propagation depends on the material through which the crack passes. However, for a given material, it is virtually insensitive to changes in microstructure and mechanical properties, unless they give rise to changes in fracture characteristics such that the 'normal' fatigue process (striation formation) is accompanied or replaced by a 'static' mode of fracture (Refs. 13,14). In the context of welded joints in steel, the uniformity of crack propagation behavior in a number of weld metals [yield strengths ranging from 386 to 636 N/mm² (56 to 92 ksi)], heat affected zones and base metal [C-Mn steel with yield strength of 375 N/mm² (55 ksi)] is shown in Fig. 2 (Ref. 15). These data include a hard HAZ (1100 C heat treatment which gave hardnesses up to 369 HV5)

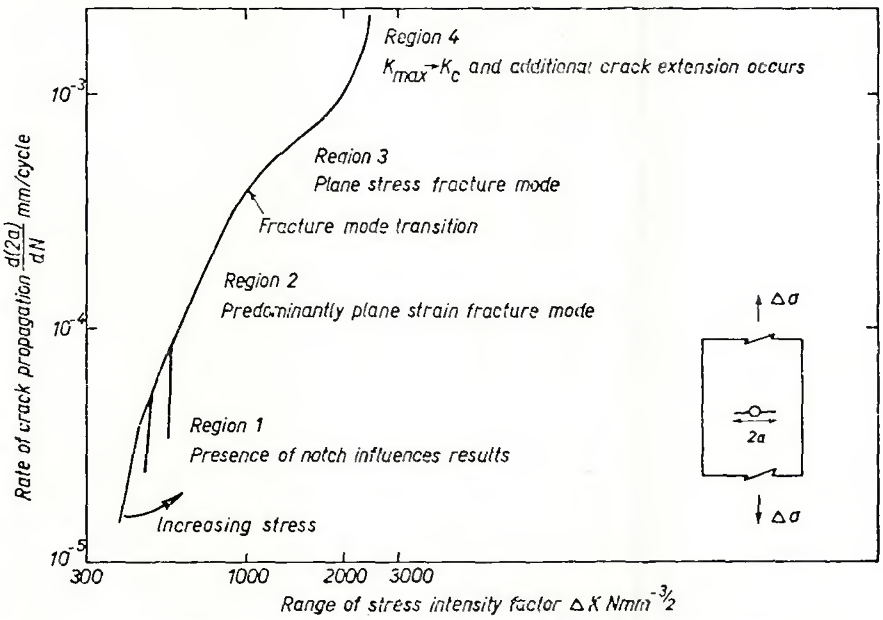

Fig. 1 — Relation between crack propagation curve and fracture characteristics for a center-cracked plate

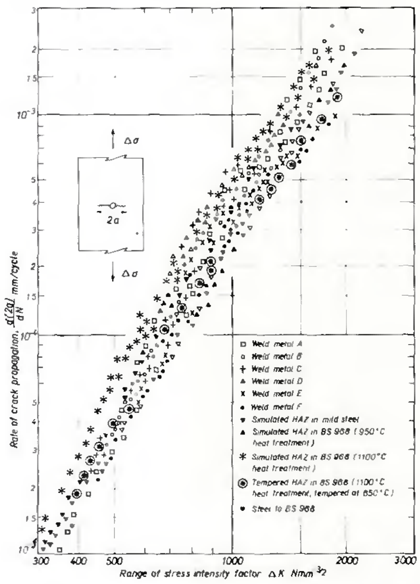

Fig. 2 — Fatigue crack propagation data for structural C-Mn steel weld metals, HAZs and base metals (see Reference 15 for description of materials)

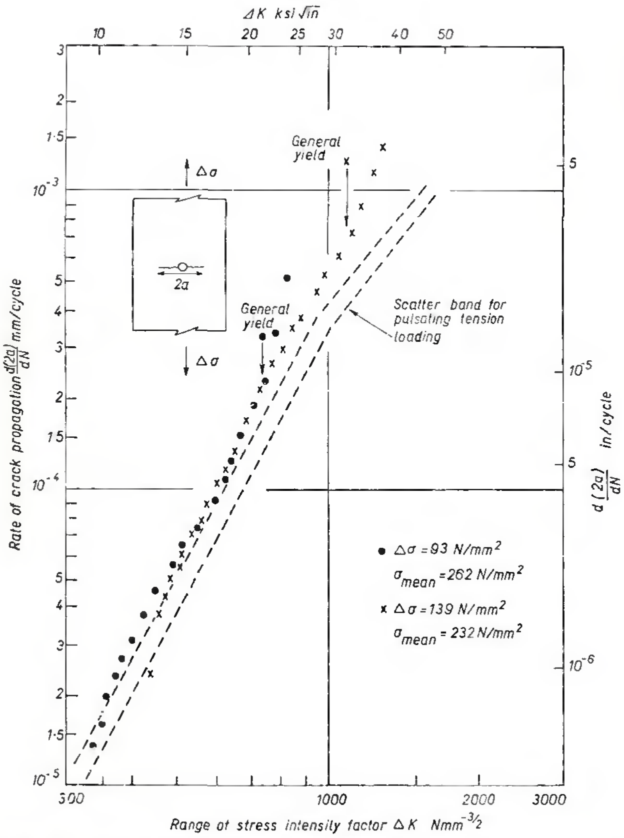

Fig. 3 — Effect of high maximum stress (corresponding to 0.8 × yield) on fatigue crack propagation in C-Mn steel to BS 4360 Grade 50B

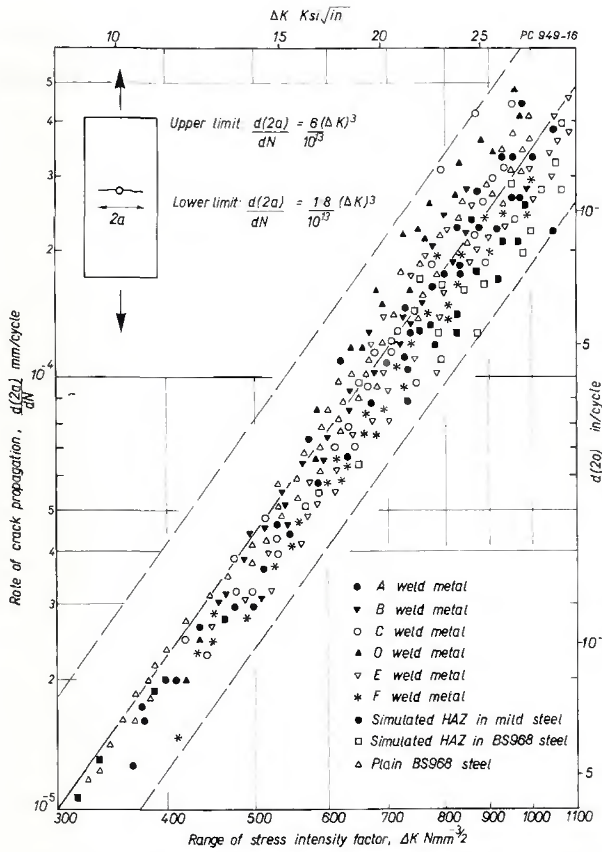

Fig. 4 — Plane strain fatigue crack propagation data (from Fig. 2)

which illustrates the possible effect of microstructure (Ref. 16). It fractured partly in an intergranular manner, as a result of the presence of martensite, giving a higher rate of crack propagation than the same material after tempering. Although in good welding practice every attempt is made to ensure that hard HAZs are not produced, their occurrence may be unavoidable. Recently, a similar effect was observed in the context of fatigue cracks propagating from lamellar tears in the plane of rolling (unpublished). Bursts of lamellar tearing during fatigue cycling increased the rate of crack propagation. Finally, some weld metals have exhibited 'static' fracture modes during fatigue and given increased rates of crack propagation (Ref. 17). As well as providing an increase in rate of crack propagation (i.e., affecting C in Eq. 1), the occurrence of 'static' fracture modes may increase m (Ref. 11).

Effect of Mean Stress Ratio on Residual Stress

Although fatigue crack initiation may depend critically on mean stress, generally fatigue crack propagation does not, particularly for fully tensile loading (Ref. 18). This is demonstrated convincingly for a C-Mn structural steel by the data in Fig. 3 (Ref. 9). However, in the context of steels, recent work indicates that the feature which introduces sensitivity to microstructure, discussed above, also introduces sensitivity to mean stress in crack propagation (Refs. 10,11). The 'static' fracture mode — intergranular, microvoid coalescence, cleavage, lamellar tearing, etc. — depends on K_{max}, so that overall rate of crack propagation depends on mean stress. If rate of propagation is sensitive to mean stress, the form of the crack propagation relationship may not adopt the simple form of Eq. 1; a number of relationships which include allowance for mean stress sensitivity have been proposed (Refs. 18-21).

In the context of as-welded structures, the inevitable presence of high tensile residual welding stresses (Ref. 22) simplifies consideration of applied mean stress. Residual stresses are often of yield stress magnitude and acting in the direction of subsequent stressing. Thus, they are superimposed on any applied stress cycle and, in simple terms, the result is that the effective stress consists of the applied stress range cycling downwards from yield (Ref. 1). Clearly, for such welded structures, only crack propagation under high tensile mean stresses is relevant. Thus, assuming that microstructural effects do not introduce 'static' modes of fracture, a single law of crack propagation of the form of Eq. 1 should be widely applicable in the context of fatigue of welded joints.

Effect of Environment

Although the subject is not discussed, it must be noted that fatigue crack propagation can be extremely sensitive to environment. Most fatigue test data are obtained in air at room temperature; in some environments it would be unsafe to use such data to predict fatigue behavior.

Outline of Fracture Mechanics Analysis of Fatigue

In the light of the above discussion, the simple crack propagation law, Eq. 1, should be widely applicable to fatigue of as-welded joints in structural steels. The constants m and C would be determined from crack propagation tests of the relevant material.

ΔK is proportional to $\Delta\sigma\sqrt{\pi}a$, where $\Delta\sigma$ is the nominal stress range remote from the crack, and may be written

$$\Delta K = Y\Delta\sigma\sqrt{\pi}a \quad (2)$$

where Y is a correction term, which may or may not be constant, depending on the geometry of the cracked detail. The crack length term, a, is the total length of an edge or surface crack or half the length of a central or buried crack.

Combining Eq. 1 and 2

$$da/dN = C(\Delta\sigma)^m(Y\sqrt{\pi}a)^m \quad (3)$$

This is a differential equation which

can be integrated between initial and final crack sizes to provide a mathematical description of any part of the fatigue life of a crack (Ref. 23). Thus

$$\int_{a_i}^{a_f} \frac{da}{(Y\sqrt{\pi a})^m} = C(\Delta\sigma)^m N \qquad (4)$$

If Y, m and C are known, Eq. 4 can be used to calculate the fatigue behavior ($\Delta\sigma$ v N relationship) of a flawed weld (a_i known) which fails at a known crack size (a_f).

It will be noted that if a_i and a_f are constant for a particular weld detail, the correction term Y is also constant, so that the integral becomes a constant. Thus, Eq. 4 becomes

$$(\Delta\sigma)^m N = \text{a constant} \qquad (5)$$

This is the form of the equation of an S-N curve for a welded joint (Ref. 1) which would not be regarded as flawed in that the defects present, such as weld toe crack-like intrusions (Refs. 2,3), are unavoidable. A recent analysis of fatigue test data for welded joints (Ref. 1) confirmed the expected approximate equality between the index m in Eqs. 1 and 5, where Eq. 1 referred to crack propagation in structural steels and Eq. 5 referred to S-N data for steel fillet welded joints which failed from the toe. As already noted, the fact that the parameters a_i, a_f and Y are not constants for a given weld type, contributes to scatter in fatigue test data.

The term Y is never a function of absolute crack size but may depend on the ratio of crack size to section size. Thus, Eq. 4 may be expressed generally as

$$\int_{(a/B)_i}^{(a/B)_f} \frac{d(a/B)}{\left(Y\sqrt{\frac{\pi a}{B}}\right)^m} = C(\Delta\sigma)^m B^{(m/2)-1} N \qquad (6)$$

where B is the total section width for an edge or surface cracked plate or half the section width for a central or buried cracked plate, in each case measured in the same direction as 'a.' Thus, a/B = 1 represents through-section cracking in each case.

In addition to the analysis of individual flaws, Eqs. 4 and 6 may be used to derive a general analysis of any kind of flaw, in any joint, for any failure criterion. Such a method of analysis provides the means of correlating fatigue data obtained from dissimilar welds, for example welds with different flaws. Referring to the integral as I, Eq. 6 may be rewritten

$$\left[\Delta\sigma\left(\frac{B^{(m/2)-1}}{I}\right)^{1/m}\right]^m N = 1/C = \text{const} \qquad (7)$$

or $(\Delta\sigma^*)^m$ N = a constant

$$\text{where} \quad \Delta\sigma^* = \Delta\sigma\left(\frac{B^{(m/2)-1}}{I}\right)^{1/m} \qquad (8)$$

and is referred to as the generalized stress parameter (Ref. 24).

The similarity between Eqs. 5 and 8 will be noted. The slope of a $\Delta\sigma^*$ - N plot would be the same as that for an S-N plot but the method of presentation of results would be superior because a greater number of relevant variables ($\Delta\sigma$, a_i, a_f and Y) can be specified. Furthermore, in theory for a given material (m and C constants, with C adopting values corresponding to the limits of scatter of crack propagation data) a single $\Delta\sigma^*$ v N scatterband would be applicable for any geometry of welded joint which failed from a flaw, including those low fatigue strength joints, chiefly fillet welds failing from the weld toe, for which the life consists of propagation (Refs. 23,24). Clearly, in practice $\Delta\sigma^*$ could only be used in cases where the initial flaw size and the factors affecting Y could be determined with reasonable accuracy.

Examples of Analysis

In order to apply the general theory outlined in the previous section in a practical situation, two pieces of information are required: a law of crack propagation (constants m and C in Eq. 1) and the means of calculating ΔK for the crack of interest (Y in Eq. 2).

Fatigue cracks associated with welded joints may propagate through any of the three regions, weld metal, heat-affected zone and base metal, and different crack propagation laws may be appropriate for each region. However, in the context of welded joints in structural C-Mn steels, the results in Fig. 2 indicate that a single crack propagation law could be used for any of the regions. This simplifies the analysis considerably. The plane strain data (region 2 in Fig. 1) in Fig. 2, excluding the results for hard HAZ, are given in Fig. 4. Scatter limits drawn at a slope of m=3 are also shown. In fact, regression analysis of the data indicated that m was 3.07 (Ref. 15) but approximating this to 3 simplifies the analysis without seriously affecting the final results. Comparing Figs. 3 and 4, it will be noted that the results obtained at high mean stresses fall within the scatter limits. Thus, they should be sufficiently wide to allow for the presence of tensile residual welding stresses. The resulting crack propagation law is

$$da/dN = C(\Delta K)^3 \qquad (9)$$

where $C = 0.9 \times 10^{-13}$ and 3×10^{-13} (da/dN in mm/cycle, ΔK in $Nmm^{-3/2}$).

A severe limitation to the application of fracture mechanics for fracture analysis of actual structures is that the relationship between stress intensity factor, loading and geometry is unlikely to be known accurately. This problem arises partly because of the difficulty in determining the relevant details about the size and geometry of flaws detected in structures. However, from the fundamental point of view, the main problem is that a solution from K may not exist. In welded joints, flaws may be irregularly shaped and situated in complex stress fields like the toe of a

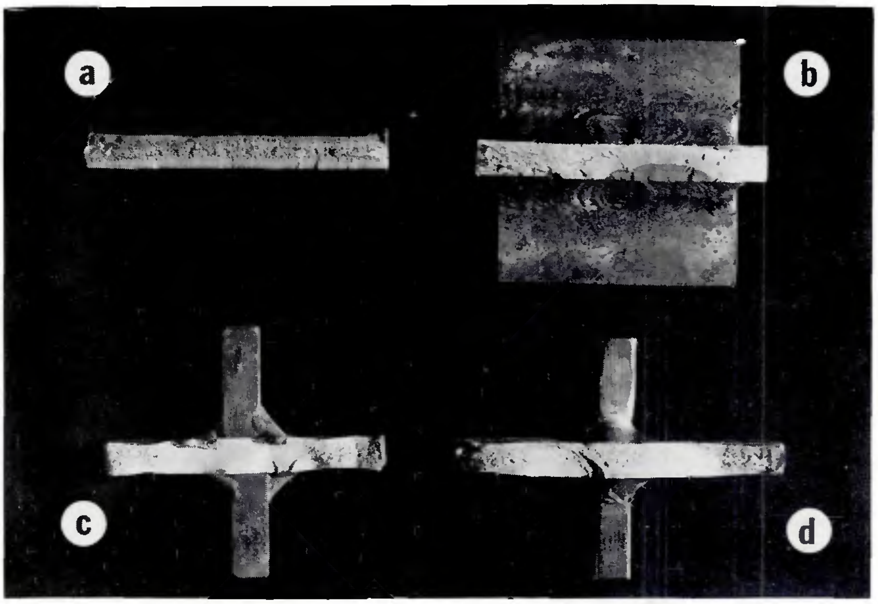

Fig. 5 — Fractures in joints which fail from the toe of the weld

weld. In contrast, solutions for K in the literature are confined mostly to regular shaped crack situations in simple stress fields. An additional complication is that flaws associated with welds are part embedded (surface) or fully embedded (buried) and few solutions for such cracks are available. The triaxial crack tip stress field associated with embedded cracks increases the difficulties of determining K. Relevant solutions are introduced in the analyses which follow.

Fatigue Failure from Toe of Fillet Weld

Fatigue cracks which propagate from the toe of a weld, and indeed from most surface stress concentrations, adopt a semi-elliptical profile. Some examples of actual cracks are shown in Fig. 5, while Fig. 6 shows a sketch of a crack. In Reference 25 a solution for the stress intensity factor of a semi-elliptical surface crack situated at the toe of a weld was determined. The correction term Y (see Eq. 2) depends on the crack shape a/2c, the crack depth a/B and the stress concentration effect of the weld geometry. The general solution may be written

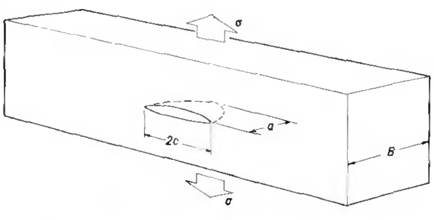

Fig. 6 — Semi-elliptic surface crack in a plate in tension

$$K = \frac{M_s M_t M_k}{\Phi_o} \sigma\sqrt{\pi a} \qquad (10)$$

where

M_s is the correction for crack front shape and depends on a/2c

Φ_o is the complete elliptic integral (obtained from standard tables) and also depends on a/2c

M_t is the correction factor for crack front position and depends on a/B and a/2c

M_k is the correction to take account of weld profile stress concentration

It will be noted that no correction is made for crack tip plasticity. In fatigue it is rare that such a correction is required since the cyclic crack tip plastic zone is small, due to reversed yielding (Ref. 26).

Figure 7 shows the curves relating $M_s M_t/\Phi_o$ and a/B. Curves relating M_k and a/B as calculated for 30 and 45 deg fillet welds and estimated for 60 deg welds, (Ref. 25) are given in Fig. 8.

From Figure 7 it will be noted that if a/2c is greater than 0.2, $M_s M_t/\Phi_o$ could be regarded as a constant. However, in practice surface fatigue cracks associated with welded joints adopt a/2c values which, at least in the important early stages of crack propagation, are lower than 0.2 (Ref. 24).

The above solution for K was used in conjunction with the crack propagation law with m=3 to correlate fatigue data obtained from welded joints which contained cracks of known sizes. The specimens were plates with longitudinal stiffeners which were fillet welded around their ends (joint (d) in Fig. 5) loaded with R=O. Failure, defined as through-thickness cracking, occurred after crack propagation from the toe of the weld at the end of the stiffener. The specimens were precracked and the fracture surface marked with soap solution. Stains produced in this way are shown on fracture surfaces (b) to (d) in Fig. 5. Crack depths between 0.1 and 4.8 mm in specimens 12.7 mm (0.5 in.) thick were produced. $\Delta\sigma$ *was calculated for each specimen, taking into consideration the weld angle, which was measured, and the expected crack shape variation based on a close study of failure in this specimen type (Ref. 9,24). The results are given in Fig. 9, together with predicted scatter limits based on the crack propagation data in Fig. 4. It will be seen that all the data fell within the predicted limits. Thus, individual test results could have been predicted using Eq. 6, to an accuracy corresponding to the scatter limits, and test data obtained from unlike specimens (initial crack size, crack shape and weld profile varying) are successfully correlated using $\Delta\sigma^*$.

Since K was not based on a closed-form solution, the integration in the above analysis was carried out graphically. The resulting calculations were simplified with the aid of a diagram relating the integral, I, with the initial crack depth $(a/B)_i$ for $(a/B)_f=1$. Such a diagram, which includes curves for constant values of a/2c, 0 and 0.1, and 45 deg fillet welds, is shown on logarithmic scales in Fig. 10. If a different failure criterion is applicable, for example failure at a/B less than 1, the same diagram could be used, since for $(a/B)_i=x$, I for $(a/B)_f=y$ is equal to I for $(a/B)_i=x$ minus I for $(a/B)_f=y$. It will be noted that as long

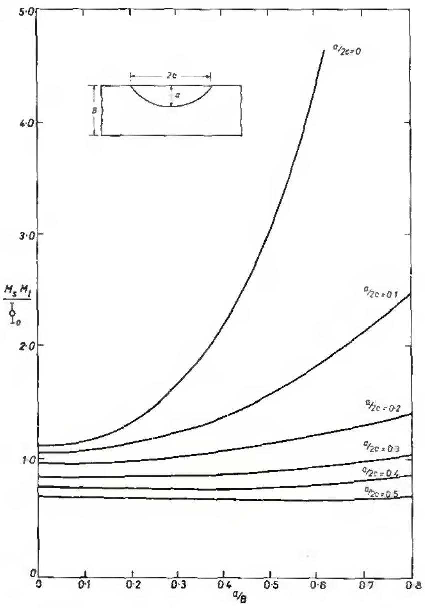

Fig. 7 — Stress intensity correction term $M_s M_t/\Phi_o$ for surface crack as a function of crack depth

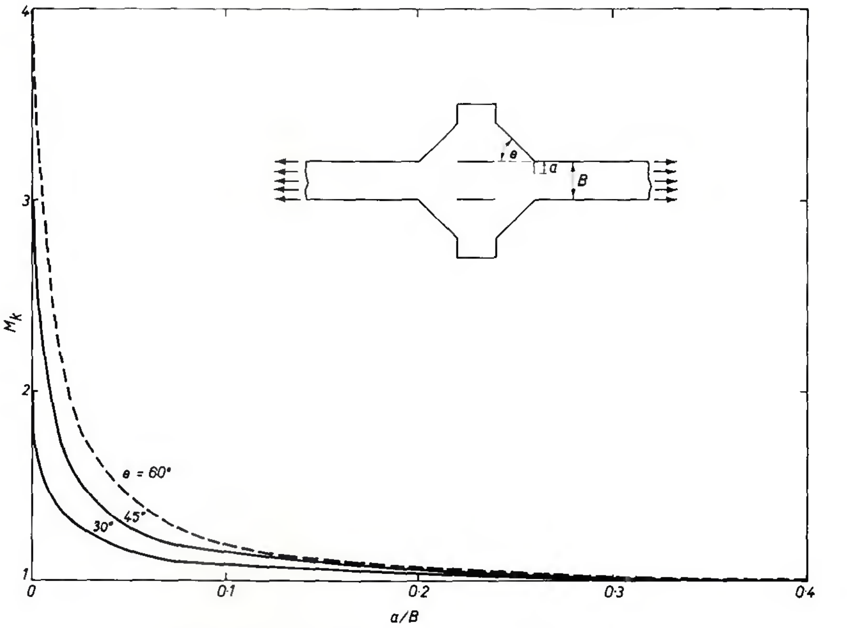

Fig. 8 — Stress intensity magnification due to stress concentration, M_k. Solid line from finite element analysis (Ref. 9), broken line estimated (Ref. 25)

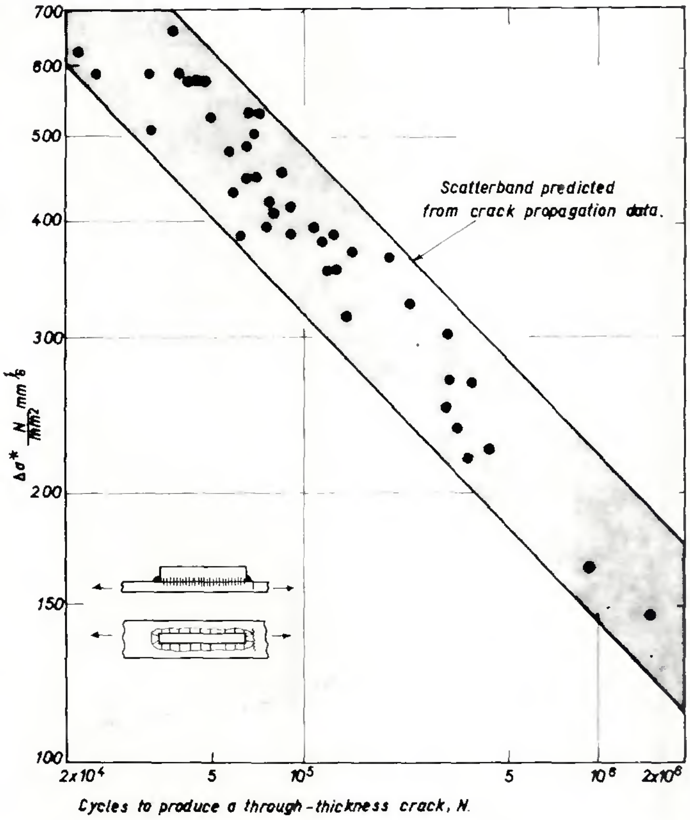

Fig. 9 — Comparison of predicted and actual fatigue results for fillet welds containing weld toe cracks

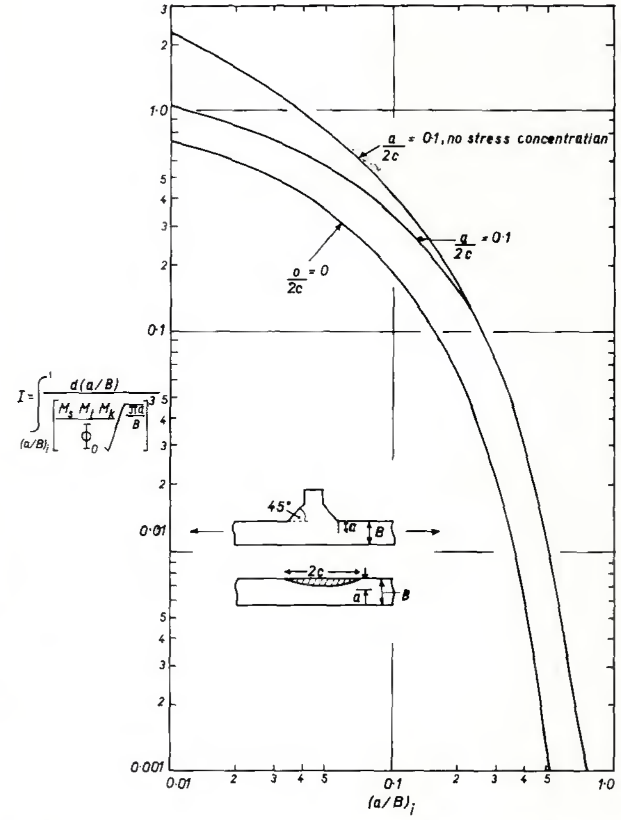

Fig. 10 — Crack propagation integral for surface cracks as a function of initial crack size

as $(a/B)_f$ is not close to $(a/B)_i$, the choice of a failure criterion is not critical. Figure 10 also shows a curve for the case of a surface crack which is not situated in a field of stress concentration. Comparison with the curve for a crack at the toe of a fillet weld illustrates the effect of the stress concentration which, as would be expected, is greatest for small initial crack depths.

Fatigue Failure from Root of Transverse Load-Carrying Weld

Transverse load-carying welds of the type which occur in tee or cruciform joints can fail in the longitudinally stressed plate, from the weld toe, or in the weld from the root. In the former case, S-N data exist (Ref. 1). However, in the case of weld failure, fatigue strength depends on the weld size to plate thickness ratio and the depth of weld root penetration. Thus, the use of $\Delta\sigma^*$ offers the best means of correlating fatigue data.

Harrison (Ref. 27) analyzed cruciform joints using fracture mechanics some years ago. However, his analysis was based on approximations regarding both the solution for K and the crack propagation law which are known to be incorrect.

Frank (Ref.28) determined a solution for K specifically for a crack at the root of a weld using the finite element method. For the geometry shown in Fib. 11 with θ = 45 deg, the normal assumption in design, and $T_p = T_c$ he found that the following closed form solution gave a reasonable approximation for K for $0.2 < (H/T_p) < 1.2$

$$K = \frac{\sigma_p}{1 + (2H/T_p)} \left[A_1 + A_2\,(a/W)\right] \left[\pi a \sec(\pi a/2W)\right]^{1/2} \qquad (11)$$

where σ_p is the stress in the plate and the coefficients A_1 and A_2 are

$A_1 = 0.528 + 3.287(H/T_p) - 4.361(H/T_p)^2 + 3.696(H/T_p)^3 - 1.874(H/T_p)^4 + 0.415(H/T_p)^5$

$A_2 = 0.218 + 2.717(H/T_p) - 10.171(H/T_p)^2 + 13.122(H/T_p)^3 - 7.755(H/T_p)^4 + 1.785(H/T_p)^5$

The solution used by Harrison was similar to Eq. 11 without the term $[A_1 + A_2\,(a/W)]$. This term can be of the order of 2 so that Harrison's solution underestimated K.

The crack propagation law based on the data in Fig. 4, that is with m = 3, should be applicable in the present analysis. Harrison assumed m = 4; at that time it was thought that a value of m = 4, as originally proposed by Paris and Erdogan (Ref. 29), applied for a large number of materials.

Substituting Eq. 11 into Eq. 6, with B replaced by W, and integrating between the limits, $(a/W)_i$ and $(a/W)_f$,

$$\int^{f} (U/V)^3 d(a/W) = I \qquad (12)$$

$$= C\sqrt{W}(\Delta\sigma_p)^3 N$$

where

$U = 1 + 2H/T_p$ and

$V = [A_1 + A_2(a/W)]\,[(\pi a/W)\sec(\pi a/2W)]^{1/2}$

and

$$\Delta\sigma^* = \Delta\sigma_p\,(\sqrt{W}/I)^{1/3} \qquad (13)$$

The integral was evaluated by computer for various values of H/T_p and $(a/W)_i$ for $(a/W)_f = 1$. The results are presented graphically in Fig. 12.

Results in the literature (Refs. 28,30-38) were used to determine $\Delta\sigma^*$, N data and they are plotted in Fig. 13. The figure also shows the predicted scatter limits, as already given in Fig. 9. The results are confined to those obtained under fully tensile loading; most results were obtained with R = 0 (minimum stress = 0). In addition, results obtained under loading which gave stresses close to yield in the weld metal or for which $(a/W)_i$ was greater than 0.7, the limit for Frank's theoretical analysis, were rejected.

It will be seen that over a large range of endurances there is good correlation between the data in Fig. 13 and that the predicted and actual results are in close agreement.

Approximating Fatigue Strength of Fillet Weld

Both analyses described above made use of detailed information about crack sizes and shapes in order to determine K. Their value lies in the fact that they illustrated that the theory was sound and that there was potentially a good correlation between crack propagation data and the fatigue behavior of joints containing flaws. In practice, it would be uncommon to have precise details about the geometry of the crack whose significance in service is to be assessed.

In order to illustrate the accuracy with which predictions can be made with limited knowledge of the cracked joint, Eq. 6 has been used to calculate the S-N curve for the latter joint (above) for the case of failure from the weld toe.

In this case it may be expected that the crack, which initiates along a length of weld toe, will adopt a low a/2c value. The fracture shown in Fig. 5(b) illustrates this. In the present analysis it will be assumed that a/2c lies between 0 and 0.1. In addition, the investigation of crack-like weld toe defects, which exist in as-welded joints of the kind considered here, by Signes et al (Ref. 2) indicated that the average depth was 0.15 mm. The welds examined were on 12.7 mm (0.5 in.) thick plates. Thus, in the present analysis it is assumed that $a_i = 0.15$ mm, B = 12.7 mm, $(a/B)_i = 0.12$, a/2c = 0 or 0.1 and the weld profile angle is 45 deg. Thus, using Eq. 6, with m = 3 and C taking the value corresponding to the centerline of the data in Fig. 4 (1.64×10^{-13}), and Fig. 10, the resulting S-N curves are

$(\Delta\sigma)^3 N = 1.2 \times 10^{12}$ for a/2c = 0

and $(\Delta\sigma)^3 N = 1.72 \times 10^{12}$ for a/2c = 0.1

These curves are plotted with actual S-N data in the literature, as given in Reference 1, in Fig. 14. It will be seen that, although the mean S-N curve is underestimated by both predicted equations, the correct order of magnitude is predicted.

Practical Applications

The results presented in Fig. 9 and 13 show that the generalized stress parameter $\Delta\sigma^*$ provides the means of correlating the fatigue behavior of flaws. In practice it may be necessary to compare the fatigue behavior of two different flaws, such as a crack at the toe of a transverse fillet weld and a region of incomplete penetration at the root of a fillet welded cruciform joint. Suppose that in the case of the cruciform joint the relevant parameters (see Fig. 11) are

$$T_p = 25 \text{ mm}, H = 15 \text{ mm} \therefore H/T_p = 0.6$$

$$2a_i = T_p W = 27.5 \text{ mm} \therefore a_i/W = 0.45$$

Thus, from the results in Figue 12, I = 0.16 and $\Delta\sigma^* = 1.46\ \Delta\sigma_p$. Suppose that in the case of the toe crack, the plate thickness (B) is 25 mm and the crack depth (a_i) is 5 mm. Then, $(a/B)_i$ = 0.2 and, from Fig. 10 for a/2c=0.1, I = 0.15 and $\Delta\sigma^* = 1.49\ \Delta\sigma_p$. Thus, $\Delta\sigma^*$ is approximately the same in each case. In other words the crack-like flaw 25 mm deep in the cruciform joint is equivalent to a 5 mm deep crack at the toe of a fillet weld for the crack shape assumed.

The comparison of flaws considered above may be extended to provide information which could be used to determine the optimum design of transverse load-carrying welds. The joint may fail in the weld or the plate, as already discussed. In simple terms, the mode of failure depends on the relative dimensions of the weld and plate cross-section; the greater the ratio of weld load-carrying area to plate cross-section, the smaller the chance of failure in the weld. The maximum fatigue strength for the joint corresponds to plate failure and appears to be relatively insensitive to weld size. Therefore, it follows that the optimum design of weld is such that there is an equal chance of failure taking place in the plate and weld. Using the analyses developed earlier, it is an easy matter to determine this condition by considering the two equations which represent the fatigue strengths for the two modes of failure and solving them for a given stress and endurance.

In the case of weld failure, the fatigue strength of the joint is given by Eq. 7, i.e.,

$$(\Delta\sigma^*)^3 N = 1/C \qquad (14)$$

Taking the upper limit value of C, which approximately corresponds to the lower limit of data in Fig. 13, and expressing $\Delta\sigma^*$ in terms of $\Delta\sigma_p$ (see Eq. 13), Eq. 14 becomes

$$(\Delta\sigma_p)^3 N (\sqrt{W}/I) = 3.33 \times 10^{12} \qquad (15)$$

In the case of plate failure, the fatigue strength of the joint is represented by the equation of the S-N curve, as plotted in Fig. 14. Regression analysis of the data (Ref. 1) gave a slope of 2.92 compared with the expected value (corresponding to m, as discussed earlier) of 3. The corresponding mean S-N curve and 95%

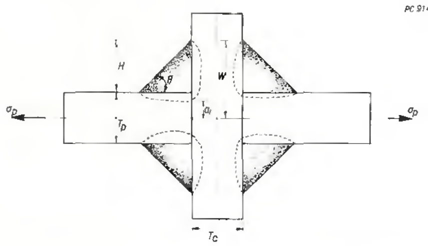

Fig. 11 — Geometry of cruciform welded joint

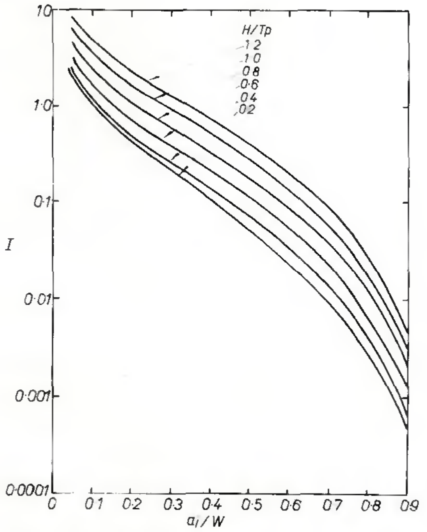

Fig. 12 — Crack propagation integral for weld crack in cruciform joint as a function of initial flaw size

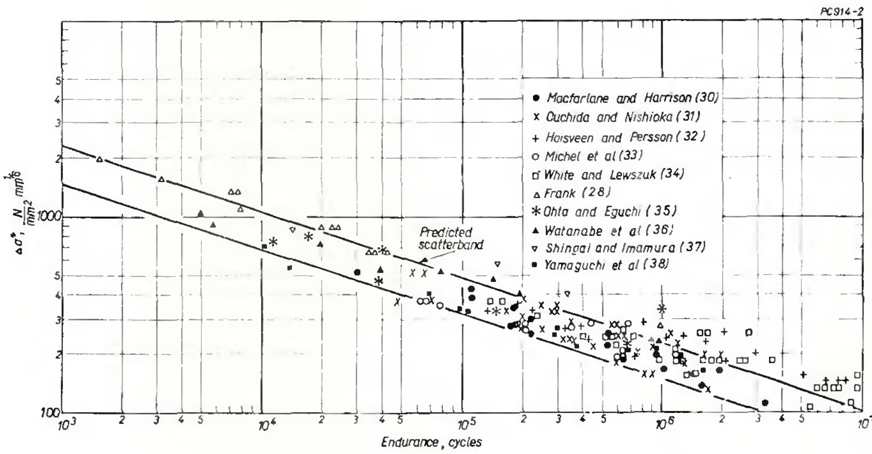

Fig. 13 — Comparison of predicted and actual fatigue data for cruciform joints failing in the weld

confidence limits are given in Fig. 14. For the purposes of the present analysis, a slope of 3 is assumed and a new lower limit with a slope of 3 has been drawn in Fig. 14. The equation corresponding to this approximate lower 95% confidence limit is

$$(\Delta\sigma_p)^3 N = 6.3 \times 10^{11} \quad (16)$$

The optimum weld design is that which allows both Eq. 15 and Eq. 16 to be satisfied for given values of $\Delta\sigma_p$ and N, i.e.,

$$3.33 \times 10^{12} (l/\sqrt{W}) = 6.3 \times 10^{11} \quad (17)$$

or $\sqrt{W}/l = 5.3$

Equation 17 defines the requirements regarding geometry for the optimum design of transverse load-carrying weld. It will be noted that the same value of $\sqrt{W}/l$ would be obtained if, for example, a safety factor had been applied to the fatigue strength relationships, providing the same factor were used in each case.

In practice, it may or may not be possible to use Eq. 17 directly, depending on the information available. For example, if the plate thickness (T_p) and weld size (H) are known, it is an easy matter to determine a_i/W, and hence the required weld root penetration, using Fig. 12. However, if the weld penetration is specified, together with the plate thickness, it is not possible to determine the weld size, H. In order to increase the scope of the present analysis, the information given in Fig. 12 can be re-analyzed in the light of Eq. 17 to give curves relating H/T_p and a_i/W for given values of T_p, in the following way.

1. From Fig. 11, $W = H + T_p/2$
2. Specify H/T_p and hence obtain W in terms of T_p
3. Specify T_p and hence obtain W
4. From Eq. 17, $l = \sqrt{W}/5.3$, so that l can be calculated.
5. From Fig. 12, a value of a_i/W corresponding to the calculated value of l and the specified value of H/T_p can be determined.
6. Hence, one point on a graph of H/T_p v. a_i/W for a given value of T_p is obtained. By repeating the procedure for other values of H/T_p and T_p a series of curves can be plotted.

For practical convenience it may be desirable to determine a_i in terms of the plate thickness, T_p, rather than W, for example as the ratio of total initial crack size to plate thickness $2a_i/T_p$. Values of a_i/W may be converted to this ratio using the following equation

$$2a_i/T_p = [(2H/T_p) + 1](a_i/W)$$

Values of $2a_i/T_p$ for the range of H/T_p values considered in Fig. 12 have been calculated for a range of plate thicknesses and smooth curves drawn, as shown in Fig. 15.

For comparison, a curve which is based on Harrison's analysis for a 25 mm (1 in.) thick plate is also included in Fig. 15. It will be seen that the results are in quite close agreement.

The results in Fig. 15 extend to weld geometries in which $2a_i$ exceeds the plate thickness. The purpose of including such geometries is not to encourage the designer to specify such welds, but to allow for the fact that such a situation might arise in practice.

In practice, although the use of optimum weld design could be preferable, it may not be possible. For example, avoidance of the risk of lamellar tearing may limit the permissible fillet weld size. In such cases, Eq. 15 could be used to calculate the fatigue strength for a particular weld geometry.

Conclusions

The fracture mechanics crack propagation law provides a powerful method of dealing with the problem of flaw assessment in welded joints. The analyses developed allow the designer much greater flexibility than present design methods based on simple S-N data, because significant variables other than applied stress and general geometry can be taken into consideration. Essentially, it allows the fatigue process to be entered (variation in initial flaw size) and left (variation in failure criterion) at will and variations in geometry and crack shape to be taken into account. The value of the method has been confirmed by accurately predicting the fatigue behavior of fillet welds containing flaws on the basis of the generalized stress parameter $\Delta\sigma^*$. The accuracy of the analysis relies heavily on the accuracy of the stress intensity factor solutions and future work is required to refine existing solutions and provide new ones for crack types not considered.

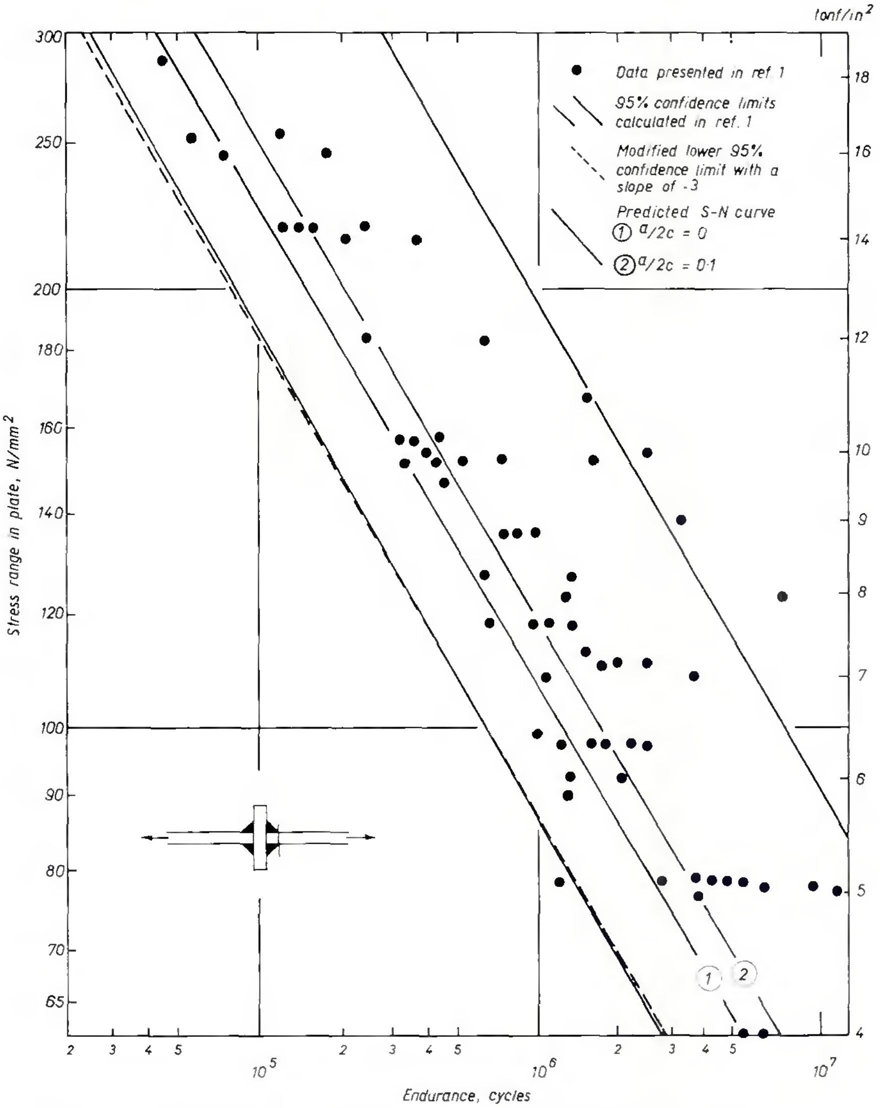

Fig. 14 — Fatigue test data for cruciform joint failing in the plate from the weld toe

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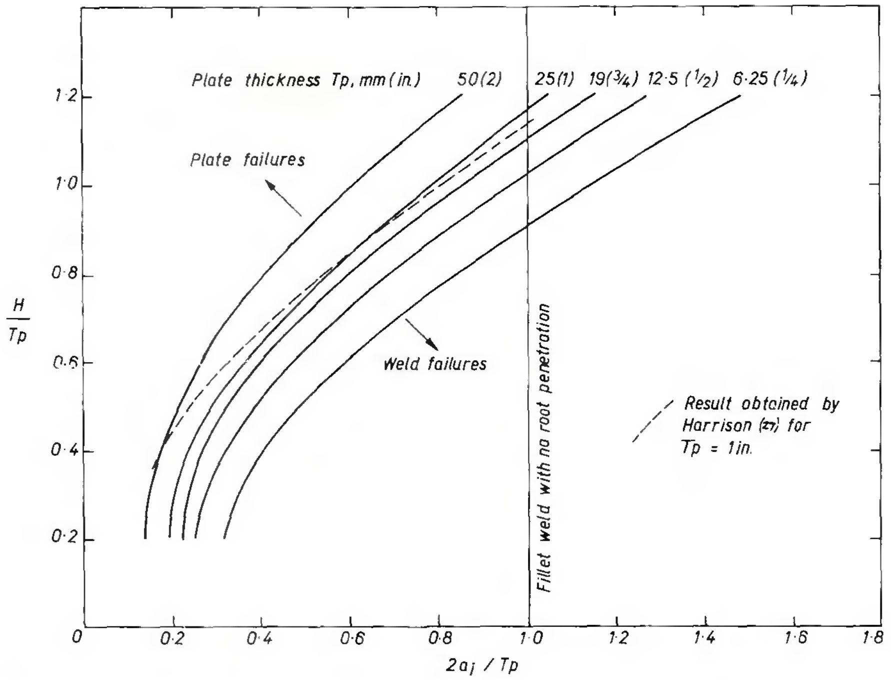

Fig. 15 — Geometric conditions for optimum design of transverse load-carrying welds in a cruciform joint

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