Calculation of Thermoelastic Deformations and Stresses in Two-Component Structures

Two models serve to illustrate how the finite element method can be used to predict elastic stresses and strains

BY J. F. STELZER

SUMMARY. This paper deals with the temperature dependent elastic behavior of structures consisting of different materials joined by brazing or welding. Appropriate joining techniques find application mainly in nuclear and chemical engineering fields and in rocket fabrication. The purpose is to make use of expensive materials only in those regions where their properties are required by conditions such as neutron flux, corrosive media or high temperatures. At the edges of such regions one will provide transitions to less expensive materials as soon as possible.

Introduction

The joint partners of a two-component structure have different Young's moduli, thermal expansion coefficients and Poisson's ratios. We may restrict ourselves in this context to two components. It is no problem to expand the considerations to three or more joining partners for treating the influence of intermediate layers.

When two-component structures are exposed to temperature changes the connection plane prevents the free movement of the adjacent metals. Strains and stresses are produced, sometimes leading to rupture.

Therefore the demand arose for a calculation system which allows prediction of the thermal behavior of such compound bodies and certain material couples. Such a system will be introduced in the following. It allows the complete solution of spatial deformation and stress fields according to arbitrary temperature distributions.

As examples two cases will be treated: (1) temperature induced deformations of a plane structure shown by the example of a bimetal strip and (2) adequate deformations and stresses in a spatial structure demonstrated by a brazed or diffusion welded tube.

Fundamentals

We get the solution by the method of finite elements (Refs. 1,2). The base of the calculation model is a variational statement for a confined area, a finite element. This statement reflects the potential energy stored in one element as a consequence of the prevailing strain. As the deformations of any structure under load will obey the least constraint condition, our finite element calculus meets this demand by minimizing the statement of potential energy within each element of the structure under consideration. The results occur in the form of displacements of structure points according to the reaction of thermal expansion to structural stiffness. The strains result from the displacements by differentiation and the stresses from the strains by Hooke's law.

The ensuing assumptions are valid due to the compound structure:

1. The requirement of structure coherence demands, that displacements of common nodes of adjacent elements must be equal, even in the case of different materials.
2. Each element equation set, which is usually written in matrix form, contains the physical properties due to its material.

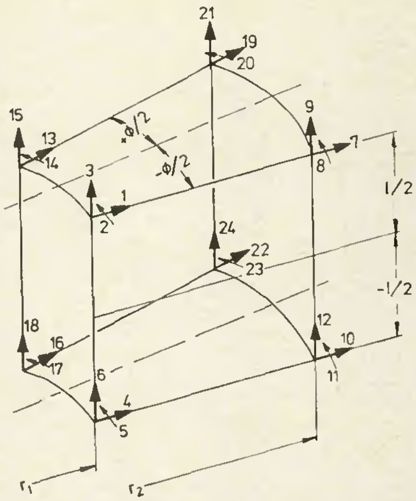

Fig. 1 — Finite element of the cylinder wall type as used in the calculation. It has 24 degrees of freedom

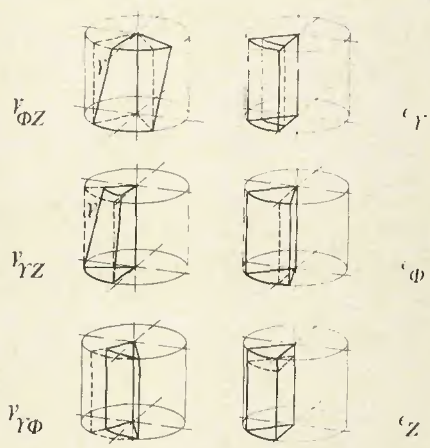

Fig. 2 — The six possible basic strains of the cylinder

J. F. STELZER is with Kernforschungsanlage Jülich, W. Germany Zeutralabteilung Allgemeine Technologie.

Paper was presented at the 7th International AWS Brazing Conference held at St. Louis, Missouri, during May 11-13, 1976.

As a consequence of statement 1, the stresses in the materials on both sides of the connecting plane will generally be different.

Since the prevention of the free thermal expansion is caused by the joining plane, the highest stresses will appear close by. Therefore the probability of an occurring crack is not so very high in the connecting plane as in the overstrained material in the vicinity.

To fulfill statement 2 the connection plane also must be the separation plane between the finite elements of different materials. The connection layer itself consisting of hard solder or of an alloy (in the case of diffusion welding) is neglected because of its small extension.

Calculations

The structure is subdivided into a number of finite elements the type of which should harmonize with the geometry. Commercial finite element programs offer a variety of element shapes.

Let us for example consider the case of a composite tube structure. Here we employ elements of the cylinder wall type (see Fig. 1) generated by Prij (Ref. 3) which have 8 nodal points and 3 degrees of freedom per node. The nodal point displacements are expressed by a 24-term polynomial which approximates the displacements within the element and gives a relation between the latter and the nodal point displacements. This polynomial is inserted into the variational statement. All element statements are then put together respecting common nodes. Thus we come to a system of linear equations which are usually written in matrix form. With its aid we can analyze structure deformations according to arbitrary temperature fields. For the assembling and the solution of the equation system we preferably employ the FEABL software (see Ref. 4). The next calculation step deals with the strain field movement. We obtain 6 basic strains (Fig. 2) by direction dependent derivation of the displacements.

Now applying Hooke's law, the distribution of the 6 basic stresses is also obtained. This makes it possible to introduce direction-dependent Young's moduli for regarding nonisotropic material properties. Usually, however, the engineer prefers the knowledge of only the comparison stress distribution. Thus an adequate calculation step has to follow, using preferably the deformation energy hypothesis. A certain disadvantage of this hypothesis is that only positive stresses result.

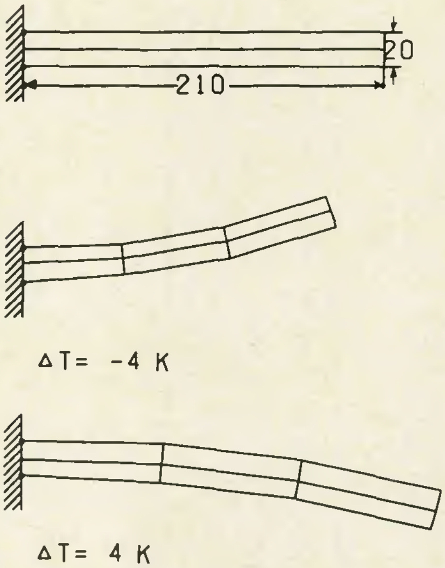

Fig. 3 — Calculated deformation of a bimetallic strip

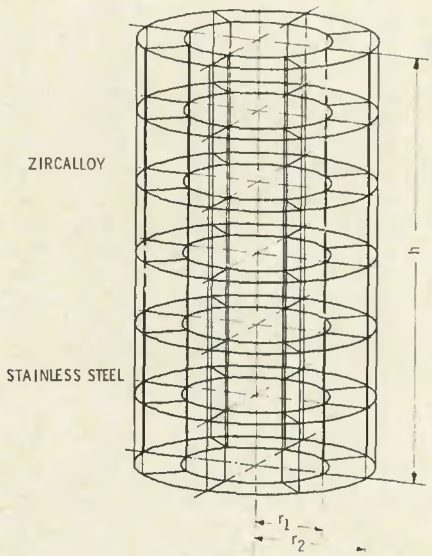

Fig. 4 — Division of the tube into 48 finite elements

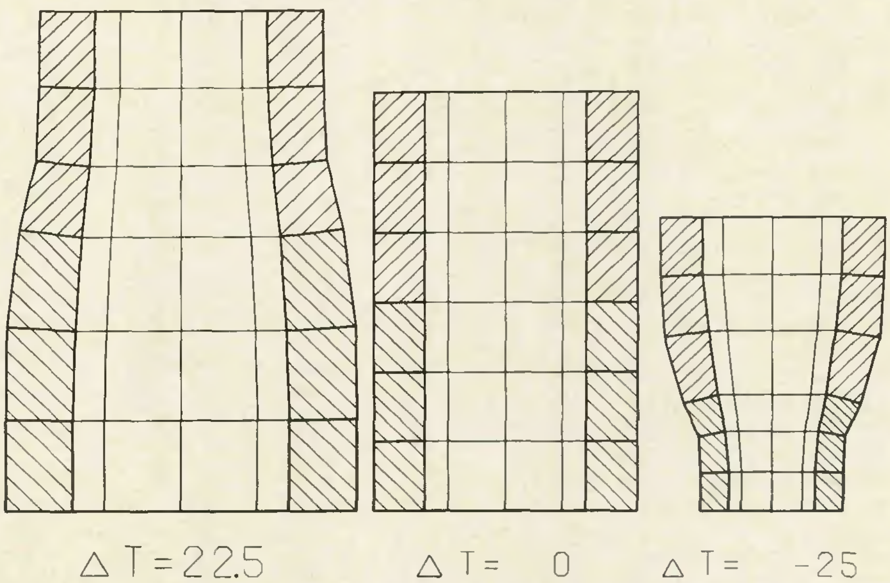

Fig. 5 — Deformations of the composite tube upon heating and cooling

Results

Temperature Induced Deformations of a Bimetallic Strip

Figure 3 shows the calculated temperature dependent deformations. To enhance the impression, the displacements are exaggerated by the factor 1000. Displacement exaggeration is an efficient means for realizing results of this kind. The subdivision into 6 elements is for clarity. In spite of the rather coarse partition the finite element model works quite well. The underlying material properties are given in Table 1.

Temperature Dependent Behavior of a Two-Component Tube

We consider a diffusion welded tube composed of Zircalloy and stainless steel, an important connection in the nuclear field.

The structure is subdivided into 48 elements (Fig. 4) with the middle plane being the joining area.

In Fig. 5 the calculated tube deformations in a longitudinal section are to be seen. Again the nodal point displacements are exaggerated by a factor 1000, as well as in the following deformation pictures. This exaggeration is conveniently done by using calculation temperatures of a thousandfold value compared to the true ones. In Fig. 5 one recognizes the enlarged expansion of the lower tube half (stainless steel) compared to the upper half (Zircalloy). This is because of its threefold thermal expansion coefficient.

Figure 6 shows the field of curves of equal comparison stresses. When heating is started the stress lines are directed out and away from the connection plane and spread out over the whole wall, gradually diminishing. This stress field of the isothermal structure allows us to predict all other isothermal stress fields because of the linear influence of the temperature, i.e., twofold temperature produces twofold stress values.

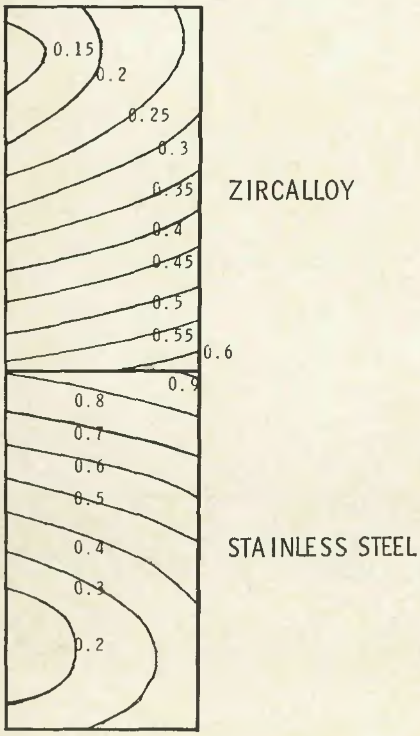

Fig. 6 — Result of calculating reference stress fields in daN/mm² (1 daN/mm² = 14.5 psi) upheating rate 10 K, entered in a radial longitudinal section of the pipe wall. Inner surface on the left

In practice a diffusion welded structure with its entire body at joining temperature will be free of internal stresses. Higher temperatures should never occur. Therefore the main interest lies in the cooling situation. Certain joints of metal pairs are so highly stressed at room temperature that a small force will cause a crack.

The next results stem from deformation calculations due to arbitrary temperature distributions. Figure 7 shows the tube with hot bottom and cold head end, with an intermediate linear temperature pattern. Figure 8 shows the reverse state. The thickened waist is the result of the expansion of the steel in the lower half, which despite its lower temperature, has grown to a larger diameter than the neighboring Zircalloy.

If the temperature exhibts different values over the circumference, the deformations show the pattern indicated in Fig. 9, e.g., when cooled unilaterally.

These calculated effects were compared to experimental results and the accordance was quite good.

Figures 5 to 9 are records of a computer controlled screen. Utilizing computer screen and film records gives the advantage of visualizing and studying the thermal behavior of a structure when it is being designed.

Further Applications

Programs of this kind are also applicable for predicting stresses induced by welding heat input. In this case it is advantageous to couple the deformation calculation program with a forerunning temperature field calculation program, whereby the same element mesh is used. The energy supply is introduced as an element heat source.

In connection with fast temperature changes, as in the case of hardening, it seems possible to predict deformations of shrinkage and distortion. For this the deformation and stress calculation program is combined with a transient temperature field program.

Summary

Improvements in the practice of joining different metals by welding, soldering and brazing prompted the desire to obtain a computational method which permits the thermal behaviors of such composites to be predicted.

The finite element method is a suitable means for solving this problem. Computational fundamentals and results are shown for the case of a bimetal strip and a diffusion welded pipe. The random temperatures are assigned to the nodes of the finite ele-

Table 1 — Material Properties Under Study

Property	Zircalloy	Stainless steel
Young's modulus in N/cm²	$9.8 \cdot 10^6$	$1.75 \cdot 10^7$
Coefficient of thermal expansion in K^{-1}	$6.5 \cdot 10^{-6}$	$1.8 \cdot 10^{-5}$
Poisson's ratio	0.38	0.3

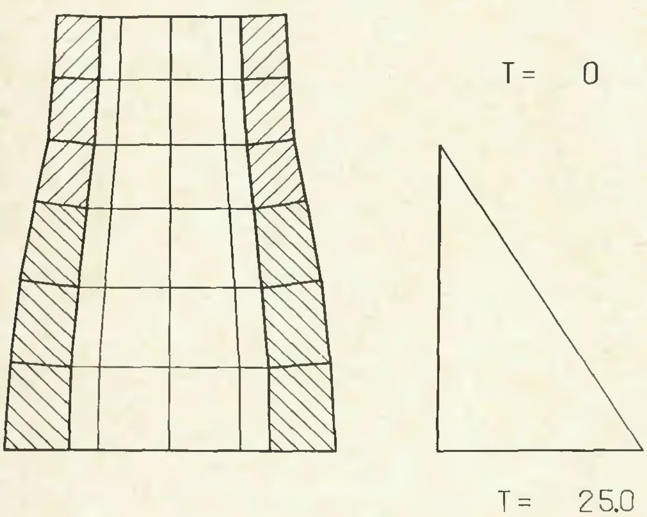

Fig. 7 — Deformation of the pipe with bottom hot and head end cold, linear axial temperature distribution

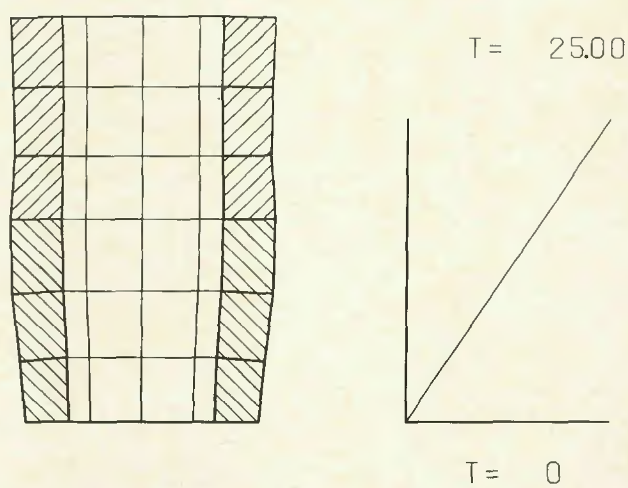

Fig. 8 — Deformation of the pipe with head end hot and bottom cold

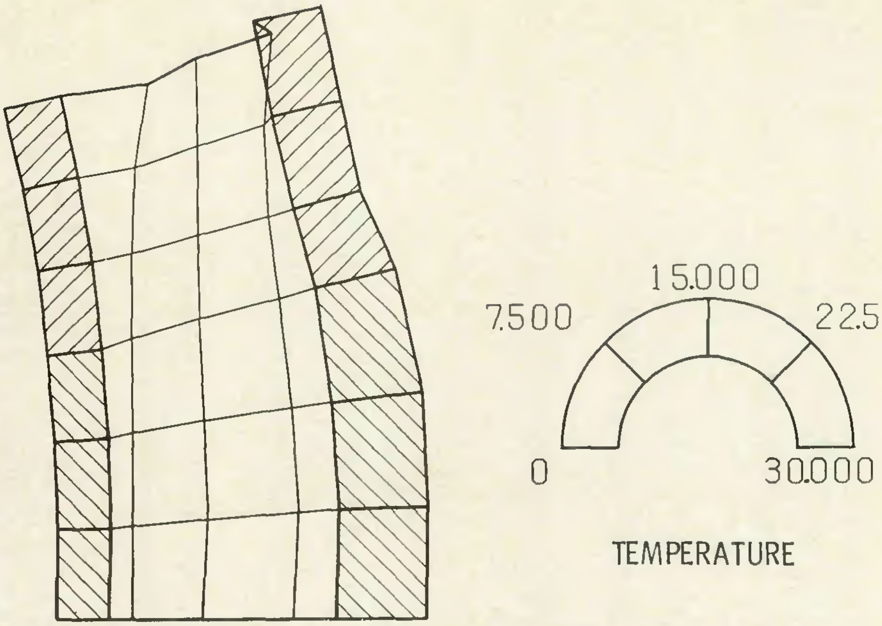

Fig. 9 — Deformation of the pipe with linear temperature distribution over the circumference (unilateral cooling)

ment network. It is thus possible to carry out the deformation and stress calculation using the same breakdown into finite elements.

Acknowledgment

The author wishes to acknowledge the productive co-work done by R. Welzel, A. Sievers and F. J. Schönebeck, all KFA Jülich.

References

1. Martin, H. C., Carey, G. F., *"Introduction to Finite Element Analysis,"* McGraw-Hill, New York, 1973.
2. Stelzer, J. F., "Practical Handling of the Finite Element Method," *Kerntechnik*, 17 (7) 1975, pp. 324-328, and 17 (8) pp. 364-370.
3. Prij, J., "Formulas due to Fields of Temperatures and Stresses," *Report 909-GR7*, Reactor Centrum Nederland, Petten, 1971.
4. Orringer, O., French, S. E., "FEABL User's Guide," AFOSR TR, ASRL TR 162-3, Massachusetts Institute of Technology, 1972.