Finite Element Analysis of Welded Structures

By varying certain welding parameters, weld joint strength can be optimized when a certain metallurgical structure is achieved

BY G. W. KRUTZ AND L. J. SEGERLIND

ABSTRACT. The object of this work was to predict the thermo-history of a welded joint using finite element analysis and provide design engineers with a reliable method of optimizing weld joint strength.

A computer model was constructed, and results indicate the cooling of metal adjacent to the weld pool is the critical metallurgical location. Variables affecting this cooling time include metallurgical constitutents, radius of the heat flux, velocity of the arc, thermal conductivity of molten steel. The model includes thermal conductivity and specific heat as functions of temperature, latent heat of fusion, radiation, convection and a gauss distributed heat flux.

Introduction

Welding is probably the most popular manufacturing process for joining metals used in structural applications. It is a common method used in fabricating farm equipment, being critical enough to have to the size of components determined by the size of weld area. Because of its wide and flexible use, weld strength has been the concern of designers and researchers who have studied it both analytically and empirically in an effort to maximize the joint strength. This maximization when achieved would reduce the weight of machines, structures, components and save material cost and processing energy.

The goal of the work described in this paper was to develop a general numerical model of the welding process which is capable of predicting the strength of a weld joint via the metallurgical viewpoint. Since many variables enter the process and this was the first attempt at a particular metallurgical model, the material was limited to low carbon steel. Low carbon steel is variable enough since its properties do vary from piece to piece as a result of process variations.

The problems of distortion, residual stresses, and reduced strength of the material in the joint area are results of the large amount of heat energy applied at the weld site over a short period of time. This heat gradient at elevated temperatures causes dynamic changes in the weld metallurgy and affects the joint toughness. The finite element method was used to predict the transient thermo-history of a welded composite and thus the joint strength.

It is hoped that this method will provide practicing engineers with a technique that is widely used in weld-joint design.

Past Theory

The problem of fast cooling rates and high temperatures that change the metallurgy and distort the joint have been concerns of the engineers for years. D. Rosenthal[1] developed the mathematical theory of heat distribution in welding during the late 1930's. By using the conventional heat transfer differential equation for a Quasi-Stationary State, Rosenthal developed equations for point, linear and plane sources of heat. These equations closely approximated actual tests and presented a method of predicting time and rate of cooling with some accuracy for a wide variety of steel thicknesses, ranges of temperature and welding conditions; an equation for Quasi-Stationary State is as follows:

$$\frac{\delta^2 T}{\delta x^2} + \frac{\delta^2 T}{\delta y^2} + \frac{\delta^2 T}{\delta z^2} = \rho c \frac{dT}{\delta t}$$

Further work on time-temperature relationships was still needed because Rosenthal's work was primarily applicable to butt joints assuming many constant inputs and boundaries.

Most of these analytical theories were summarized by Meyers, *et al.*,[2] where he states the assumptions which make their usefulness limited. The assumptions are:

1. The material is solid at all times and at all temperatures, no phase changes occur, and is isotropic and homogenous.
2. The thermal conductivity, density, and specific heat are constant with temperature.
3. There are no heat losses at the boundaries, i.e., the piece is insulated.
4. The piece is infinite except in the directions specifically noted.
5. Conditions are steady with time, i.e., in the middle of a long weld, heat input, travel speeds, etc., are steady.
6. The source of heat is concentrated in a zero-volume point, line or area.
7. There is no Joule (I^2R electric) heating.

Because of these limitations researchers of the 1970's have gone to numerical methods which can handle the above seven items. The finite element method is one such tech-

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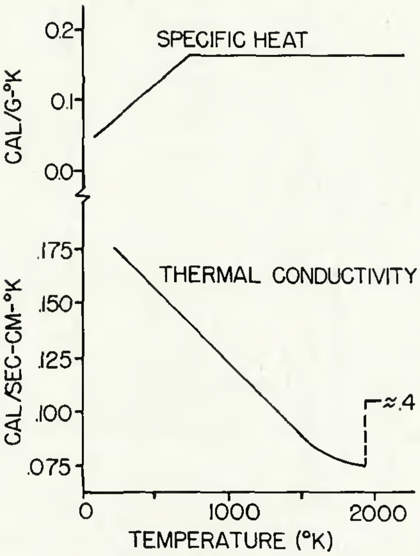

Fig. 1—Temperature dependent properties of low-carbon steel

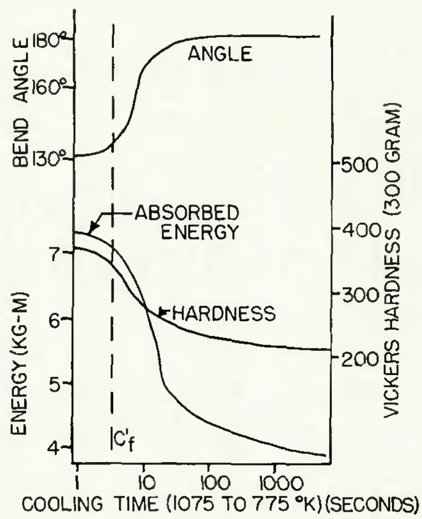

Fig. 2—Inagaki's experimental tests depicting critical cooling time vs. toughness and hardness

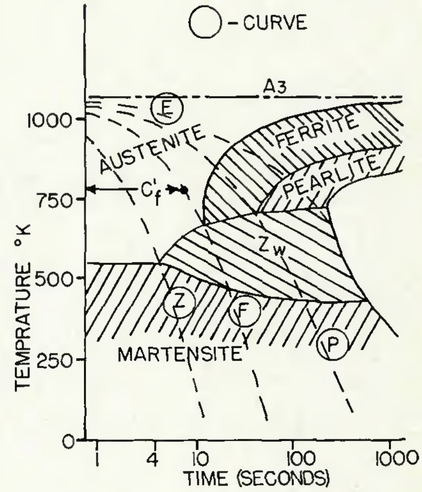

Fig. 3—Continuous cooling diagram

nique that can be applied to this problem.

Model's Assumptions

The model of the welding process is done in a two part analysis: first, the temperature is determined as a function of location and time; subsequently, all points in the heat-affected zone are checked against the critical cooling time from which revisions to the process will be recommended or final data acquired.

For the thermo portion of this study there are several boundary conditions and physical phenomena that are complicated in this non-linear problem. These are discussed, pointing out which ones are essential to the model.

1. The materials are subjected to a wide range of temperatures. All properties must then be considered as functions of temperatures with the exception of density. Figure 1 depicts these properties for low-carbon steel over the temperature range in question. The important properties affected are thermal conductivity and specific heat. It was assumed that the thermal history (i.e., thermal expansion) would have little or no effect on the shape of the elements. In actuality the molten and plastic zones deform due to shrinkage and restraints.

2. The phase changes are another important phenomenon affecting this work. The heat of transformation and latent heat of fusion represent a storage of heat during the welding process and subsequently greatly affect the cooling time and microstructure metallurgy. This heat is large enough that it may create favorable recrystallized fine-grained ferrite/pearlite or unfavorable brittle structures. An implicit heat absorption feature to simulate the phase change over a temperature range was included. Most models assumed that the phase change takes place at a specific temperature, which is not the case for alloys.

3. Shrinking distortion and residual stresses are existent and noticeable in the workpiece, but considerable modeling progress must be achieved to provide accurate and meaningful results. All current methods are unable to predict cracking, a common occurrence in many types of welds, and therefore this model has taken a metallurgical approach.

4. Input heat flux has been passed over lightly by some researchers as voltage times current and accounting for some efficiency factor. A more complete description of the heat flux as a function of position and time is presented. The addition of filler metals does affect heat input, but these were not added to the finite-element grid because of the difficulty in element modeling.

5. Boundary conditions must be employed to account for radiation (quartic Stefan-Boltzman) and linear Newton convective cooling. No forced convection was assumed and the effect of gas diffusion in the weld pool was not considered. Also, slag formation and its effects was assumed small and negligible because the width of slag is dependent on the process or rod coating and its formation at lower temperatures should have a reduced effect on convection and element conductivity.

6. Weld pool size was determined by the location of solidus line. The model was two-dimensional and because of symmetry only one half needs to be constructed and analyzed.

The second part is the determination of the critical cooling time factor. Inagaki, et al.[3] did a large amount of experimental work determining a relationship between energy absorbed during a weld joint bend test and the cooling time of the weld joint-fusion line.

In their study total bend angle and Charpy impact tests were done on samples to verify that cooling time longer than C'_f created larger bend angles and more absorbed energy—Fig. 2. Also, these curves show that hardness decreases with cooling times greater than C'_f. Therefore, to keep the toughness at a maximum use C'_f as minimum, but to keep the strength of the steel up (related to hardness), attempt to keep C'_f as the maximum. It then can be concluded that C'_f is a cooling time to optimize weld joint strength for various loadings and steels. From experimental results, Inagaki[2] determined the minimum cooling time from the equivalent carbon content C_{eq}:

$$\log C'_f = 8.59\, C_{eq} - 1.69$$

where:

$$C_{eq} = \%\,C + \frac{1}{12}\,MN\% + \frac{1}{24}\,Si\%$$

It is known that the A_3 transformation point is affected by the metallurgy of the steel and the continuous cooling curve transformation (CCT) diagram affected by how long the structure is heated above the A_3 temperature. To determine C'_f, the time to cool from 1075 to 775° K was considered to be the only general practical boundaries—Fig. 3.

The A_3 transformation temperature point is approximately 1075° K for mild and high tensile steels. For various other alloys the cooling time would have to be determined over a different range, with 775° K still being the lower bound by definition of C'_f. Different welding processes did not affect a typical hardness-cooling curve. The hardness at C'_f has been found to be equivalent to Hv = 350 which is a maximum guideline set up by Welding Standards. At this hardness no underbead cracks result in the heat-affected zone, and weld joint exhibits enough ductility.

Now a weld joint can be optimized by varying the input parameters to achieve the critical cooling time C_f' which will give maximum hardness at the fusion line without sacrificing bend tests angle. This can be done in the finite element program. For metallurgy determination away from the fusion line, CCT diagrams will have to be referred to.

Finite Element Theory

The finite element used to model any joint must be of an order one less than the governing differential equation. For this transient two-dimensional heat transfer quasi-stationary state equation the order needed is one and linear triangles match this criterion. Minimizing the function of this governing differential equation the following system of equations result:

$$[C]\ \{\dot{T}\} + [K]\ \{T\} + \{F\} = 0$$

[C] = conductance matrix
[K] = stiffness matrix
{F} = force vector
{T} = nodal temperatures

A solution was achieved by using an implicit Crank-Nicolson Method which yields:

$$\left([K] + \frac{2}{\Delta t}[C]\right)\{T_1\} = \left(\frac{2}{\Delta t}\ [C] - [K]\right)\{T_o\} - 2\left(\frac{\{F_1\} + \{F_o\}}{2}\right)$$

where subscripted 1's are values at time t + Δt and subscripted 0's are values at the initial time t. Details of the development of [K], [C] and { F } are available in Segerlind's[4] text.

Certain variables in the system of equations are temperature dependent or vary with position. These are thermal conductivity, radiation, heat flux, specific heat, and latent heats. The implementation of these temperature and position dependent variables is a significant contribution to the model but each does increase the solution computational time.

For many materials such as low carbon steels varying thermal conductivity linearly with temperature provides a good approximation. This variation can be written as

$$K_A^e = K_o^e + K_1^e T$$

where K_o^e and K_1^e are constants for an element (e). If the material was homogenous K_o and K_1 would be constant for the entire body. Values for K_o and K_1 are determined from Fig. 1.

Assuming the temperature variation is linear within a triangular element, another convenient representation is expressing the temperature as the arithmetic average of the node temperatures. The following equation results:

$$K_A^e = K_o^e + K_1^e\left(\frac{T_i + T_j + T_k}{3}\right).$$

Now, using thermal conductivity as a function of temperature the process of minimizing the variational form will yield another term in the global stiffness matrix [K]. Now $[K^e]$ is:

$$[K^e] = [K_A^e] + \frac{1}{6}[T_B]^T[K_B^e]$$

When thermal conductivity is a function of temperature the terms are:

$$[K_A] = [B]^T\begin{bmatrix} K_o + K_1\left(\frac{(T_i + T_j + T_k)}{3}\right) & 0 \\ 0 & K_o + K_1\frac{T_i + T_j + T_k}{3}\end{bmatrix}[B]$$

$[T_B]^T$ is a 3 × 3 matrix expanded to be:

$$[T_B]^T = \begin{bmatrix} T_i & T_j & T_k \\ T_i & T_j & T_k \\ T_i & T_j & T_k\end{bmatrix}$$

[B] = gradient matrix (function of element coordinates), and:

$$[K_B] + [B]^T\begin{bmatrix} K_1 & 0 \\ 0 & K_1\end{bmatrix}[B]$$

The second term in $[K^e]$ is not a symmetric matrix and therefore cannot be combined with $[K_A^e]$. More detailed derivations are described in Krutz.[5]

An estimate of the second term's influence on temperature results was done. Analysis showed:

1. A minor (5%) importance when nodal temperatures vary by more than 300° K per element.
2. Decreased importance for little nodal temperature difference.
3. Further decreased importance for small elements.
4. Increased computer costs and execution time because of [K] becoming unsymmetric (4 times or more).

Using these results, especially non-symmetry costs, the additional second term was assumed negligible and not included in constructing [K].

Now the finite element model incorporates thermal conductivity as a function of temperature given by three relationships:

$$K_{xx} = .795 - .0004^* \text{ TAVE for } T \leq 1300° \text{ K}$$

$$K_{xx} = .3\left(\frac{\text{watts}}{\text{cm °K}}\right)$$

for 1775° K > T ≥ 1300° K

$$K_{xx} = .6 \text{ for } T \geq 1775° \text{ K}$$

where TAVE = average of element nodal temperatures

$$\text{TAVE} = \frac{T_i + T_j + T_k}{3}$$

These equations are subject to change when varying the material modeled or welding under other convective patterns in the molten pool. A single constant value for thermal conductivity was calculated for each element.

A similar second term results when specific heat is a function of temperature. Defining $\alpha = \rho c$ where density is constant, then the variation of α with temperature can be expressed as two governing equations when using low carbon steel.

$$\alpha_A = \alpha_1 + \alpha_2\left(\frac{T_i + T_j + T_k}{3}\right)$$

for T < 775° K

and α_B = constant for T ≥ 775° K

For triangular elements the addition to the [C] matrix becomes:

$$\frac{\alpha_A A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2\end{bmatrix} + \underbrace{\frac{\alpha_2 A}{36}\{T\}^T\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2\end{bmatrix}}_{\text{second term}}$$

Again, because of non-symmetry and a similar importance analysis yielding negligibility for the second term, specific heat was not included in constructing [C] of this model. Specific heat was modeled as:

$$\rho c = 1.3 + .0053^* \text{ TAVE for } T < 775° \text{ K}$$

$$\rho c = 5.0 \text{ for } T \geq 775° \text{ K}$$

In the weld pool vicinity, heat is radiated to the surrounding environment because of the high difference in surface and ambient temperatures. This radiation must be an accounted for boundary condition.

When setting the derivative of the variational portion equal to zero we see that one term can be added on to the global stiffness matrix [K] and the other term is added to the force vector {F}. Factoring out a $\{T_i\}$ from the first term gives the following form:

$$\frac{dI_i}{d\{T\}} = [\sigma\epsilon T_i^3]\left\{T_i\right\} - \sigma\epsilon\, T_{1\infty}^4 = 0$$

or sometimes this is shown in the matrix form:

$$[R]\{T_i\} - \{r\} = 0$$

σ = Steftan Boltzmann constant = $5.672 \times 10^{-12} \frac{watt}{cm^2\ K^4}$

where:

= emissivity;

T_1 = absolute temperature (°K);

[R] = is the addition to the stiffness matrix [K];

{r} = is the addition to the force vector {F}

Using area coordinates, radiation from one side of a triangular finite element may be expressed as:

$$\frac{\sigma\epsilon T_1^{3} L_{ij}}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & 0\\ 0 & 0 & 0\end{bmatrix}$$

where T_1 is $\left(\frac{T_i + T_j}{2}\right)^3$

Heat flux is an addition to the force vector and has been expressed by Friedman[6] as a function of radius and time:

$$q_i(r,t) = \frac{3Q}{\pi \bar{r}^2}\ EXP[(-3r/\bar{r})^2]$$

$$EXP\ \ [-3\left(\frac{V\ t-\tau}{\bar{r}}\right)^2]$$

where: r = distance from center for a particular q_i (m); v = velocity of the arc (m/s); Q = total heat input (watts); t = time (s); $q_i \doteq$ watts/m at point i; $\bar{r}$ = maximum radius; τ = lag factor = $\bar{r}$/V (seconds, s).

In this planar analysis it has been assumed the speed of the arc is high compared to the diffusion rate of the material so that the amount of heat conducted ahead of the arc is relatively small compared to the total heat input. In order to make this a two-dimensional problem it has been assumed that the heat flow across the plane in the third dimension (direction of electrode travel) is again very small and negligible. Therefore, the

$$\frac{d}{dz}\left[K\frac{\delta T}{\delta Z}\right] \text{term goes to zero.}$$

Heat conduction in welding must include a change of phase. The dominant latent heat is that of fusion. The latent heat transformation has a value for steel of approximately 1/10 that of fusion and is considered minor in importance. Fusion usually occurs in a temperature range for alloy steels from about 1700 to 1755° K and, therefore, must be considered as a factor of heat input or withdrawal over that range.

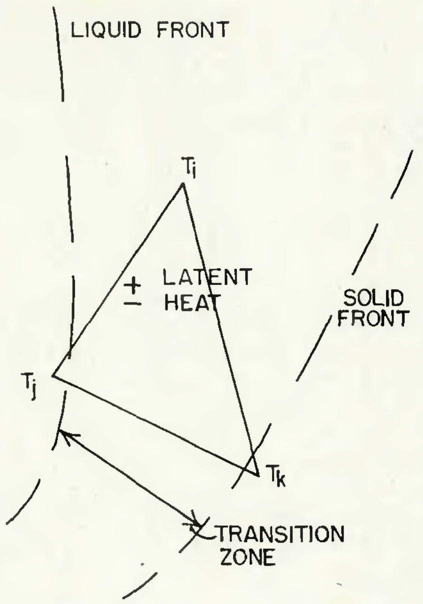

Fig. 4—Latent heat affecting a triangular element

During the welding process as heat is input from the arc there exists a liquid front surface, a solid front surface, and the in-between transition zone where latent heat is being absorbed—Fig. 4.

The opposite phenomenon takes place during the solidification process. Heat is given off in the transition zone as the metal is cooled which will increase the cooling time. The solid surface is represented by a 1700° K isotherm for steels while the liquid surface consists of the 1755° K isotherm. A method to incorporate latent heat must be developed for the triangular element.

The possibility of both the liquid and solid front intersecting a triangular element exists. This would create a complicated internal boundary problem to be solved for latent heat effects. By decreasing element sizes in this latent affected zone the number of elements affected by this internal boundary problem can be minimized. Then only a solid front or liquid front would be intersecting a single element. This then again would create a complex computational problem along with all the increased computer calculations used with temperature dependent properties. Therefore a compromising assumption is made to approximate the true latent heat effects. The latent heat generated or given off within an element was allotted equally to all three nodes of the triangle. To determine if an element is affected by latent heat an average temperature for the element is calculated:

$$T^e_{ave} = \frac{T_i + T_j + T_k}{3}$$

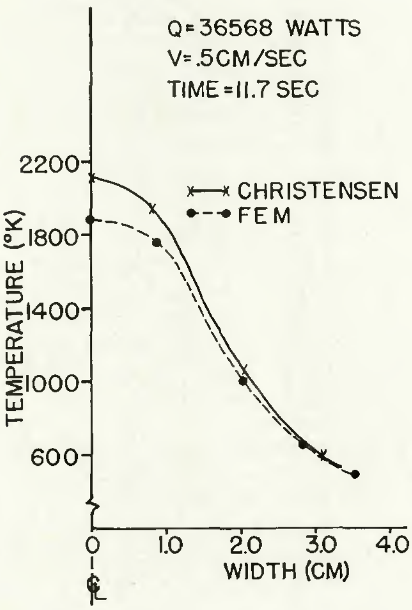

Fig. 5—Verification of FEM with Christensen's work

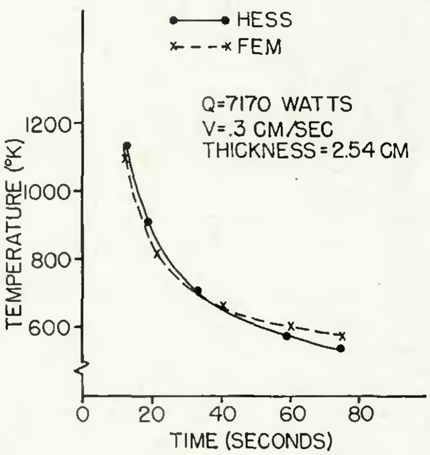

Fig. 6—Verification of FEM with Hess' work

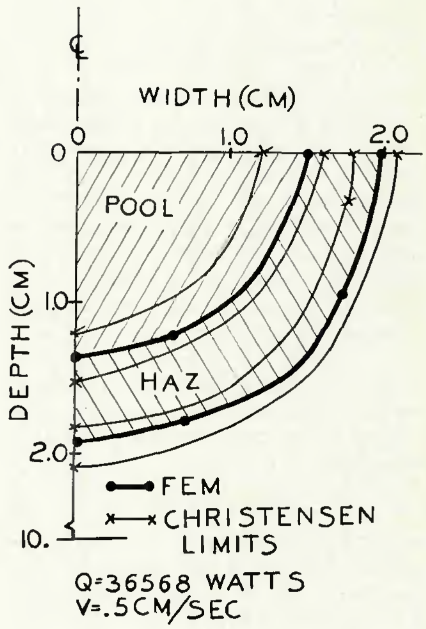

Fig. 7—Size of weld area

If T^{e}_{ave} falls within the latent heat range of 1700 to 1755° K, this particular element would have a term added or subtracted to the force matrix. The sign of this term is either negative or positive corresponding to heat absorption and heat generation respectively and the computer program was coded to a sign convention which was regulated by increasing or decreasing average element temperatures.

Assuming constant thickness when using triangular elements this latent heat term is given as:

$$\frac{pLA}{3\Delta t}\begin{Bmatrix}1\\1\\1\end{Bmatrix}$$

where: p = density (for steel = 7.87 g/cm); L = latent heat (for steel = 65.5 cal/gram); Δt = length of a time step (seconds, s).

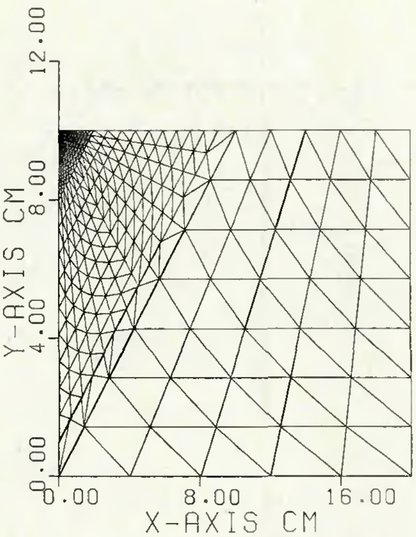

Fig. 8—Finite element grid used to model a weld joint

Model Verification and Sensitivity Analysis

In order to be confident that this finite element model is approximating an actual butt weld, a comparison with actual thermocouple experiments was done. Two groups of researchers—Hess[7] and Christensen[8]—have done extensive recording of time-temperature relationships for a range of welding applications. Their data was used to verify the accuracy of this numerical model. Three comparisons which show close agreement between experimental and finite element approximation are given in Figs. 5–7.

Figure 6 is a comparison of thermocouple readings and the finite element model (Fig. 8) in the heat-affected zone. After a considerable time-lapse (50 s from arc passing) the two begin to differ by as much as 10% with the experimental readings being lower. This could very well be the effect of the thermocouple's drilled hole creating a heat loss boundary lowering local temperatures. The key area of the curve, critical cooling time, correlates well and the values of C_f' are almost equal. These graphs show that this finite element model closely approximates actual welding conditions.

A sensitivity analysis was done on the influence of $\bar{r}$ on the finite element solution because of the unsurety of its value. This analysis was done by varying $\bar{r}$ from 0.5 cm to 1.5 cm using Christensen's welding parameters. A node in the heat-affected zone was chosen to determine the effects of various welding arc radii.

Figure 9 shows a plot of time-temperature values and how they are affected by varying $\bar{r}$. From this graph it can be depicted that $\bar{r}$ affects the size of the weld pool, size of the heat affected zone and critical cooling time. Therefore, the welding arc must be accurately modeled for meaningful results to be obtained from numerical methods.

Since little research has been done in arc modeling and its shape highly influences finite element results, more work is needed. Possible use of inverse conduction methods might provide accurate arc dimensions and heat transfer efficiencies of welding processes.

Again, a heat-affected node was used as the basic location for estimating the sensitivity of the FEM solution to variations in weld pool thermal conductivity.

Three different values of thermal conductivity were used in the finite element model, and the effect on temperature relationships is shown in Fig. 10. As K varied from 0.2 watts/cm °K to a value of 1.2 watts/cm °K, the temperature history experienced by the surface location varied from one extreme of not even getting hot enough to enter the HAZ to the other extreme of reaching melting temperatures.

The relationship of higher thermal conductivity values creating higher maximum temperatures makes FEM solutions very sensitive to this variable.

The element grid used to compare Christensen's work is shown in Fig. 8. This grid model has a fine mesh in the area of high thermal gradients. A comparison of results with a coarser grid (Fig. 11) is graphed in Fig. 12. The coarser grid does not adequately transmit heat in the weld area and results in a lower temperature curve.

The choice of grid size is an engineering judgment when using finite element approximations. It plays an important part in getting accurate results with small elements—a must criterion for high gradient areas. The sensitivity of other variables (convection coefficient, emissivity, and time step) on the results showed little influence.

Application to Welding Design

In static loading of welded joints, the total joined area regulates the strength. This area can be determined from the finite element model. The only variable needed in static load calculations not determined by the

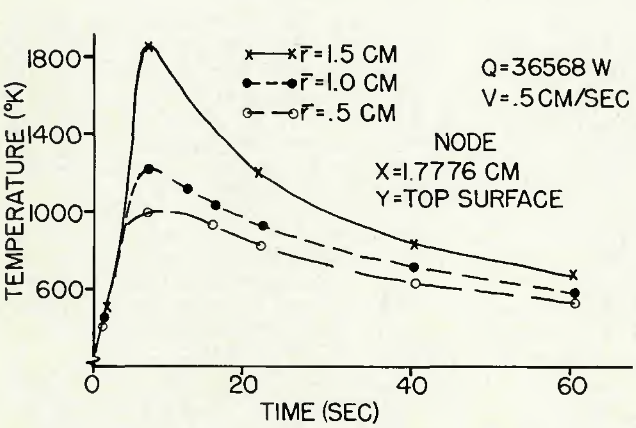

Fig. 9—Sensitivity of finite element solution to various arc radii

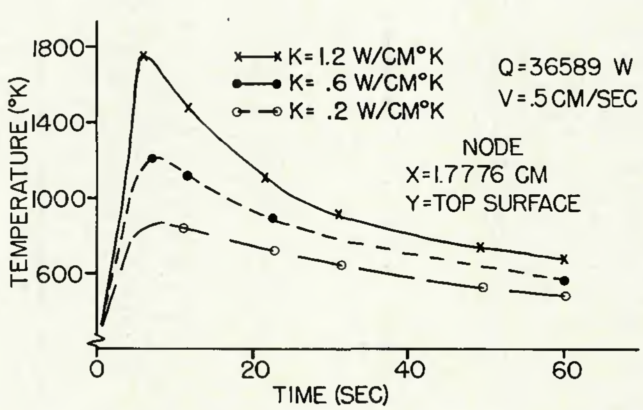

Fig. 10—Sensitivity of finite element solution to molten thermal conductivity

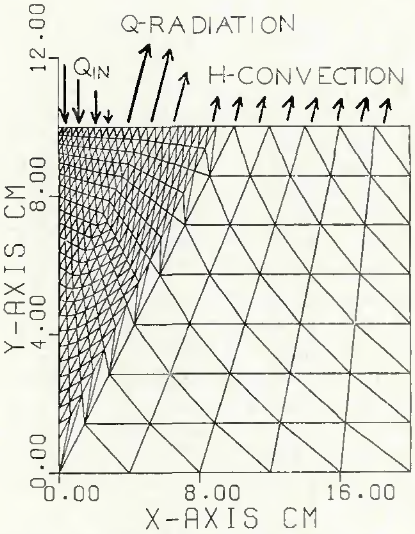

Fig. 11—Coarse grid triangular element weld joint model

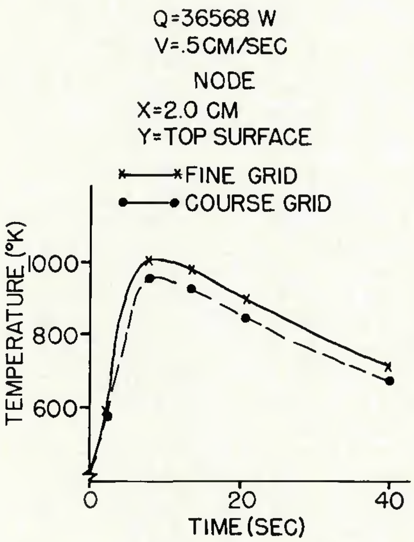

Fig. 12—Grid differences affecting results

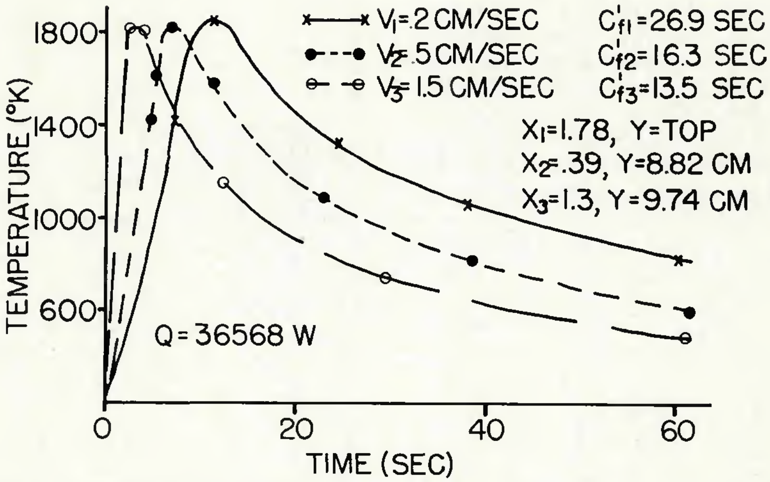

Fig. 13—Effect of velocity on C_f'

FEM model is the weld reinforcement stress concentration. Stress concentration is considered a state of the art function related heavily to type of welding rod, angle of the arc, and arc velocity.

For dynamic loading cases such as fatigue the toughness of a joint regulates its strength over time. Inagaki's work lays the ground work for maximizing the energy absorbed by a joint (toughness) in his critical cooling time factor. This C_f' is the optimum for maximum strength at maximum toughness. The finite element model will calculate the cooling time for a proposed set of welding parameters.

If the cooling time varies from Inagaki's optimum, numerical welding parameters can be varied to optimize the joint strength by meeting the C_f' criteria. Some parameters that will affect C_f' are arc velocity, size of the electrode, type of welding process, backup plates, physical size of the base metal, and material constituents. The advantage of using the finite element model in optimizing C_f' is the ease of changing variables and getting results of this complicated non-linear heat transfer problem in a matter of minutes.

Effect of Arc Velocity on Cooling Time

To depict how one of the aforementioned variables can be changed to optimize C_f', the arc velocity effect was looked at. The effects of different velocities on C_f' temperature histories for fusion line nodes were plotted in Fig. 13. It can be concluded from these plots that increasing C_f' can be accomplished by slowing the speed of travel. The speed of travel also interacts with melt depth, where slower speeds result in larger depths. Before use of these conclusions becomes a recommendation economic factors must be considered. Items such as welding time costs might lead to a compromise in final joint conditions.

Conclusions

The finite element method was used to model welded joints determining time-temperature relationships throughout the structure. This transient thermal history affects the metallurgy in the heat-affected zone. By varying certain welding parameters, the weld joint strength can be optimized when a certain metallurgical structure is achieved.

This computer program used simplex triangular elements, included specific heat and thermal conductivity as functions of temperature, and incorporated latent heat of fusion, convection and radiation. The following conclusions can be drawn from this study:

1. This FEM non-linear model closely approximates actual welding conditions but must be used cautiously because the results are sensitive to arc radius and the thermal conductivity of molten steel.
2. The fusion line-surface intersection is an acceptable location to use in estimating critical cooling time.
3. Slowing the arc velocity will increase cooling time of the fusion line.
4. Heat-affected zone and penetration can be determined with this method. If joint strength is insufficient in weld depth, the correct parameters can be arrived at in the computer program.

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