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Analysis and Control of Distortion in Welded Aluminum Structures

A three-year investigation at the Massachusetts Institute of Technology emphasizes the prediction and control of longitudinal bending distortion of built-up beams and the buckling distortion of thin aluminum plates

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ABSTRACT. The development of analytical means for predicting and controlling distortion in welded aluminum structures is summarized in Part I. Experimental and analytical studies on transverse shrinkage, longitudinal distortion, out-of-plane angular distortion and buckling distortion are presented. Methods for reducing weld distortion, such as elastic prestraining, clamping and differential heating are briefly discussed.

Part II covers an in depth examination of the investigations, both experimental and analytical, conducted in the past three years at M.I.T. on the subjects of longitudinal bending distortion of built-up beams and buckling distortion of thin aluminum plates. Comparisons between experiment and analysis are presented. Methods for reducing these kinds of distortion are proposed. They include the method of "differential heating" which consists of heating one of the two components to be joined to some suitable predetermined temperature.

The application of a uniform external tensile load during welding of thin aluminum plates is also proposed to control the buckling distortion.

Introduction

One of the most troublesome problems a welding fabricator is facing today is that of weld distortion. The nonuniform heating and cooling cycle, which occurs in the weld and the adjacent base metal, causes the development of complex strains during welding. Their respective stresses combine and react to produce internal forces that can cause bending, rotation, and/or buckling. These dimensional changes, collectively known as welding shrinkage distortion, are shown in Fig. 1.

In the past 30 years most critical structures, such as ships, submarines etc., were built using steel. As a consequence, a large amount of empirical information was gathered on the various kinds of welding distortion encountered during the fabrication of steel structures. At the same time, analytical investigations were carried out in an effort to gain better understanding of the problems.

Lately, however, the interest in using aluminum extensively as a structural material has increased. This has occurred due to the many desirable properties aluminum possesses; these include low weight-to-strength ratio, excellent resistance to salt water corrosion, etc. Unfortunately, however, the distortion problems in welding aluminum are much more severe than those of steel because of the following reasons:

1. Compared with steel, aluminum has a higher heat conductivity.
2. Aluminum has about twice the coefficient of thermal expansion of steel.
3. The modulus of elasticity of aluminum is one-third that of steel.

No information can, accordingly, be transfered directly from the knowledge of welding distortion of steel structures.

The above facts gave the incentive to investigators at the Massachusetts Institute of Technology (M.I.T.) to carry out a thorough fundamental study on distortion in welded aluminum structures. A three-year investigation was first directed towards an understanding of the problems through the conduction of experiments and the development of analytical means to predict distortion. Having successfully completed this task, the effort was then directed towards

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finding and analyzing ways of distortion control. Several methods were tried such as clamping, elastic-plastic prestraining and differential heating. Part I of this paper summarizes the findings of the investigation.

Part II of the paper is devoted to a detailed presentation of the results obtained in the cases of longitudinal bending distortion of welded aluminum T-beams and buckling distortion of welded thin aluminum plates.

Part I

This part summarizes the most important results obtained by investigators at M.I.T. in the past 3½ years on the subject of analysis and control of distortion in welded aluminum structures. Part of the investigations related to transverse shrinkage, longitudinal bending distortion, out-of-plane distortion and buckling distortion were funded jointly by the Aluminum Alloys Committee of the Welding Research Council and the National Science Foundation.[1] The Office of Naval Research supported a three-year research contract which included the following two tasks:

Task 1: Development of monograph for predicting stresses, strains, and other effects produced by welding.

Task 2: Prediction and control of distortion in welded aluminum structures.

Details of the results of this study can be found in the literature.[2] In the discussion to follow it is assumed that the reader is familiar with or has access to WRC Bulletins 149 and 174 written by Masubuchi.[3,4]

Development of a Monograph

A monograph entitled "Analysis of Design and Fabrication of Welded Structures" was developed by Masubuchi in 1977.[5] The monograph, which is divided into sixteen chapters and is about 1,300 pages long, deals with the prediction of stresses, strains and other effects produced by welding.

It contains information about heat flow during welding, development and calculation of transient thermal stresses and residual stresses, distortion in weldments, etc. The strength of welded structures is examined as related to notch toughness, brittle fracture, fatigue fracture, stress corrosion cracking, hydrogen embrittlement and buckling strength. Finally, matters related to joint restraint and weld defects are considered.

The authors feel that this monograph will be a valuable source of information for people dealing with welded structures.

Development of Computer Programs

Computer programs capable of analyzing heat flow, transient thermal stresses, residual stresses and distortion in weldments were developed. Specifically the following programs are now available:

1. Computer programs for the analysis of heat flow in weldments.[6] Some programs are analytical solutions while others use the finite element method. They can treat one-dimensional, two-dimensional, and three-dimensional cases.
2. Computer programs for the one-dimensional analysis of thermal stresses during welding.[7] Only the stress component parallel to the weld line is taken into account.
3. Computer programs capable of analyzing stresses and strains in a two-dimensional stress field either in the plane stress or in the plane strain condition.[8] These programs are based on the finite element method.
4. Computer program for analyzing longitudinal stresses in the middle part of a large, heavy weldment. Bending is not taken into account.
5. Extension of the above program, to take bending into account. This program is suitable for analyzing longitudinal bending distortion.
6. One-dimensional computer program for analyzing thermal stresses and metal movement during welding fabrication of a built-up beam.[9]

These computer programs have been, and are continuously, tested against available experimental data in an effort to improve their efficiency and predictability. As an example, it is mentioned that an investigation is currently under way aimed towards the incorporation of phase transformation effects which are believed to be of big importance for the correct simulation of welding of high-strength steels.

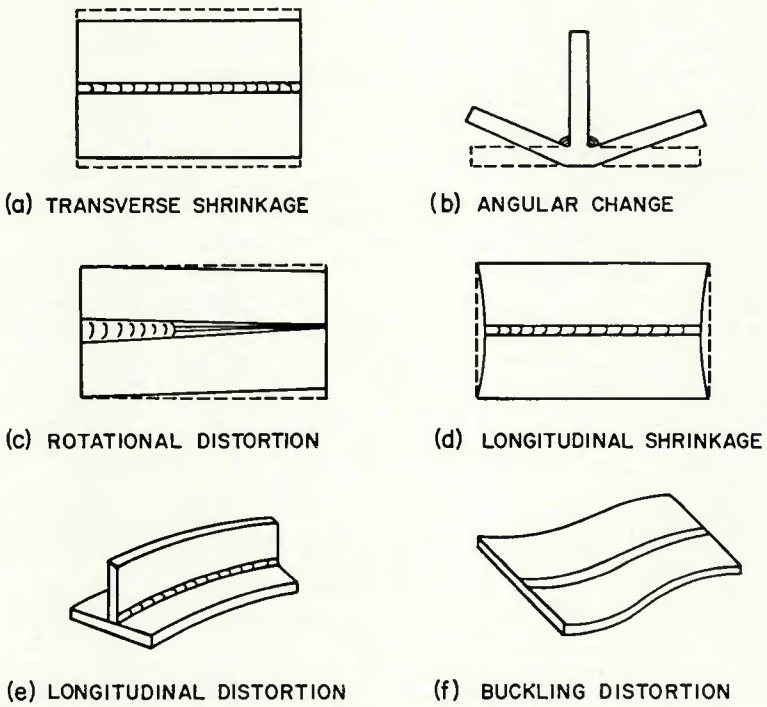


Fig. 1—Various types of weld distortion

Transverse Shrinkage of Aluminum Butt Welds

Transverse shrinkage is the shrinkage that occurs perpendicular to the weld line—Fig. 1(a). This kind of distortion was investigated by Iwamura.[10] Mathematical models were developed to express transverse shrinkage of a butt weld. The analytical results agreed well with experimental data. They also revealed that, in order to reduce transverse shrinkage, it is essential to reduce temperature in the base plate when the weld metal solidifies and starts to contract. Cooling after joining is accomplished has little effect on transverse shrinkage.

Experiments were conducted in order to study the feasibility of reducing shrinkage by use of cryogenic cooling. Unfortunately, however, this procedure was not found to be effective, because it did not produce temperature distributions as described above. It is finally mentioned that the restraint employed in the specimens produced a significant reduction (about 30%) in transverse shrinkage.

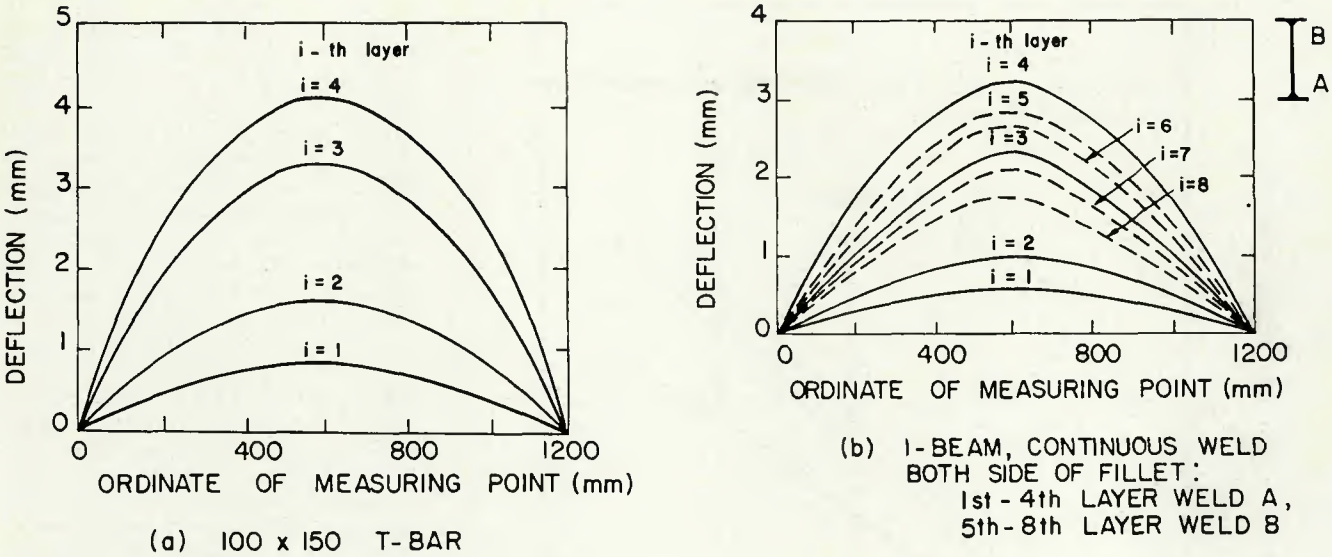


Fig. 2—Longitudinal deflection of steel due to fillet welding

Longitudinal Distortion of Built-Up Beams

Longitudinal distortion occurs when the weld line does not coincide with the neutral axis of a weld structure—Fig. 1(e). Yamamoto[11] conducted an experimental investigation of the subject, and a considerable amount of data on temperature, strains, and distortion during welding was collected. Convex deflections were observed during the welding process, while concave deflections resulted during the cooling stage. A limited effort was made to study whether longitudinal distortion can be reduced by clamping the weldment to a fixture during welding. It was found that reduction of longitudinal distortion by additional restraint is rather difficult. In order to fabricate a built-up beam with zero longitudinal distortion, one must produce negative distortion of an adequate amount, something very difficult since it requires a considerably large force.

Serrotta[12] studied the method of "differential heating" in an effort to reduce longitudinal distortion. In this method, a web or a flange is heated to a certain temperature before welding to compensate distortion. It was found that by properly selecting the temperature differential, it is possible to fabricate beams with little or no longitudinal distortion.

The above experimental results were analyzed by Nishida[9] using a computer program he developed. Good correlation between experiment and analysis was achieved. A more detailed account for all these results can be found in Part II of this paper.

Out-of-Plane Distortion of Fillet-Welded Aluminum Structures

General information about out-of-plane distortion of fillet welds—Fig. 1(b)—can be found in the literature.[3,4] The emphasis here is on methods for reducing this kind of distortion. An experimental investigation to study the effectiveness of mechanical prestraining was undertaken by Henry.[13] His method was based on the idea that the angular distortion of a fillet weld can be reduced if an initial distortion is provided in the opposite direction. Investigation showed that the idea was correct. As a result, curves have been generated on the optimum values of prestraining to produce zero distortion for various thicknesses.

Beauchamp[14] extended the above work to cover panel structures. Instead of clamping the plate at its four corners as Henry did, he used a steel channel section to distribute the clamping force along the plate's edge. In this case the plate can be modeled as being made up of a finite number of simple beams with a uniform strain pattern. The specimens examined consisted of two stiffened panels joined by a butt weld. All fillet welding was carried out using elastic prestraining.

It has been concluded that elastic-plastic prestraining is an effective method of reducing out-of-plane angular distortion in aluminum structures. The method can easily be incorporated into welding fabrication of various panel structures.

The results do not agree with the general belief by most welding engineers that welding under restraint should be avoided since high residual stresses may remain. On the contrary, the results suggest that one way to reduce distortion is to weld under restraint and remove the restraint after welding. Residual stresses may be quite high when welding is completed, but the residual stresses may be reduced considerably when the restraint is removed. Further study is needed on this subject.

Finally, a study was conducted by Brito[15] to experimentally determine the out-of-plane distortion in welded aluminum panel structures (i.e., structures in which longitudinal and transverse stiffeners are fillet welded to a plate), and compare the data with Navy specifications. An analytical procedure was also developed for predicting out-of-plane distortion caused by angular changes along the fillet welds. Detailed description of these results can be found in his thesis and the literature.[2]

Buckling Distortion

When thin plates are welded, residual compressive stresses occur in areas away from the weld and cause buckling—Fig. 1(f). This type of distortion is one of the most troublesome, and it seems impossible to avoid, hard to understand, and is extremely expensive to correct.

The subject was primarily investigated by Pattee,[16] both analytically and experimentally. Further study was undertaken by Papazoglou[17] who was able to develop analytical solutions for both elastic and plastic buckling. As an end result of those studies, a computer-aided system has been developed for prediction and control of buckling distortion—Fig. 19.

Details of the above investigations can be found in Part II of this paper.

Part II

In the second part of this paper technical details are given for two specific kinds of distortion, namely:

1. Longitudinal (bending) distortion

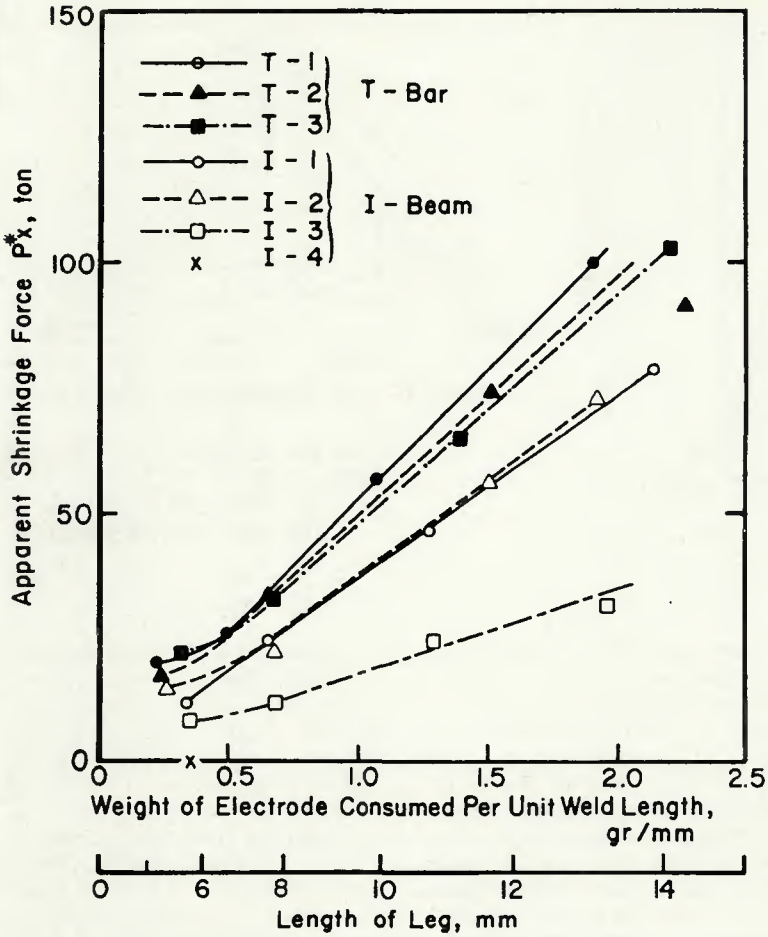


Fig. 3—Increase of longitudinal distortion during multipass welding (Sasayama, et al.)

of welded aluminum T-beams.

2. Buckling distortion of welded thin aluminum rectangular plates.

Longitudinal Distortion

When the weld line does not coincide with the neutral axis of a weld structure, the longitudinal shrinkage of the weld metal induces bending moments, resulting in longitudinal distortion of the structure. This type of distortion is of special importance when fabricating T-bars and I-beams.

Previous Investigations. Sasayama, et *al.*[18] investigated distortion of steel caused in the fillet welding of various T-bars and I-beams in low-carbon steel. Figure 2 shows the experimental results obtained. In a T-bar the deformation gradually increases as the welding progresses—Fig. 2(a).

In an I-beam the phenomena are somewhat different—Fig. 2(b); the deformation increases with the welding of the underside fillet, and it decreases with the welding of the other side. However, since the deformation due to the welding of the second fillet is generally smaller than that of the first, the residual deformation remains, even when the weight of the deposit metal of both fillet welds is equal and the geometry of the joint is symmetric.

This occurs because the effective resisting area of the joint differs between the two—that is, the upper flange does not effectively constrain the deformation during the welding of the underside of the fillet, since the upper flange is only tack welded to the web plate. On the contrary, both flanges effectively constrain the welding of the upper side fillet, since the lower flange has already been welded to the web.

In analyzing their experimental results, Sasayama, et *al.* developed a theory similar to the bending-beam theory.[18] They obtained a relation between the weight of electrode consumer per weld length and the apparent shrinkage force, P^*_x, as shown in Fig. 3. The apparent shrinkage force is taken from the relation with the curvature of longitudinal distortion 1/R. The following formula was proposed from the experiment:

$$\frac{1}{R} = \frac{P^*_x l^*}{EI} \qquad (1)$$

where, l^* = distance between the neutral axis of the beam and the acting axis of apparent shrinkage force, I = moment of inertia about the neutral axis, and E = Young's modulus.

Ujiie, et *al.*[19] of Mitsubishi Heavy Industries investigated distortion in aluminum structures and proposed a twin-GMA double-fillet welding technique that would reduce longitudinal distortion. The method seems to be effective if used for T-bars thicker than 20 mm (0.79 in.).

Experimental Investigation at M.I.T. Yamamoto[11] conducted a series of experiments at M.I.T. trying to analyze the longitudinal distortion mechanism in a built-up beam, to provide experimental data for the development of a computer program on longitudinal distortion, and to investigate a method of reducing longitudinal distortion. Experiments were performed in the following phases:

1. Simple rectangular plates were welded or heated along one edge by automatic GMA or GTA (no filler metal) welding processes.
2. T-section beams with the same web depth as the plates of the previous phase were welded by the automatic GMA welding process under the same supporting condition used in phase 1.
3. A clamped T-section beam was welded by the automatic GMA welding process.

The material used in the experiments was the aluminum-magnesium 5052-H32 structural alloy, strain hardened and non-heat treatable. Filler metals 4043 and 2319 were used, the selection based on ease of welding and material on hand.

The plates used were 12.5 mm (½ in.) thick and 1.2 m (3.9 ft) long, so that two-dimensional characteristics could be obtained. Welding conditions were changed during the various passes so that good penetration and the minimum weld length for adequate joint strength could be obtained.

Figure 4 shows a typical test specimen. It also provides dimensions as well as strain gage and thermocouple locations. Figure 5 shows the general arrangement of the experimental equipment for the simply supported test specimen.

Similar trends were observed in the development of longitudinal deflection of the simple rectangular plate using GMA and GTA welding. Convex deflections were observed during the welding process, while concave deflections resulted during the cooling stage—Fig. 6. Upon comparison of GMA and GTA welding (no filler metal) it was found that the effect of deposit metal in increasing the longi-

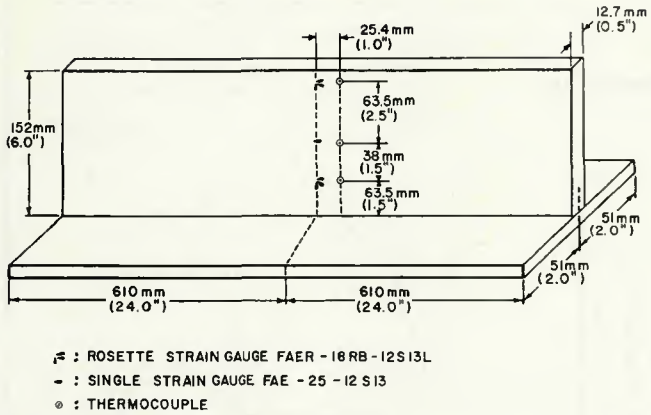


Fig. 4—Arrangement of strain gauge and thermocouple for T-section beam

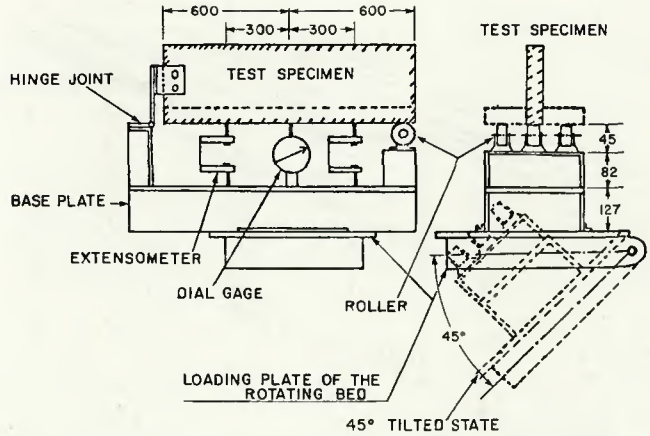


Fig. 5—General arrangement of experimental equipment for simply supporting test specimen (dimensions in mm where 1 in. = 25.4 mm)

tudinal distortion can be estimated at about 10%, assuming that thermal input is proportional to the maximum deflection.

Yamamoto[11] included in his thesis an exhaustive list of data concerning temperature distribution, strains and stresses measured during the experiments. The collection of this data was mandatory, so that a comparison with predicted results from analytical formulations of the problem could be obtained. The interested reader is referred to this thesis for further information.

Yamamoto tried to relate the experimental results with an analytical method for predicting the longitudinal deflection. Modelling the simple beam welded at its edge by a simply supported beam with the shrinkage force P_x^* applied at the welded edge as an external force, he found that the deflection y is given by:

$$Y = \frac{M_o L^2}{4EIu^2}\left[1 - \frac{\cos(u-kx)}{\cos u}\right] \qquad (2)$$

where:

$$u = \frac{kL}{2}$$

$$k = \left[\frac{P_x^*}{EI}\right]^{1/2}$$

$M_o = -L_{na} \cdot P_x^*$

L_{na} = distance between the neutral axis of the beam and the weld line

L = beam length

I = moment of inertia of beam

E = Young's modulus

P_x^* = shrinkage force

At mid length:

$$y = -\frac{M_o L^2}{8EI} \cdot \frac{2(\sec u - 1)}{u^2} \qquad (3)$$

Shrinkage force is calculated for both GMA and GTA welding processes using the values:

$I = 4.2 \times 10^5$ mm^4; $E = 70 \times 10^3$ MN/m^2; $L_{na} = 75$ mm

So:

$$P_x^* = \begin{cases} +\ 62 \text{ kN, for GMA} \\ +\ 40 \text{ kN, for GTA} \end{cases}$$

(+ sign means compression)

The stresses are then calculated using:

$$\sigma = \frac{M_o}{I/y} - \frac{P_x^*}{th} \qquad (4)$$

where: t = thickness; h = breadth.

It is found:

$$\sigma = \begin{cases} 4{,}641y - 4{,}641, \text{ for GMA} \\ 3{,}008y - 3{,}008, \text{ for GTA} \end{cases}$$

Comparison of the stresses calculated above with the experimental ones showed good agreement everywhere, except in the region near the weld where the discrepancy is expected due to the plastic region. Therefore, Yamamoto believes that the use of the shrinkage force obtained by the beam theory may be a useful tool in predicting longitudinal deflection.

The shrinkage force calculated above is now applied to fillet welding of a T-section beam. For a 150 × 100 mm (5.9 × 3.9 in.) T-beam, $I = 9 \times 10^5$ mm^4 and $L_{na} = 44$ mm (1.73 in.). Substituting these values into equation (3) we find:

$$Y_{cal} = 0.8 \text{ mm } (0.03 \text{ in.})$$

The corresponding experimental value was:

$$Y_{exp} = 0.6 \text{ mm } (0.02 \text{ in.})$$

Thus, deflection is overestimated by approximately 33%. Yamamoto proposed the use of a modification factor to take into account the difference between the shrinkage force P_x^* of a simple beam (edge welding) and a T-section beam (fillet welding). This modification factor was based on the observation that the heat intensity of the simple beam was 1.5 times that of the fillet welding in the T-section beam, which results in a value of 2/3 for the factor. The calculated deflection thus becomes $y_{cal} = 0.528$ mm (0.021 in.). Comparing analytical and experimental results for other cases, too, he finally proposed the following formula, which gives a reasonable approximation of longitudinal deflection at the mid-point of a T-section beam:

$$y = -\frac{M_o L^2}{4EI} \cdot \frac{\sec u - 1}{u^2} \qquad (5)$$

where

$$u = \frac{kL}{2}$$

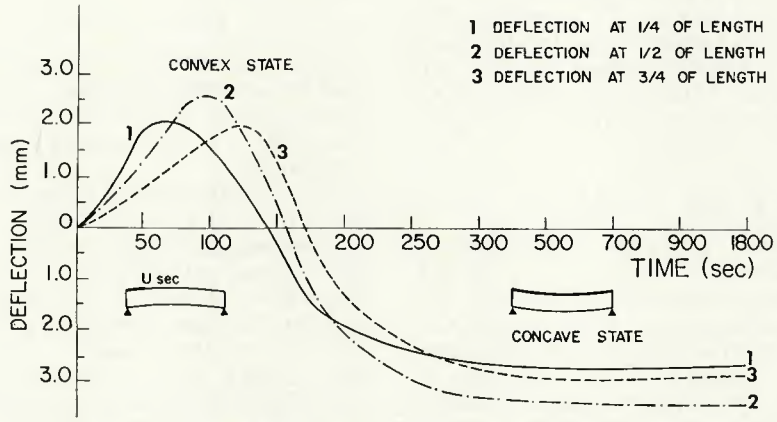


Fig. 6—Longitudinal deflection at different longitudinal points (GTA weld)

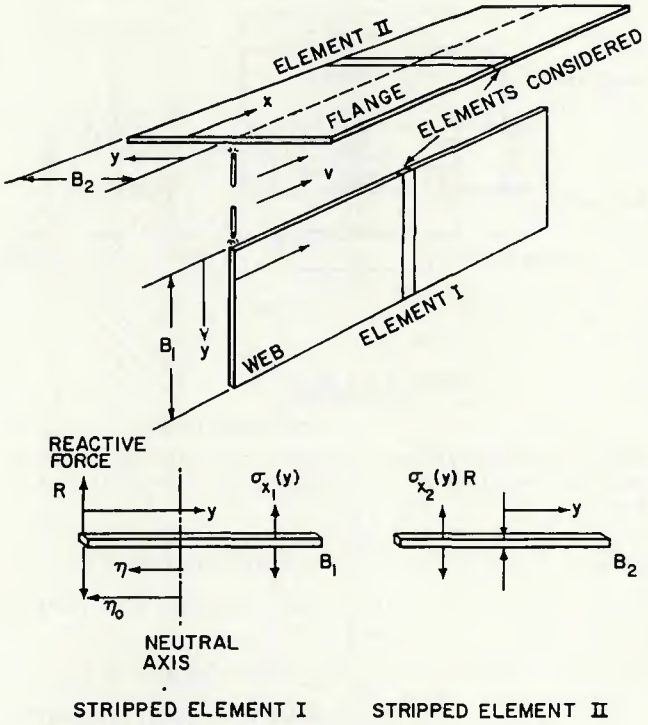


Fig. 7—Elements of T-shaped built-up beam

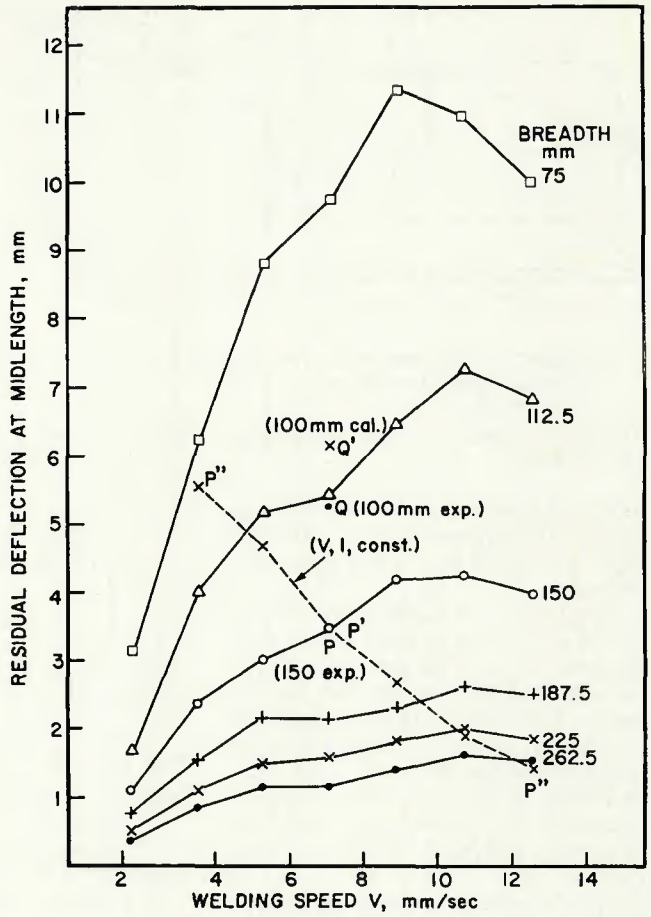


Fig. 8 (right)—Example of parameter study

$$k = \left[\frac{P_x^* C}{EI}\right]^{1/2}$$

$M_o = -L_{na} \cdot P_x^*$

$C = 2/3$, coefficient for fillet welding of a T-section beam.

Computer Analysis. The complexity of the problem of longitudinal distortion of built-up beams makes the application of complete analytical methods almost impossible. Driven by this statement, Nishida[9] tried to apply one-dimensional analysis to the problem.

In one-dimensional analysis, only one-directional stresses are considered—namely, those parallel to the fillet weld. Nishida assumes the quasi-stationary state, so that the Rosenthal solution for the temperature distribution can be used.[20-22] A narrow strip element perpendicular to the weld line is cut, both edges of which are assumed to remain straight (simple beam theory assumption). Then, the stress-strain relation is:

$$\epsilon_x = \frac{1}{E}\sigma_x + \alpha T + \epsilon_x^p \qquad (6)$$

where ϵ_x = total strain in x-direction; σ_x = stress in x-direction; E = Young's modulus; α = thermal expansion coefficient (average); T = temperature change from reference temperature; ϵ_x^p = plastic strain in x-direction

Since no external forces are present, the following equilibrium conditions hold:

$$\int_0^B \sigma_x A\, dy = 0 \qquad (7)$$

$$\int_0^B \sigma_x A\, y\, dy = 0 \qquad (8)$$

where, B = plate width. By assuming $\pi_{xy} = \sigma_y = \sigma_z = \emptyset$ the compatibility equation becomes:

$$\frac{\delta^2 \epsilon_x}{\delta y^2} = 0 \qquad (9)$$

which results in:

$$\epsilon_x = a + by \qquad (10)$$

Using equations (6) through (10) and an iteration procedure in cases where plastic deformation has occurred, stress σ_x, and strain ϵ_x can be determined.

Nishida extended the above procedure to a T-beam, treating each element separately (Fig. 7) so that one-dimensional analysis can still be applied. Note that in this case an unknown reaction force R is present so that the equations become a little more complicated. A computer program is included in his thesis to carry out all the computations necessary.

Once the transient thermal strains are calculated, it is then possible to calculate the transient deflections of weld plates and built-up beams. Curvature, p, at a given time is equal to the quantity b in equation (10). Since the simple beam theory is assumed, the following relation holds:

$$p = \frac{d^2W(x)}{dx^2} \qquad (11)$$

where W is the deflection in the y-direction at location x. The shape of deflection, W, can then be obtained by integrating the known p-curve twice along the x-direction.

The sensitivity of the results to material properties at the high temperature region was found to be great. Therefore, Nishida suggested that the precise value of high-temperature properties should be used in the above calculations.

Based on the above analysis which as is seen later was proven to be adequate for analyzing the problem of longitudinal distortion, Nishida conducted a parametric study to investigate the effect of various factors on values of residual deflection (deflection after welding is completed and the specimen has cooled to room temperature) at midlength.

Figure 8 shows an example of this parametric study, in which the value of heat input was kept unchanged. It can

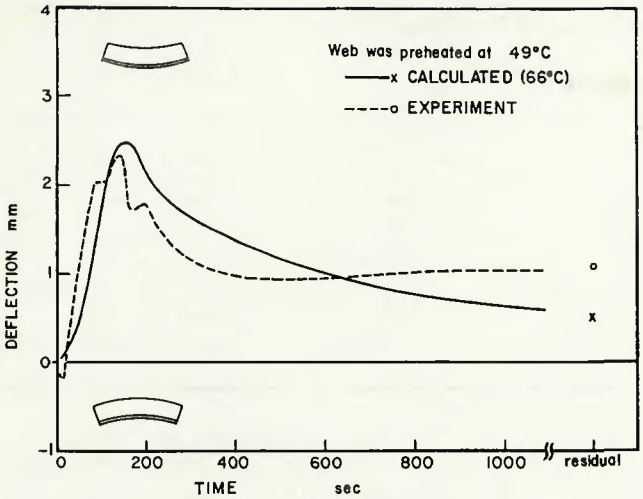


Fig. 9—Changes of deflection during welding fabrication of a T beam (the web plate was preheated to 120 F, i.e., 49 C)

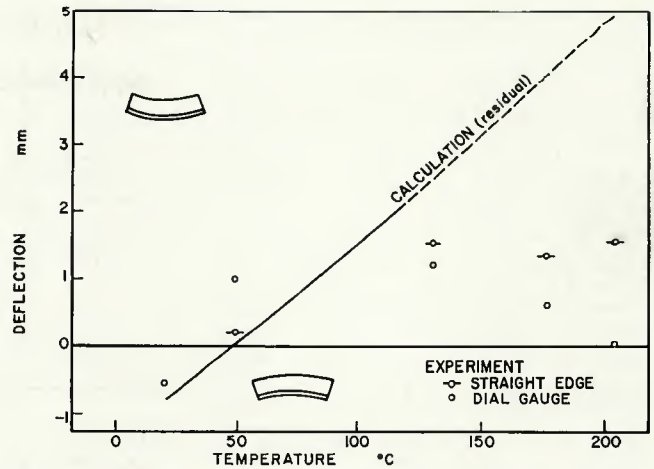


Fig. 10—Calculated final center deflection vs. preheating temperature of the web

be seen that when welding speed is increased while current, I, and voltage, V, are kept constant, the amount of distortion decreases rather drastically. On the other hand, when welding speed is increased while heat input is kept unchanged, residual distortion increases. There is also an indication that a welding speed exists where distortion becomes maximum.

Distortion Reduction. In an effort to reduce longitudinal distortion two methods were used: the clamping method and the differential heating method.

The clamping method is widely used in industry, but it is not helpful in reducing residual distortion in every situation (sometimes only transient distortion is prevented). The experiments conducted by Yamamoto[11] showed that the method is not effective in reducing the final deflection of T-shaped built-up beams. Nishida[9] simulated the clamping conditions using the one-dimensional program previously mentioned. His results confirmed the experimental findings. The authors, however, feel that in some cases clamping might be beneficial. It is therefore proposed that, before attempting any trial-and-error sequence, it would be useful to run the computer program and predict the effectiveness of the method for the particular situation in hand.

The term "differential heating" refers to a powerful technique for distortion reduction. One of the two components to be joined is heated to some pre-determined temperature. The parts are then joined and allowed to cool. The preheated part cools and contracts more than the part initially at room temperature. The thermal stress that is generated can offset residual bending stresses that would be generated if both parts had been at the same temperature when joined. This results in a distortion reduction.

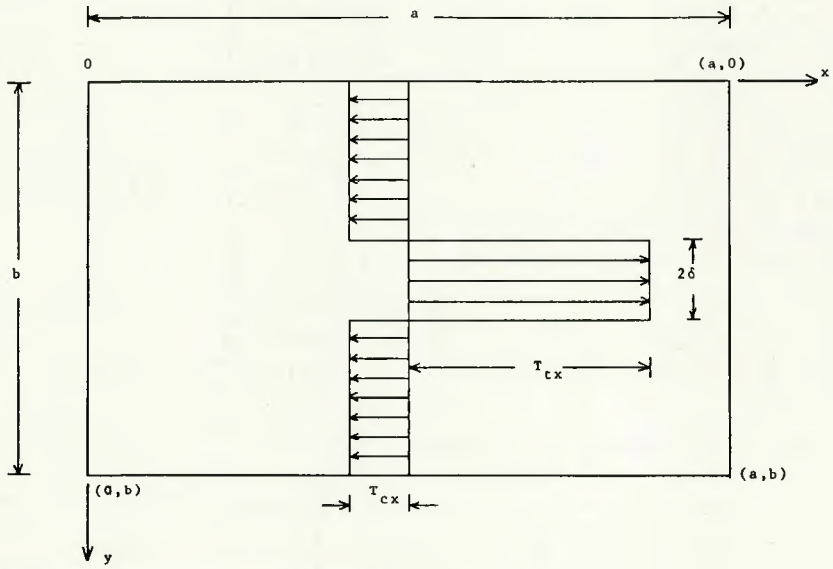


Fig. 11—Residual stress representation

A series of experiments were conducted by Serotta[12] to investigate the effectiveness of the method. The same material and specimen sizes were used as in Yamamoto's experiments, to permit ready comparison. The preheating was accomplished using electric resistance heaters.

Figure 9 shows the results of a representative experiment, as they compare with results obtained using the one-dimensional computer program. The matching of the two curves is good. Figure 10 shows how the deflection after welding changes with the preheated temperature of the web. Discrepancies between experimental data and analytical results appear when the web is heated to a relatively high temperature. It is believed that the experimental data for high preheating temperatures are not accurate, since the temperature differential will be reduced by conduction. Figure 10 shows that zero deflection can be achieved by heating the web to around 50 C (122 F).

When the welding is done in two passes, the best technique is to produce a slightly positive distortion after the first welding pass by using a higher preheating temperature—75 C (167 F), for example—so that distortion after the second pass will be close to zero.

The computer program developed by Nishida can be used to determine the optimum welding and preheating conditions for joining T-beams of various sizes.

Buckling Distortion

When thin plates are welded, residual compressive stresses occur in areas away from the weld and cause buckling. Buckling distortion occurs when the specimen length exceeds the critical length for a given thickness in a given specimen size. In studying weld

distortion in thin plated structures, it is important to first determine whether the distortion is being produced by buckling or by bending. Buckling distortion differs from bending distortion in that:

1. There are more than one stable deformed shapes.
2. The amount of deformation in buckling distortion is much greater.

Basic information about buckling distortion and its effect on the service performance of structures can be found in references 3, 4, and 5, written by Masubuchi. In this section, the analytical and experimental investigations carried out recently at M.I.T. are summarized.

Analytical Investigation. Pattee[16] carried out an analytical study of buckling distortion, in an effort to determine the critical load of a plate under various boundary conditions. For simplicity he assumed that longitudinal residual stresses (paralled to the weld line) exist only. Furthermore, he assumed the presence of a uniform tension zone of width 2γ and magnitude T_{tx} and a uniform compression zone of magnitude T_{cx}. If the plate width is b, the equilibrium condition can be written as:

$$T_{tx} \cdot (2\gamma) = T_{cx} \cdot (b - 2\gamma) \qquad (12)$$

where the compressive stresses are taken as positive—Fig. 11.

Assuming linear elasticity, the governing differential equation can then be written as:

$$\frac{\delta^4 w}{\delta x^4} + 2\frac{\delta^4 w}{\delta x^2 \delta y^2} + \frac{\delta^4 w}{\delta y^4} + \frac{1}{D}(N_x + n_x)\frac{\delta^2 w}{\delta x^2} = 0 \qquad (13)$$

where

w = deflection (mm)
D = flexural rigidity of the plate = $\frac{Eh^3}{12(1-\nu^2)}$
E = Young's modulus [N/m²]
ν = Poisson's ratio
h = plate thickness (mm)
N_x = mid-plane load (N/m)
n_x = residual stress distribution (N/m)

Equation (13) can be solved either by assuming a solution of the form

$$w = f(y) \cdot \sin\frac{m\pi x}{a} \qquad (14)$$

where a = plate length (mm); m = number of half-lengths in x-direction or by using the so-called "energy method."

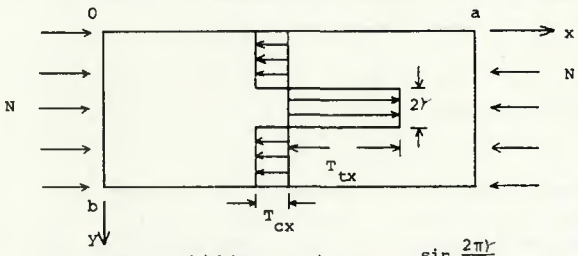
Pattee solved the equation for the four boundary conditions shown in Table 1 in the case of a butt weld. Due to the complexity of the calculations, however, the solutions could not be found in closed form.

Papazoglou[17] tried to solve the

Table 1—Boundary Conditions and Their Analogs

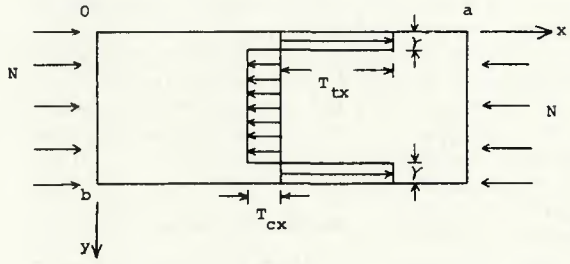
Boundary Conditions	Analogs
1. 4 edges simply-supported	1. Butt-welded plates which are large in both directions
2. Loaded edges simply-supported opposite edges free and free	2a. Long, narrow unrestrained plates 2b. The unclamped test plate
3. Loaded edges simply-supported opposite edges clamped and free	3a. The "outboard" section of a stiffened panel 3b. The partially unclamped test plate
4. Loaded edges simply-supported opposite edges clamped and clamped	4a. Plate clamped during welding 4b. Sections of stiffened plates between stiffeners

(a) The center of plate y = b/2 is welded



$$\sigma_{cr} = \frac{E\pi^2}{12(1-\nu^2)}\left(\frac{h}{b}\right)^2\left(m\frac{b}{a} + \frac{1}{m}\frac{a}{b}\right)^2 + \sigma_c \cdot \frac{\sin\frac{2\pi\gamma}{b}}{\frac{2\pi\gamma}{b}}$$

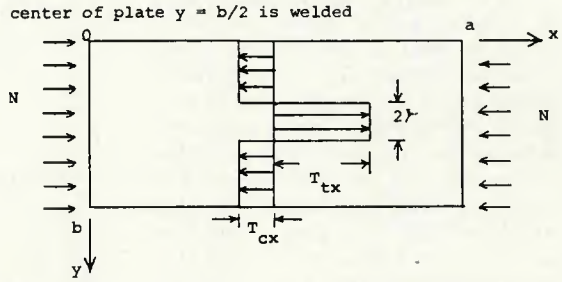
(b) Both sides y = 0 and y = b are welded



$$\sigma_{cr} = \frac{E\pi^2}{12(1-\nu^2)}\left(\frac{h}{b}\right)^2\left(m\frac{b}{a} + \frac{1}{m}\frac{a}{b}\right)^2 - \sigma_c \cdot \frac{\sin\frac{\pi(b-2\gamma)}{b}}{\frac{2\pi\gamma}{b}}$$

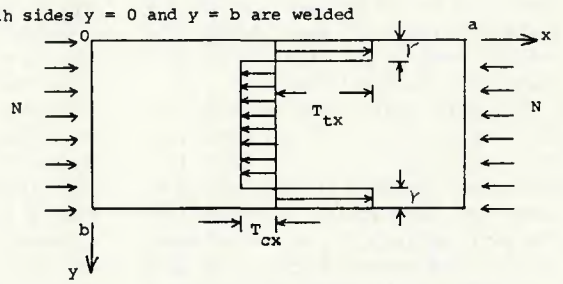
Fig. 12—Buckling of simply supported rectangular plate under uniaxial compressive stresses

(a) The center of plate y = b/2 is welded



$$\sigma_{cr} = \frac{E\pi^2}{144(1-\nu^2)} \cdot \left(\frac{h}{b}\right)^2 \cdot \left[48\, m^2\left(\frac{b}{a}\right)^2 + 48\,\frac{1}{m^2}\left(\frac{a}{b}\right)^2 + 32\right] + \sigma_c \cdot \left[\frac{2b}{3\pi\gamma} \cdot \sin\frac{2\pi\gamma}{b} + \frac{b}{12\pi\gamma} \cdot \sin\frac{4\pi\gamma}{b}\right]$$

(b) Both sides y = 0 and y = b are welded



$$\sigma_{cr} = \frac{E\pi^2}{144(1-\nu^2)} \cdot \left(\frac{h}{b}\right)^2 \cdot \left[48\, m^2\left(\frac{b}{a}\right)^2 + 48\,\frac{1}{m^2}\left(\frac{a}{b}\right)^2 + 32\right] - \sigma_c \cdot \left[\frac{2b}{3\pi\gamma} \cdot \sin\frac{2\pi\gamma}{b} - \frac{b}{12\pi\gamma} \cdot \sin\frac{4\pi\gamma}{b}\right]$$

Fig. 13—Buckling of clamped rectangular plate under uniaxial compressive stresses

equation in closed form. Using the method developed by Yoshiki, *et al.*[23] he transformed the partial differential equation into the following integral equation.

$$w(x, y) = -\int_0^a \int_0^b [(N_x + n_x) \frac{\delta^2 w}{\delta E^2}] \cdot G(x, y; E, n)\, dEdn \qquad (15)$$

where $G(x, y; E, n)$ = appropriate Green's function to satisfy the boundary conditions.

The integral equation was solved for the cases of butt welds and edge welds

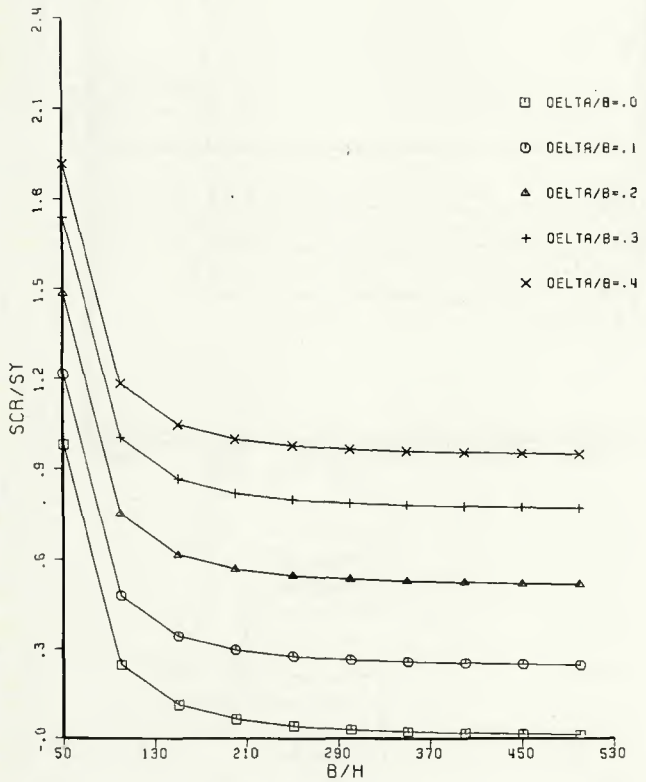


Fig. 14—Critical stress for a simply supported square plate with bead-on-plate weld

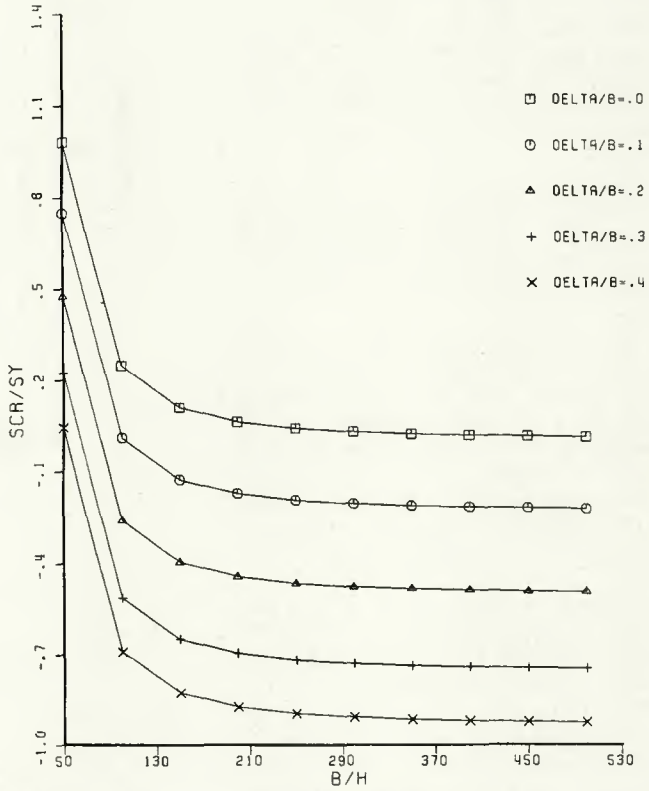


Fig. 15—Critical stress for a simply supported square plate with two edge welds

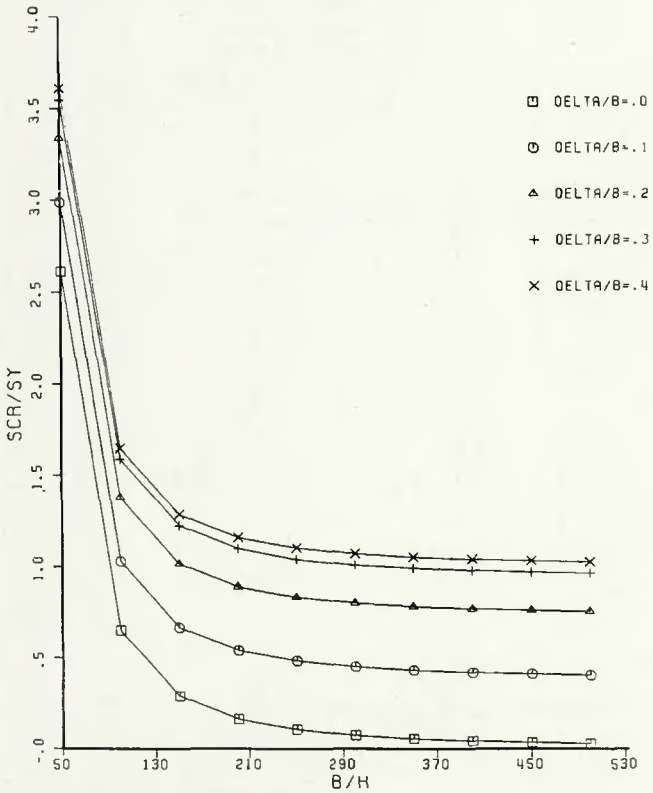


Fig. 16—Critical stress for a clamped square plate with bead-on-plate weld

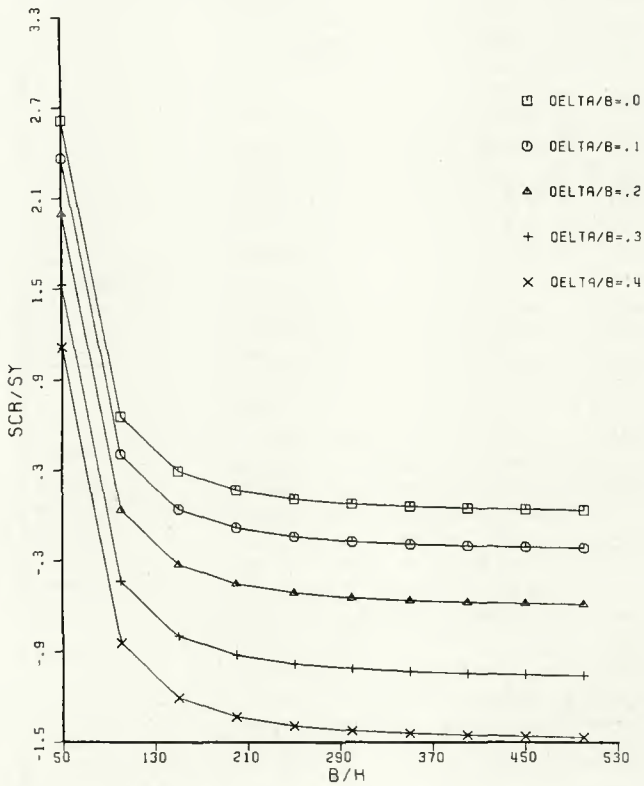
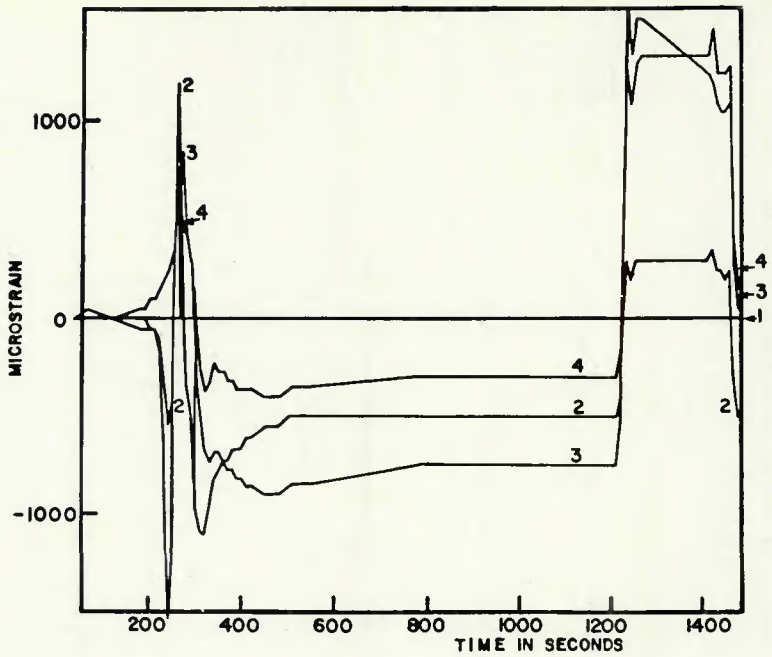


Fig. 17—Critical stress for a clamped square plate with two edge welds

Fig. 18—Test I-B strain



for both simply supported and clamped boundary conditions.

Figures 12 and 13 show the results of the analysis for the cases of simply supported and built-in loading edges respectively. Note that the results are very similar to the ones obtained by investigators at Kawasaki Heavy Industries.[15]

Figures 14 through 17 show plots of non-dimensionalized critical stress, σ_{cr}/σ_y (σ_y = yield stress of material used), as a function of b/h and γ/b for the case of square plates (a = b) and m = 1.

Several notes should be made regarding the results obtained. First, one should note that similar results occur in the case of bead-on-plate welds for both simply-supported and built-in boundary conditions. The only difference is that somewhat higher critical loads are found in the clamped case for the same values of the parameters.

A second observation is that in the case of edge welds things are somewhat reversed. The critical load again decreases with increasing breadth-to-thickness ratio, b/h, but it now increases as γ/b decreases—Figs. 15 and 17. A possible physical explanation is that the compressive residual stresses are now confined in one region only (the middle region of the plate) and hence a lower external compressive load is required for buckling as the value of these compressive stresses increases with increasing tensile zone width.

Finally, it should be noted that, in the case of edge welds, negative values of the critical load are observed. This means that it is possible for the plate to buckle due to the residual stress distribution only (no external loads) or even if tensile external stresses, lower than critical ones, are present. Although this fact may seem strange, it was actually observed during the conduction of experiments at M.I.T.[16] The above result may find practical applications in fabrication. To avoid buckling, it may be enough to stretch the plates during welding and/or even have them in a prestressed condition during service.

One should be very careful when utilizing the above analytical results. It is well established that, for a given plate, critical dimensions exist which will prevent buckling. A problem in buckling, however, has five variables—namely, plate material, length,

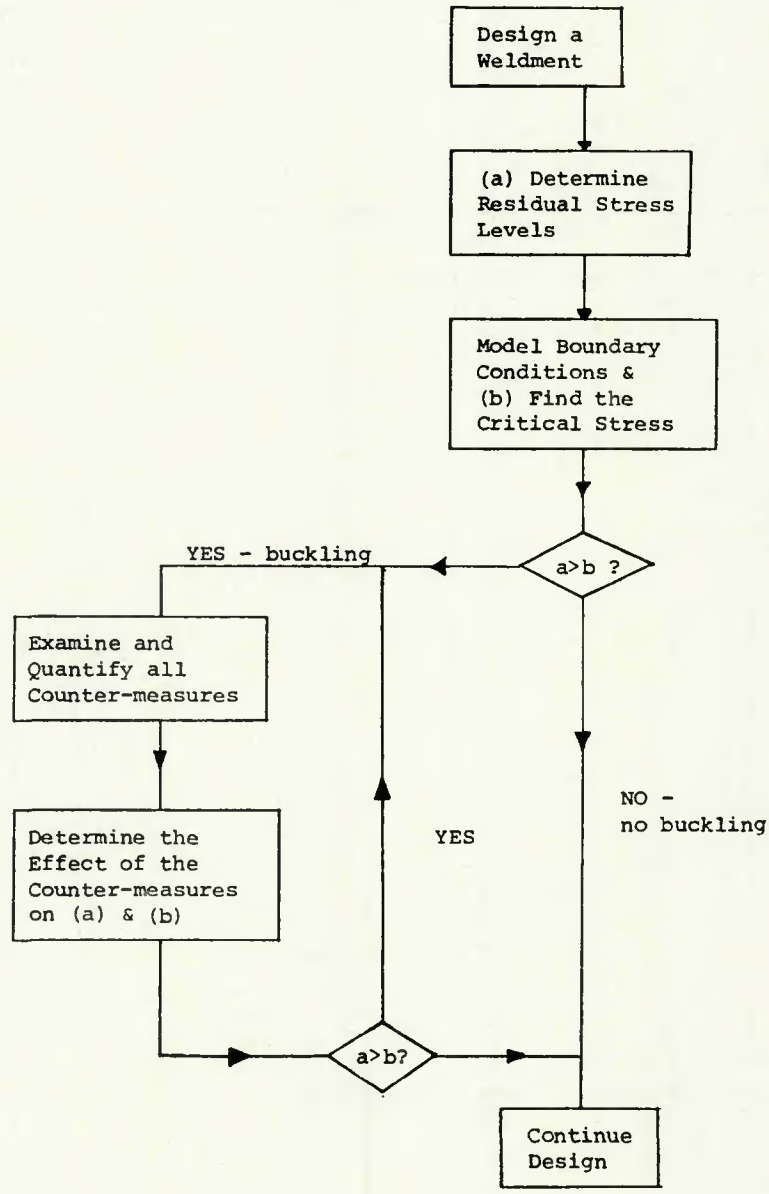


Fig. 19—Flow chart for the "System"

width, thickness, and critical stress. Hence, it is necessary to specify four of these in order to find the fifth, or three in order to find a ratio between the other two (e.g., aspect ratio).

A normal design sequence will define material choice and plate dimensions first. Then a critical stress will be found. If one wishes to know another variable's value, one must predict the welding stress. This becomes an iterative procedure best suited to preliminary design.

The main reason for caution is a misinterpretation of what critical dimensions will allow. One might suppose that any smaller length or width and any greater thickness is always good. This is not entirely true.

The critical load, for uniform compression, can be calculated by the general formula:

$$(N_x)_{cr} = k\frac{\pi^2 D}{b^2} \qquad (16)$$

where k is a function of a/b and the boundary conditions. As the width decreases, the critical stress rises. Hence, one can make b so small relative to the length that the plate behaves like a beam; and it is well known that beams are not so stiff as plates.

If the length is decreased, the critical load will also rise, but not greatly. Also, increasing the plate thickness is always safe, but not necessarily economical.

Plastic Buckling. Papazoglou[17] further expanded the study of compressive strength of welded plates to cover plastic buckling. As a first step he generalized the analysis developed by Fujita and Yoshida[24] and which was based on Stowell's deformation theory of plasticity.[25] Only the cases of simply supported rectangular plates were investigated with welds made along the center and along the two edges. For each of these two cases three subcases were considered:

1. The whole plate is in the plastic region.
2. Only the portion of the plate where compressive residual stresses exist is in the plastic region.
3. The whole plate is in the elastic region (in which case the results obtained by the linear elasticity analysis were recovered).

The assumptions regarding the residual stress distribution which were made in the elasticity analysis were also made here. Linear strain hardening was assumed. Closed form analytical solutions were found and the interested reader is referred to Papazoglou's thesis.[17]

The results obtained were checked vs. available experimental data, and generally good correlation was observed. Papazoglou thinks that this is strange because the deformation theory of plasticity is approximate and cannot be justified mathematically. On the other hand, results obtained by other investigations using the incremental theory of plasticity on ordinary plates (unwelded) appear to be in considerable disagreement with experimental data. This may be due to the high sensitivity that the more accurate incremental approach has on initial distortion and the difficulty in obtaining perfectly flat plates in the experiments.

Despite this drawback, Papazoglou decided to carry an analysis using the incremental theory of plasticity as developed by Handelman and Prager. Results of this highly complicated mathematical analysis are available.[17]

Experimental Investigation. Pattee[16] conducted a series of experiments to determine the buckling behavior (during and after welding) of variously dimensioned aluminum plates with a number of different boundary conditions. Material used was the 5052-H32 aluminum alloy. Eighteen specimens were tested with thickness of 1.5, 3, and 4.5 mm (0.06, 0.12, and 0.18 in.). All specimens were 1,800 mm (5.9 ft) long, with widths 300, 600, and 1,200 mm (0.98, 2.0 and 3.94 ft).

Two specimens were tested for each combination of the above parameters. Thermocouples and strain-gages were located on each of them in order to measure temperature and strain distributions. An automatic GTA system was chosen for the welding, using 5456 alloy filter metal. Since welding of thin plates can cause many problems (local buckling, "burn-through," etc.), the backing plates and the test specimens were preheated and continuous heating was applied in front of the arc. The welding quality was mixed (good and bad), but generally speaking acceptable.

During the experiments four types of data were collected: temperature, strain, stress and photographic. Figure 18 shows a typical curve for strain versus time. Note that only the longitudinal strain ϵ_x was measured and hence the longitudinal stress σ_x could only be calculated.

The temperature curves looked much as expected. The traces from the thermocouples nearest the weld-line show very steep slopes as the arc approaches. Those further from the weld are not as steep or high. The temperature approaches room temperature asymptotically during cool-down.

The stress and strain curves had four distinct regions, as can be seen from Fig. 18. In region 1 the welding has started but few effects are noted in the center of the plate. In region 2 the arc is approaching and there are large strains in the plate. The arc has passed in region 3. The metal is cooling and residual stresses are forming. Finally, in region 4 the plates are unclamped.

Local buckling was a major problem while welding the 1.5 mm (0.06 in.) thick plate. Whenever the plate became too hot, the surface would rear up and cause numerous problems. Arc instability would result. Without the presence of the backing plate, burn-through would occur. The wave length of this buckling was about 150 mm (5.9 in.). The occurrence of this local buckling was not predicted by the analysis discussed in the previous paragraph, since the measured welding stresses were much smaller than the analytically predicted critical ones. It is felt that the reason for this is that the elastic analysis does not hold and so this particular problem should be examined using plasticity.

The correspondence between theory and experiments was very good in all other cases. Buckling occurred whenever the theory predicted it.

Systematic Prediction and Control of Buckling. As was pointed out in the first paragraph of this section, any plate with given dimensions has some critical buckling load. To avoid failure, the welding stresses must remain below this level. This can be achieved by welding less, using less heat or removing the heat.

The surest way to weld less is to use intermittent welding. As a rough estimate one can say that by halving the amount of welding, the critical load is doubled. Another way is to decrease the weld-bead size, which results in smaller heat requirement during welding and hence in lower stress levels. As a third way, removal of welding heat from the plate using chill bars, water-cooled backing plates, etc., also results in reduced stress levels. Unfortunately, however, this quenching can produce brittle fractures.

One can see from the above that, within normal operating ranges, lower heat inputs are very significant in lowering the stress levels.

Finally it is worth noting that increasing the transverse moment of inertia of a structure will give as a result an increase in its resistance to buckling. This can be achieved by a plate thickening or by a decrease in stiffener plating. Both ways, however, are not always the reliable alternative, since both require more welding and more material which result in an increase in weight and cost.

Driven by these observations and based on the analytical and experimental investigations conducted so far, Pattee proposed a systematic approach to the buckling problem. A flow chart of this "system" is shown in Fig. 19. Its components can be de-

scribed as follows:

1. Derivations that describe the buckling due to welding of thin plates with commonly encountered boundary conditions.
2. Flexible computer programs which calculate either critical load or critical dimensions.
3. A welding simulation program to predict residual stresses.
4. Calculations which determine the effect of any corrective measures (as those described in the beginning of this paragraph).

An example showing how this system works can be found in the literature.[16]

Acknowledgments

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