An Extended Rosenthal Weld Model

A moving heat source weld model can be extended to include effects of phase changes and circulations in the weld pool

BY A. C. NUNES, JR.

ABSTRACT. Moving heat source weld models of the Rosenthal type can be built up in multipolar expansion form. Within a multipolar expansion content it is proposed to model phase changes by thermal dipoles and circulations in the molten weld pool by thermal quadrupoles.

Introduction

The moving heat source weld model, which Daniel Rosenthal popularized in publications (Ref. 1-4) beginning in 1935, appears at first glance to represent an appropriate idealization with which to compare and gain an understanding of real welds. The idealization is even more compelling when calculations based on this model can be turned out quickly and effortlessly by computer.

Although it is attractive, the Rosenthal model in its pristine form has certain defects. Perhaps the chief defect is that empirical observations show that liquid metal flow patterns in the molten weld pool strongly affect the shape of the molten weld pool (Ref. 5-7). Phase changes have also been considered to exert an important effect upon molten weld pool shape (Ref. 8).

In what follows, a method is suggested to incorporate the effects of liquid metal flow in the molten weld pool and phase changes without sacrificing the elegance of the moving heat source model. It is proposed to incorporate these features by introducing higher order multipoles, namely dipoles and quadrupoles into the moving heat source model. (The writer prefers a multipole expansion rather than a strictly Taylor series expansion because of what he considers to be the greater physical impact of the former.)

In a multipole weld model, each of the multipoles required to bring the calculated temperature configuration into register with empirical observations can be assigned a physical significance. The monopole represents the power flow into the workpiece. Comparison of the monopole strength with the input power yields a process efficiency figure. Dipoles and higher order poles represent heat flows from one point to another within the weld. The latent heat is absorbed at the front of the molten weld pool and given off at the rear as the metal melts and then solidifies; this perturbs the flow of heat in the vicinity of the molten weld pool somewhat as would a pair of equal and opposite heat sources, i.e., a thermal dipole separated along the direction of welding.

The flow of heat carried from the central regions of the molten weld pool to the edge regions (or vice versa) by fluid circulations within the pool may reasonably be represented by a pair of heat sources at the pool edges balanced by a pair of heat sinks at the molten weld pool center, equivalent to a pair of equal and opposite and displaced dipoles or to a quadrupole component. Several quadrupole components would provide the basis for incorporating molten weld pool circulation effects into the moving heat source weld model.

Even with the incorporation of the proposed multipole extensions, moving heat source models still neglect property gradients, e.g., the variation of thermal conductivity with temperature. Other effects not considered here, such as geometry changes due to sagging of the root of the molten weld pool, might be accommodated in a moving heat source context. Nevertheless, keeping in mind imperfections of this kind, the extension of the Rosenthal type moving heat source weld model through incorporation of higher order multipoles appears promising as a means of representing phase change and molten weld pool circulation effects.

Multipole Expansion of a Moving Distributed Power Source

Inside an isotropic, homogeneous material of constant thermal diffusivity α, the temperature field T varies in space coordinates, x,y,z, and time t in accordance with the differential equation of heat transfer:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (1)$$

For steady state welding processes with coordinates fixed to the moving heat source, the time derivative of temperature is zero. Thus, the time derivatives of temperature in coordinates fixed in the workpiece material must be equal and opposite to the temperature changes produced by movement with the heat source. Thus, for a heat source moving in the positive x-direction with velocity V, the time derivative of the temperature at a site fixed in the workpiece is:

$$\frac{\partial T}{\partial t}=-V\frac{\partial T}{\partial x}\,. \qquad (2)$$

Insertion of equation (2) into equation (1) yields the well known differential equation:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=-\frac{V}{\alpha}\frac{\partial T}{\partial x}\,. \qquad (3)$$

A solution of equation (3) which represents the temperature field produced in an infinite conductive continuum at x,y,z by a unit strength heat source at position ξ, η, ζ is:

$$U=\frac{1}{4\pi k}\frac{e^{-\frac{V}{2\alpha}(r+x-\xi)}}{r} \qquad (4)$$

where k is the thermal conductivity of the workpiece and:

$$r=\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}\,. \qquad (5)$$

Equation (3), being linear, permits the superposition of its solutions. If we wish to represent the temperature distribution about a weld, and if we know the distribution of the power input density p:

$$p=\frac{dP}{dv} \qquad (6)$$

where P is power input and v, volume; then:

$$T-T_o=\int U(\xi,\eta,\zeta)\,p(\xi,\eta,\zeta)\,d\xi d\eta d\zeta \qquad (7)$$

where T_o is the base temperature of the workpiece.

U may be expanded in a Taylor series about a point $(\xi,\eta,\zeta)=(0,0,0)$ such that:

A. C. NUNES, JR., is an Aerospace Engineer, Materials and Processes Laboratory, NASA, Marshall Space Flight Center, Huntsville, Alabama.

$$U(\xi,\eta,\zeta) = U(0) + \xi\left(\frac{\partial U}{\partial \xi}\right)_o + \eta\left(\frac{\partial U}{\partial \eta}\right)_o + \zeta\left(\frac{\partial U}{\partial \zeta}\right)_o$$
$$+ \frac{\xi^2}{2}\left(\frac{\partial^2 U}{\partial \xi^2}\right)_o + \frac{\eta^2}{2}\left(\frac{\partial^2 U}{\partial \eta^2}\right)_o + \frac{\zeta^2}{2}\left(\frac{\partial^2 U}{\partial \zeta^2}\right)_o$$
$$+ \xi\eta\left(\frac{\partial^2 U}{\partial \xi\,\partial \eta}\right)_o + \eta\zeta\left(\frac{\partial^2 U}{\partial \eta\,\partial \zeta}\right)_o + \xi\,\zeta\left(\frac{\partial^2 U}{\partial \xi\,\partial \zeta}\right)_o + \ldots \qquad (8)$$

If the expansion of U is inserted in equation (7), the multipole expansion of a moving distributed power source is obtained:

$$T - T_o = \dot{Q}\,U(0) + \dot{Q}_\xi\left(\frac{\partial U}{\partial \xi}\right)_o + \dot{Q}_\eta\left(\frac{\partial U}{\partial \eta}\right)_o + \dot{Q}_\zeta\left(\frac{\partial U}{\partial \zeta}\right)_o$$
$$+ \dot{Q}_{\xi\xi}\left(\frac{\partial^2 U}{\partial \xi^2}\right)_o + \dot{Q}_{\eta\eta}\left(\frac{\partial^2 U}{\partial \eta^2}\right)_o + \dot{Q}_{\zeta\zeta}\left(\frac{\partial^2 U}{\partial \zeta^2}\right)_o$$
$$+ \dot{Q}_{\xi\eta}\left(\frac{\partial^2 U}{\partial \xi\,\partial \eta}\right)_o + \dot{Q}_{\eta\zeta}\left(\frac{\partial^2 U}{\partial \eta\,\partial \zeta}\right)_o + \dot{Q}_{\xi\zeta}\left(\frac{\partial^2 U}{\partial \xi\,\partial \zeta}\right)_o + \ldots \qquad (9)$$

$\dot{Q}$ represents the total power input:

$$\dot{Q} = \int p(\xi,\eta,\zeta)\,d\xi d\eta d\zeta \,. \qquad (10)$$

The term $\dot{Q}$ U(0) is equivalent to a single moving heat source. It is the monopole component of the multipole expansion of the moving distributed power source. $\dot{Q}$ is the monopole component of the power distribution.

$\dot{Q}_\xi$, $\dot{Q}_\eta$, and $\dot{Q}_\zeta$ are the dipole components of the power distribution, and:

$$\dot{Q}_\xi = \int \xi\, p(\xi,\eta,\zeta)\,d\xi d\eta d\zeta \qquad (11)$$

etc.

$\dot{Q}_{\xi\xi}$, $\dot{Q}_{\eta\eta}$, $\dot{Q}_{\zeta\zeta}$, $\dot{Q}_{\xi\eta}$, $\dot{Q}_{\eta\zeta}$, and $\dot{Q}_{\xi\zeta}$ are the quadrupole components of the power distribution:

$$\dot{Q}_{\xi\xi} = \int \frac{\xi^2}{2}\, p(\xi,\eta,\zeta)\,d\xi d\eta d\zeta \qquad (12)$$

etc.

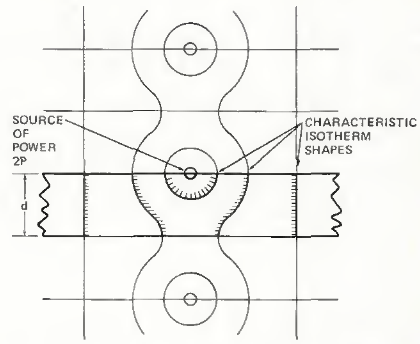

Fig. 1—Use of an array of monopole heat sources of power 2P spaced at a distance 2d to model the heat flow in a plate of thickness d subjected to a heat source of power P acting on one surface

The Rosenthal Weld Model

The classical Rosenthal weld model is based on the monopole heat source. In order to model a point source of power P applied to the surface of a plate of wide extension in the x-y plane and of thickness d in the z-direction an array of monopole heat sources of strength 2P spaced at distance 2d, as shown in Fig. 1, is required. The temperature distribution is given by:

$$T - T_o = 2P\left[U(n=0) + \sum_{n=1}^{\infty}\left(U(r_n) + U(r_n')\right)\right] \qquad (13)$$

where:

$$r_n = \sqrt{x^2 + y^2 + (z - 2nd)^2} \qquad (14)$$

and:

$$r_n' = \sqrt{x^2 + y^2 + (z + 2nd)^2} \qquad (15)$$

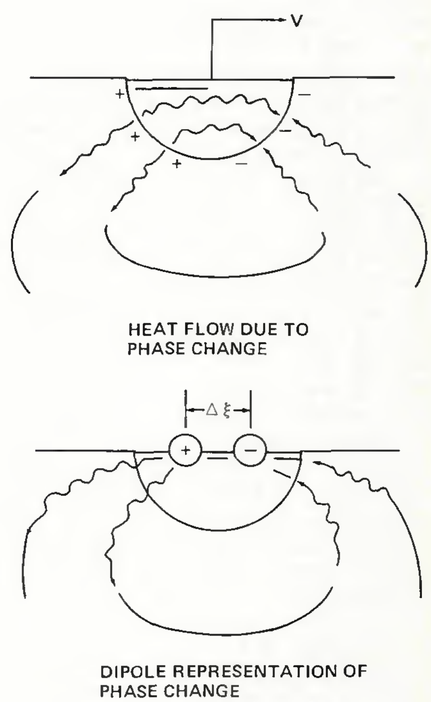

Fig. 2—Dipole field approximation of the effect of the welding phase change

Table 1—Some Symbols and Definitions in the Order of Their Usage

Symbol and definition		Initial reference equation numbers
α	—thermal diffusivity	(1), (2)
T	—temperature field	(1), (2)
x,y,z	—space coordinates	(1), (2)
t	—time	(1), (2)
V	—velocity of moving heat source	(3)
ξ,η,ζ	—heat source positions	(4), (5), (7)
k	—thermal conductivity of workpiece	(4)
p	—power input density	(6)
P	—power input	(6)
v	—volume	(6)
T_o	—workpiece base temperature	(7)
$\dot{Q}$	—total power input	(9), (10)
d	—plate thickness	(14)
K_o	—modified Bessel function, third kind, zeroth order	(16)
ρ	—weld metal density	(18)
L	—weld metal heat of fusion	(18)

wherein x, y and z locate the sites at which the temperature is calculated vs. ξ, η, and ζ which locate the heat sources. Once the heat source has been located at $\xi = 0$, $\eta = 0$, $\zeta = 0$ (i.e. at the coordinate origin) the ξ, η, ζ coordinates need not be mentioned further. Both ξ, η, ζ and x, y, z coordinates move together.

The component terms in equation (13) diminish with n and permit $T - T_o$ to converge and to be evaluated without problems by computer.

For complete joint penetration welds where the weld root and face widths are identical, the temperature distribution of equation (13) approaches a moving line source:

$$T - T_o = \frac{P}{2\pi kd} e^{-\frac{V}{2\alpha}x} K_o\left(\frac{V}{2\alpha}R\right) \quad (16)$$

where K_o is the zeroth order modified Bessel function of the third kind and:

$$R = \sqrt{x^2 + y^2} \quad (17)$$

Dipole Phase Change Representation

As a weld pool moves forward, the forward surface melts and the rear surface freezes. Under steady state conditions the temperature perturbation ΔT produced by the phase change should be

$$\Delta T = \oint_{A'} \rho LV\, U(\xi,\eta,\zeta)\, d\eta d\zeta \quad (18)$$

where ρ and L are the density and latent heat of fusion respectively for the weld metal.

When the integral is evaluated, it is necessary to add a correction which cancels out the heat flow induced through the supposedly insulating half space surface from the latent heat sources on the cooling and freezing sites on the molten weld pool surface. This is accomplished by placing a symmetrical image array of latent heat sources (i.e., a symmetrical molten weld pool interface) above the half space surface.

Integration over this symmetrical, closed surface of the solid-liquid molten weld pool interface in an infinite, unbounded continuum allows symmetry for a nonconducting plane at the half space boundary; also, it adds no additional heat, since symmetry also distributes the heat from the two sets of heat sources into two symmetrical half spaces. Thus, A′, the surface of integration, should be taken to include the symmetrical "image surface" above the boundary plane of the half space containing the molten weld pool.

Representing the above heat source distribution by a pair of equal and opposite monopoles as shown in Fig. 2 of strength 2P spaced at $+\frac{\Delta\xi}{2}$ and $-\frac{\Delta\xi}{2}$ on the $\eta = 0$, $\zeta = 0$ axis required by radial symmetry:

$$\Delta T \approx 2P\left[U\left(\frac{\Delta\xi}{2}\right) - U\left(-\frac{\Delta\xi}{2}\right)\right] \quad (19)$$

or

$$\Delta T \approx 2P\Delta\xi\left[\frac{U\left(\frac{\Delta\xi}{2}\right) - U\left(-\frac{\Delta\xi}{2}\right)}{\Delta\xi}\right] \quad (20)$$

If P is allowed to become very large as $\Delta\xi$ approaches zero so that a limit $\dot{Q}_\xi$ exists such that:

$$\dot{Q}_\xi = \lim_{\Delta\xi \to 0} 2P\Delta\xi \quad (21)$$

then as $\Delta\xi$ approaches zero,

$$\Delta T \approx \dot{Q}_\xi\left(\frac{\partial U}{\partial \xi}\right)_o \quad (22)$$

where:

$$\left(\frac{\partial U}{\partial \xi}\right)_o = U_o\left[\left(1 + \frac{x}{r}\right)\frac{V}{2\alpha} + \frac{x}{r^2}\right] \quad (23)$$

The proof of equation (23) proceeds as follows from equation (4):

$$U = \frac{1}{4\pi k}\frac{e^{-\frac{V}{2\alpha}(r + x - \xi)}}{r} \quad (4)$$

$$\frac{\partial U}{\partial \xi} = \frac{1}{4\pi k}\left[e^{-\frac{V}{2\alpha}(r + x - \xi)}\frac{\partial}{\partial \xi}\left(\frac{1}{r}\right) + \frac{1}{r}\frac{\partial}{\partial \xi}\left(e^{-\frac{V}{2\alpha}(r + x - \xi)}\right)\right] \quad (1a)$$

$$= \frac{1}{4\pi k}\left[e^{-\frac{V}{2\alpha}(r + x - \xi)}\left(-\frac{1}{r^2}\frac{\partial r}{\partial \xi}\right) + \frac{1}{r}\left(e^{-\frac{V}{2\alpha}(r + x - \xi)}\left(-\frac{V}{2\alpha}\right)\left(\frac{\partial r}{\partial \xi} - 1\right)\right)\right] \quad (2a)$$

$$\frac{\partial r}{\partial \xi} = \frac{\partial}{\partial \xi}\sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}$$

$$= \frac{1}{2}\frac{2(x - \xi)(-1)}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}}$$

$$= -\frac{x - \xi}{r} \quad (3a)$$

Substituting (3a) into (2a):

$$\frac{\partial U}{\partial \xi} = \frac{1}{4\pi k}\left[e^{-\frac{V}{2\alpha}(r+x-\xi)} \left(\frac{x-\xi}{r^3}\right) + \frac{1}{r}\left(e^{-\frac{V}{2\alpha}(r+x-\xi)} \left(-\frac{V}{2\alpha}\right)\left(-\frac{x-\xi}{r} - 1\right)\right)\right] \quad (4a)$$

$$= \frac{1}{4\pi k}\frac{e^{-\frac{V}{2\alpha}(r+x-\xi)}}{r}\left[\left(1+\frac{x-\xi}{r}\right)\frac{V}{2\alpha} + \frac{x-\xi}{r^2}\right] \quad (5a)$$

$$= U\left[\left(1+\frac{x-\xi}{r}\right)\frac{V}{2\alpha} + \frac{x-\xi}{r^2}\right] \quad (6a)$$

Setting $\xi = 0$, $\eta = 0$, $\zeta = 0$, in (6a):

$$\left(\frac{\partial U}{\partial \xi}\right)_o = U_o\left[\left(1+\frac{x}{r}\right)\frac{V}{2\alpha} + \frac{x}{r^2}\right] \quad (7a)$$

which is equation (23).

Note that:

$$U_o = \frac{1}{4\pi k}\frac{e^{-\frac{V}{2\alpha}(r+x)}}{r} \quad (8a)$$

$$\text{where: } r = \sqrt{x^2 + y^2 + z^2} \quad (9a)$$

Quadrupole Pool Circulation Representation

Circulation in a molten weld pool sometimes takes the form of a flow outward from the central regions of the pool at the top of the pool towards the edges of the pool—then down and back along the bottom of the pool to the center again. Reduced surface tension in the hotter, central regions of the molten weld pool surface would promote such a circulation; this is the Marangoni effect (Ref. 7).

The Lorentz force caused by the interaction of charge currents in the weld pool with their magnetic field produces a circulation in the opposite direction, i.e., into the center and down from the surface.

Surface flows both toward and away from the center have been observed in weld puddles. One way in which minor elements may affect penetration is by alteration of surface tension so as to promote or restrict the surface tension induced circulation component. These circulations appear to enhance very significantly the heat transfer through the weld puddle.

The effect of these kinds of circulations is to extract heat from the molten weld pool center and transfer it to the edge or to the bottom of the pool. This is as if heat sinks were placed at the weld center and heat sources towards the weld edges or center.

Since heat is being transferred from one location to another, a dipole model is appropriate. However, symmetry conditions require a second, equal, and opposite dipole on the opposite side of each plane of symmetry as shown in Fig. 3. A pair of dipoles constitutes a quadrupole. Thus, a quadrupole representation is required. By symmetry argument only the three quadrupoles $\dot{Q}_{\xi\xi}$, $\dot{Q}_{\eta\eta}$, and $\dot{Q}_{\zeta\zeta}$ are needed.

If sources of power 2P spaced at $\Delta\xi$ as shown in Fig. 3 constitute the quadrupole, then the temperature perturbation ΔT due to the quadrupole is:

$$\Delta T = 2P\,[U(\xi + \Delta\xi) - 2U(\xi) + U(\xi - \Delta\xi)] \quad (24)$$

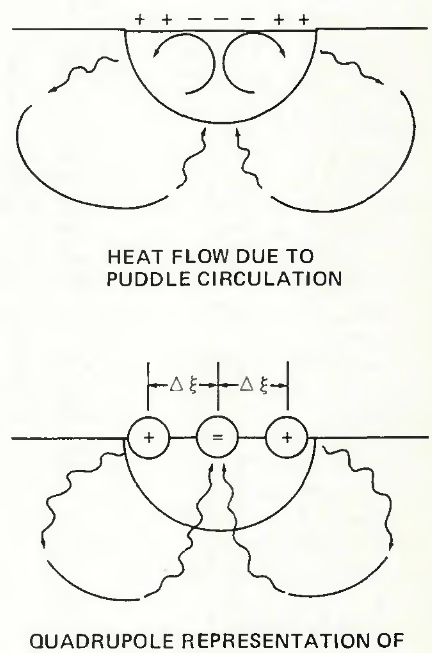

Fig. 3—Quadrupole field approximation of the effect of weld puddle circulation

or:

$$\Delta T = 2P\Delta\xi^2 \left\{ \frac{\left[\frac{U(\xi+\Delta\xi)-U(\xi)}{\Delta\xi}\right] - \left[\frac{U(\xi)-U(\xi-\Delta\xi)}{\Delta\xi}\right]}{\Delta\xi} \right\} \tag{25}$$

If P is allowed to become very large as $\Delta\xi$ approaches zero so that a limit $\dot{Q}_{\xi\xi}$ exists such that

$$\dot{Q}_{\xi\xi} = \lim_{\Delta\xi \to 0} 2P(\Delta\xi)^2 \tag{26}$$

then, as $\Delta\xi$ approaches zero:

$$\Delta T \approx \dot{Q}_{\xi\xi}\left(\frac{\partial^2 U}{\partial \xi^2}\right)_o \tag{27}$$

Similarly:

$$\dot{Q}_{\eta\eta} = \lim_{\Delta\eta \to 0} 2P(\Delta\eta)^2 \tag{28}$$

However, since the source displacements in the z-direction which create the quadrupole $\dot{Q}_{\zeta\zeta}$ need no additional image force, it is more appropriate to write:

$$\dot{Q}_{\zeta\zeta} = \lim_{\Delta\zeta \to 0} P(\Delta\zeta)^2 \,. \tag{29}$$

The nature of the circulations is such that $\dot{Q}_{\zeta\zeta}$ should be of opposite sign to $\dot{Q}_{\xi\xi}$ and $\dot{Q}_{\eta\eta}$. This is to say that, when heat is flowing outwards (positive $\dot{Q}_{\xi\xi}$ and $\dot{Q}_{\eta\eta}$) along the top of the weld pool due to fluid transport, the upward flow of fluid from the bottom of the pool superimposes a component of heat flow away from the pool bottom towards the surface (negative $\dot{Q}_{\zeta\zeta}$) onto the original heat flow pattern. The total contribution of heat flow patterns around a weld pool due to internal fluid circulations is then:

$$\Delta T = \dot{Q}_{\xi\xi}\left(\frac{\partial^2 U}{\partial \xi^2}\right)_o + \dot{Q}_{\eta\eta}\left(\frac{\partial^2 U}{\partial \eta^2}\right)_o + \dot{Q}_{\zeta\zeta}\left(\frac{\partial^2 U}{\partial \zeta^2}\right)_o \tag{30}$$

Where $\dot{Q}_{\xi\xi}$ $\dot{Q}_{\eta\eta}$, and $\dot{Q}_{\zeta\zeta}$ are independent variables (except for the above mentioned loose relation), and:

$$\left(\frac{\partial^2 U}{\partial \xi^2}\right)_o = U_o\left[\left(1 + 2\frac{x}{r} + \frac{x^2}{r^2}\right)\left(\frac{V}{2\alpha}\right)^2 + \left(-1 + 2\frac{x}{r} + 3\frac{x^2}{r^2}\right)\left(\frac{V}{2\alpha}\right)\left(\frac{1}{r}\right) + \left(-1 + 3\frac{x^2}{r^2}\right)\left(\frac{1}{r}\right)^2\right] \tag{31}$$

$$\left(\frac{\partial^2 U}{\partial \eta^2}\right)_o = U_o\left[\left(\frac{y^2}{r^2}\right)\left(\frac{V}{2\alpha}\right)^2 + \left(-1 + 3\frac{y^2}{r^2}\right)\left(\frac{V}{2\alpha}\right)\left(\frac{1}{r}\right) + \left(-1 + 3\frac{y^2}{r^2}\right)\left(\frac{1}{r}\right)^2\right] \tag{32}$$

$$\left(\frac{\partial^2 U}{\partial \zeta^2}\right)_o = U_o\left[\left(\frac{z^2}{r^2}\right)\left(\frac{V}{2\alpha}\right)^2 + \left(-1 + 3\frac{z^2}{r^2}\right)\left(\frac{V}{2\alpha}\right)\left(\frac{1}{r}\right) + \left(-1 + 3\frac{z^2}{r^2}\right)\left(\frac{1}{r}\right)^2\right] \quad (33)$$

Conclusion

For a weld made by a heat source of effective power P, i.e., the power supplied by the welding unit multiplied by a process efficiency factor, moving with constant velocity V in the x-direction on the surface (x-y plane) of a wide, long plate of thickness d, the temperature field T may be represented approximately by the expression:

$$T = T_o + P\left[F(n=0) + \sum_{n=1}^{\infty}(F(r_n) + F(r_n'))\right] \quad (34)$$

where T_o = plate base temperature; $r_n = \sqrt{x^2 + y^2 + (z - 2nd)^2}$, (equation 14); $r_n' = \sqrt{x^2 + y^2 + (z + 2nd)^2}$, (equation 15). and:

$$F(r) = \frac{1}{2\pi k}\frac{e^{\frac{-V(r+x)}{2\alpha}}}{r}\Bigg\{1 + \left(\frac{\dot{Q}_\xi}{P}\right)\left[\left(1 + \frac{x}{r}\right)\frac{V}{2\alpha} + \left(\frac{x}{r}\right)\left(\frac{1}{r}\right)\right]$$

$$+ \left(\frac{\dot{Q}_{\xi\xi}}{P}\right)\left[\left(1 + 2\frac{x}{r} + \left(\frac{x}{r}\right)^2\right)\left(\frac{V}{2\alpha}\right)^2 + \left(-1 + 2\frac{x}{r} + 3\left(\frac{x}{r}\right)^2\right)\left(\frac{V}{2\alpha}\right)\left(\frac{1}{r}\right)\right.$$

$$\left. + \left(-1 + 3\left(\frac{x}{r}\right)^2\right)\left(\frac{1}{r}\right)^2\right] + \left(\frac{\dot{Q}_{\eta\eta}}{P}\right)\left[\left(\frac{y}{r}\right)^2\left(\frac{V}{2\alpha}\right)^2\right.$$

$$\left. + \left(-1 + 3\left(\frac{y}{r}\right)^2\right)\left(\frac{V}{2\alpha}\right)\left(\frac{1}{r}\right) + \left(-1 + 3\left(\frac{y}{r}\right)^2\right)\left(\frac{1}{r}\right)^2\right]$$

$$+ \left(\frac{\dot{Q}_{\zeta\zeta}}{P}\right)\left[\left(\frac{z}{r}\right)^2\left(\frac{V}{2\alpha}\right)^2 + \left(-1 + 3\left(\frac{z}{r}\right)^2\right)\left(\frac{V}{2\alpha}\right)\left(\frac{1}{r}\right)\right.$$

$$\left. + \left(-1 + 3\left(\frac{z}{r}\right)^2\right)\left(\frac{1}{r}\right)^2\right]\Bigg\} \quad (35)$$

$\dot{Q}_\xi$ is a thermal dipole representing the phase change contribution to the temperature field. $\dot{Q}_\xi/P$ is a length which would be expected to be negative and on the order of magnitude of the size of the weld puddle.

$\dot{Q}_{\xi\xi}$, $\dot{Q}_{\eta\eta}$, and $\dot{Q}_{\zeta\zeta}$ are quadrupole moments representing the contribution of fluid circulations in the weld puddle to the surrounding temperature distribution. For circulations upwards and out at the weld center the first two terms should be positive and the third, negative. The terms $\dot{Q}_{\xi\xi}/P$, $\dot{Q}_{\eta\eta}/P$, and $\dot{Q}_{\zeta\zeta}/P$ have the dimensions of the square of a length. Wide variations in these terms may be expected depending upon widely different flow conditions in the weld pool.

Weld models employing more variables must, surely, offer possibilities for better fits to empirical data. Variables with physical significance yield insight into physical phenomena and thus are of special value. If a best fit of the above model to empirical data consistently yields process efficiencies and multipole values within expectation bands provided by theory and other kinds of empirical data, then indications will be that the model does provide the physical information which it is supposed to provide. In the meantime the model is proposed as a tentative approach to obtaining insight into heat flow in one particular kind of weld.

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