# The Melting Rate of a Single-Phase Welding Wire

*A single differential equation linking melting rate, electrode extension and current density is proposed*

BY R. L. BISH

ABSTRACT. Results of an analytical investigation of the rate at which a single-phase welding wire melts are presented. Both the heat conduction and transport terms are considered. By use of the Wiedmann-Franz law, the resulting equation may be reduced to an ordinary differential equation having constant coefficients. It is shown, by means of this equation, that Lesnewich's empirical rule may be derived, theoretically, when the wire extension is large. A method of obtaining a nearly exact relation connecting melting rate, wire extension and current density for single-phase wires is outlined.

## Introduction

Two major problems of arc welding are those of describing, in useful ways, the cooling rates in the metallic body being welded and, in the case of nonautogenous welding, the melting rate of the filler metal. Attempts to solve these problems were first made by Rosenthal (Ref. 1), who applied to them the moving heat source theory, which he developed independently of the earlier mathematical investigations of moving heat sources made by Wilson (Ref. 2).

While the solutions that Rosenthal obtained remain useful for points remote from the molten zones, they are imperfect in several ways (Refs. 3, 4), and efforts are needed to make allowance for the temperature variations exhibited by the thermophysical constants of most metals.

Indeed, a very serious obstacle in the mathematical analysis of welding-heat conduction problems is presented by the fact that, while the specific heats of most metals vary only slightly between standard temperature and the melting points, over these same ranges the thermal conductivities change by substantial amounts. This effect gives rise to a term in the governing equation of the moving heat source theory that remains proportional to the square of the temperature gradient. Since the temperature gradients encountered in welding can be very steep, this effect generally precludes the use of analytical solutions.

In the case, however, of the wire melting rate problem, there exists a remarkable simplification, which it is the purpose of this paper to describe. The method involves the use of the physical relation connecting the electrical and thermal conductivities of metals to reduce the governing equation to an ordinary differential equation having constant coefficients.

In order to achieve this, the Wilson/Rosenthal theory must first be redeveloped so that the conducted heat flux density becomes the dependent variable in the governing equation. This subject is detailed below.

## Governing Equations

In the case of a heated body that moves through space, there exists a heat flux due to transport, the density of which, referred to stationary space, equals $\rho C v \underline{n} \Phi$, where $\rho$ and C respectively, are the density and specific heat of the body, v is its speed, $\underline{n}$ is a unit vector in the direction of movement, and $\Phi$ is the temperature.

Thus, if $\underline{J}$ denotes the conducted heat flux, also referred to stationary space, then by energy conservation, if S is any fixed surface (so the mass of the body moves through it) in which $\tau$ is the volume

$$\int_S (\underline{J} + \rho C v \Phi \underline{n}) \cdot d\underline{S} = \int_\tau (\underline{j}^2 / \sigma) d\tau$$

where heating is assumed to arise from an electric current of density $\underline{j}$, and $\sigma$ is the electrical conductivity of the body.

By applying the Gauss divergence theorem and the rule for the divergence of a product of a vector and scalar (Ref.

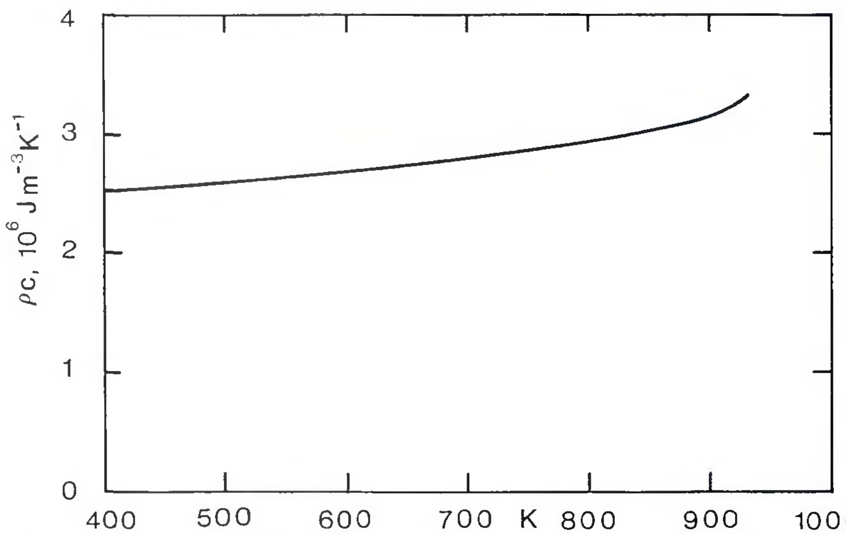


*Fig. 1 — Variation of the product of density and specific heat with temperature, for aluminum.*

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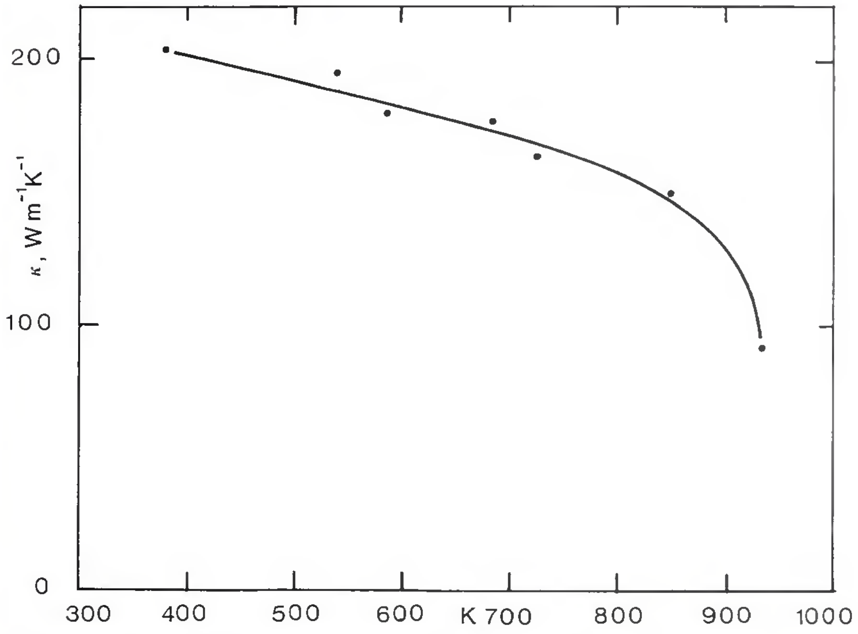


*Fig. 2 — Variation with temperature of the thermal conductivity of aluminum.*

5), we then obtain, in the usual way, assuming ρC to equal a constant,

$$\operatorname{div} \underline{J} + \rho C v \underline{n} \cdot \operatorname{grad} \Phi = j^2/\sigma$$

But, by Fourier's law

$$\underline{J} = -\kappa \operatorname{grad} \Phi \qquad (1)$$

where κ is the thermal conductivity of the body, and so

$$\operatorname{div} \underline{J} - \lambda v \underline{n} \cdot \underline{J} = j^2/\sigma \qquad (2)$$

where

$$\lambda\kappa = \rho C \qquad (3)$$

The fact that ρC is nearly constant in single-phase metal bodies is well illustrated by the data for aluminum shown in Fig. 1. In Fig. 2, on the other hand, is shown (Ref. 6) the variation of κ.

We now apply these equations to a welding wire fed at a steady speed.

**Application to a Moving Wire**

If ζ denotes distance along the welding wire measured from the feed tube exit (Fig. 3), then we may write

$$\operatorname{div} J = \frac{dJ}{d\Phi}\,\frac{d\Phi}{d\zeta}$$

while Equation 1 becomes

$$J = -\kappa\,\frac{d\Phi}{d\zeta} \qquad (4)$$

With these expressions Equation 2 then becomes

$$-\frac{J}{\kappa}\,\frac{dJ}{d\Phi} - \lambda v J = \frac{j^2}{\sigma}$$

or, on multiplying through by -κ and using Equation 3,

$$J\,\frac{dJ}{d\Phi} + \rho C v J = -\left(\frac{\kappa}{\sigma}\right) j^2$$

But, by the Wiedmann-Franz law (Ref. 7), if we express Φ in Kelvin

$$\frac{\kappa}{\sigma} = \left(\frac{\pi^2 k^2}{3e^2}\right)\Phi$$

where k is Boltzmann's constant and e is the electron charge. Thus, the governing equation for the melting rate of the wire becomes

$$\frac{dJ}{d\Phi} + \left(\frac{\pi^2 k^2 j^2}{3e^2}\right)\frac{\Phi}{J} + \rho C v = 0 \qquad (5)$$

This is an ordinary differential equation with constant coefficients, and has the solution

$$J = \mu\Phi \qquad (6)$$

where μ equals a constant, found upon substituting into Equation 5, to satisfy the quadratic equation

$$\mu^2 + (\rho C v)\mu + \left(\frac{\pi^2 k^2 j^2}{3e^2}\right) = 0 \qquad (7)$$

On the other hand, returning to Equation 4, we see, by Equation 6, that

$$\frac{d\Phi}{d\zeta} = -\frac{\mu}{\kappa}\Phi$$

and on integrating we obtain

$$\Phi = \Phi_o \exp\left(-\mu\int_0^{\zeta}\frac{d\zeta}{\kappa}\right) \qquad (8)$$

where $\Phi_o$ equals the temperature of the feed tube exit. If $\Phi_m$ is the melting temperature of the material of the wire and h is its extension then

$$\Phi_m = \Phi_o \exp\left(-\mu\int_0^{h}\frac{d\zeta}{\kappa}\right) \qquad (9)$$

and, on writing s = ζ/h, this equation becomes

$$\Phi_m = \Phi_o \exp\left(-\mu h\int_0^{1}\frac{ds}{\kappa}\right) \qquad (10)$$

which implies that μh depends only on κ(Φ), $\Phi_o$ and $\Phi_m$. Thus, for a given material and a given feed tube exit temperature

$$\mu h = B \qquad (11)$$

where B equals a constant, which may be calculated as explained below.

**Calculation of B**

Starting with the approximation $\kappa = \kappa_o$, where $\kappa_o$ is the value of κ at the feed tube exit, Equation 10 enables an initial approximation to the value of B to be found. This equals $-\kappa \ln(\Phi_m/\Phi_o)$. We may substitute this value for μh into Equation 8 to obtain an initial approximate temperature distribution along the rod. A corresponding distribution of κ(Φ) values may then be obtained from tables of thermophysical data. These values enable the integral in Equation 10 to be numerically evaluated for various points along the wire extension. Then, by Equation 10, a revised value of μh may be calculated and a revised temperature distribution calculated by means of Equation 8, leading by the same procedure to a new value for μh. By successive approximations of this

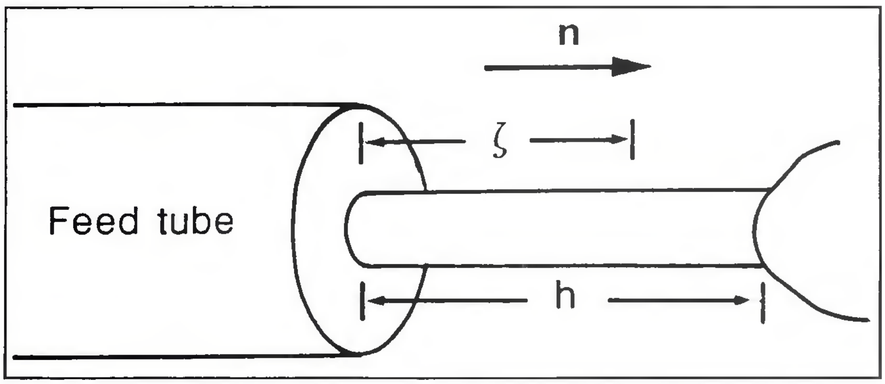


*Fig. 3 — Notation used in analysis of wire melting rates.*

kind, a sequence of $\mu h$ values is obtained, which, if the calculations have been properly performed, converges toward the constant B in Equation 11.

Calculations for aluminum wires were made as outlined, using the data in Fig. 2, and assuming a feed tube exit temperature of 400 K. The numerical integrations were carried out using Simpson's rule, with coarse and fine intervals of the subdivision of the normalized wire extension, to enable errors, due to the use of numerical integration, to be evaluated.

The sequence of values thus obtained for B was: –173, –146.5, –148.8, –148.3, W/m/K, indicating an actual value for B... close to –148.3 W/m/K.

#### Comparison with Experimental Results

Wire melting rates encountered in arc welding were the subject of extensive measurement by Lesnewich (Ref. 8), who found that the melting rate could be related to current density and electrode extension, h, by an expression of the form

$$v = Aj^2h + C'j$$

where A and C′ are constants for a particular material. Halmoy (Ref. 9) claims that this rule only holds if h is relatively large.

On the other hand, from Equations 7 and 11, it is simple to show that

$$v = \frac{B}{\rho Ch} + \left(\frac{\pi^2 k^2}{3e^2 B\rho C}\right)j^2h \quad (12)$$

where h is the wire extension, and if h is large, this tends to the same form as Lesnewich's rule. On the other hand, if the wire extension is small, Equation 12 predicts that the melting rate should pass through a minimum at an extension equaling $\sqrt{3e\,B/\pi kj}$.

## Conclusions

By incorporating the Wiedmann-Franz law into the Wilson/Rosenthal moving heat source theory, an equation has been obtained that, in the case of a wire of any single-phase metal, becomes an ordinary differential equation with constant coefficients. This equation takes into account both the heat transport and heat conduction terms and makes proper allowance for the variations with temperature of the electrical and thermal conductivities of the material of the wire. Moreover, the solution of this equation leads to the conclusion that Equation 11 holds, and in case of long electrode extensions, this leads to a relation between melting rate, welding wire extension and current density of the same form as that found experimentally.

For very short electrode extensions, on the other hand, the theory predicts that the welding wire melting rate should pass through a minimum at an extension that varies inversely with the current density. Halmoy's results suggest that this effect is real.

The constant B in Equations 11 and 12 may be calculated by an iterative procedure involving Equations 8 and 10, and by this means has been found to equal, for aluminum electrodes, about –148.3 W/m/k, assuming a feed tube exit temperature of 400 K.

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