Prediction of Welding Thermal History by a Comprehensive Solution

Thermal histories, cooling times and shapes of heat-affected zones can be predicted by a comprehensive heat conduction solution.

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ABSTRACT. A comprehensive solution has been developed to examine heat conduction in three different welding cases. Case I is a quasi-steady-state model to predict the shape of the heat-affected zone (HAZ), and the thermal histories and cooling times between 800° and 500°C (t_{85}) in wide plates. Case II is an instantaneous heat source model to predict cooling time from solidification to 100°C (t_{100}) in small test pieces. Case III is an unsteady heat model to predict t_{100} in locally preheated plates. Rosenthal's and Tanaka's formulas can be derived from a solution of Case I. These three cases were applied to the predictions of thermal histories, cooling times and HAZ shapes. The calculated results have been compared with the experimental results of several different cases of arc welding and good agreement has resulted.

Introduction

Since mechanical properties of welds such as hardness and toughness are greatly affected by welding thermal history, a number of researchers (Refs. 1-18) have reported on thermal history prediction and its application to the estimation of welded joint qualities. Rosenthal (Refs. 1, 2) derived an analytical solution for a welding heat flow problem under the assumption that a point heat source is on the plate surface and that there is no surface heat transfer. This solution has been used worldwide to predict welding thermal history and cooling rates.

Tanaka (Ref. 3) solved welding heat flow without neglecting surface heat transfer. Owing to surface heat transfer, Tanaka’s solution is more complicated than Rosenthal’s. However, surface heat transfer must be taken into account in welding heat conduction because it varies from welding process to process; it is very low in submerged arc welding, due to covering flux, and is very high in electroslag and electrogas welding, due to water cooling. Tanaka also proved that Rosenthal's solution is a special case of his solution.

In the case of high heat input welding, the assumption of a surface point heat source is not valid. A heat source in this case is considered to expand below the plate surface. In order to analyze heat flow in high heat input welding, it is necessary to derive a solution under the condition that a point heat source is located inside a plate and that heat of varying extent flows at the surface.

Rosenthal and Tanaka solved a heat flow problem of a quasi-steady state. When a plate to be welded is wide enough, heat flow approaches the quasi-steady state. This is the case with most welding. When welding test pieces for cold cracking or HAZ hardness, thermal history is influenced by the test piece size and the weld bead length. Cold cracking or hydrogen-assisted cracking can be avoided by the employment of preheating, which reduces the cooling rate, especially around 200°C (392°F), to facilitate the evolution of hydrogen from a weld. Therefore, cooling time from solidification to 100°C (212°F), t_{100}, is a thermal parameter by which the hydrogen evolution is assessed. The derivation of a solution of heat flow in a small test piece is needed to determine the necessary preheating temperature from the results of cold cracking tests. This is done by comparing cooling times of welding a small test piece with those of actual welding.

Weldability test specimens are generally preheated uniformly. However, weldments in actual welding are often preheated along a welding line only, i.e., local preheating is performed. Welding cooling times differ, sometimes significantly, from local preheating to uniform preheating.

The purpose of the present study is to present analytical solutions for thermal history in wide plate welding, small test piece welding and locally preheated welding.

Heat Conduction Models and Their Solutions

Models of Heat Flow in Welding

Constant thermal properties are assumed in the present study. Thus, the following heat conduction equation holds in this case:

\[
\frac{\partial T}{\partial T} = \kappa \nabla^2 T = \kappa \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)
\]

where \( \kappa \) is thermal diffusivity (mm\(^2\)/s).
Surface heat transfer is assumed to obey Newton's cooling law, and thus the boundary condition becomes

\[ \lambda \frac{dT}{dn} = - \alpha (T - T_a) \]

where \( n \) is normal coordinate to the surface, \( \lambda \) is thermal conductance (J/°C mm s) and \( \alpha \) is surface heat transfer coefficient (J/°C mm² s). This study introduces three different types of heat flow models.

**Model I: A Quasi-Steady-State Model for Wide Plates**

In most actual welding, a plate to be welded is regarded as infinitely wide and long from the viewpoint of heat flow due to a concentrated heat source in welding. Model I is a point heat source moving inside a plate as seen in Fig. 1A, under a quasi-steady-state condition during the whole welding time, except the very beginning and end of welding. The heat flow due to an expanded heat source encountered in high heat input welding can be investigated by distributing a point heat source of Model I inside a plate.

**Model II: An Instantaneous Line Heat Model for Small Test Pieces**

The thermal history in small test piece welding is influenced by the test piece size and the weld bead length. In this case, a quasi-steady state of heat flow cannot be attained during the whole welding period. Furthermore, the cooling time to low temperatures, such as \( T_{100} \), is not influenced significantly by the shape of the welding heat source as long as the same energy is supplied as the total welding heat input. It follows that an instantaneous line heat model, as shown in Fig. 1B, has been introduced.

**Model III: A Model for Locally Preheated Plates**

When local preheating is conducted, the temperature distribution in a plate is already nonuniform at the start of welding. Therefore, the model shown in Fig. 1C is introduced to analyze the heat flow of local preheating. The temperature distribution depends on the power of preheat, \( q_p \) (J/mm² s), and its width, \( 2b_w \) (mm).

**Analytical Solution**

To solve Equations 1 and 2, temperature, \( \theta = T - T_a \) is expressed as follows:

\[ \theta = \theta(x,y) \theta(z) \theta(t) \]

In the coordinate system shown in Fig. 1, the functions on the right-hand side of Equation 3 are given by substituting Equation 3 into Equation 1, as follows:

\[
\begin{align*}
X &= A_n \sin \left( \frac{a x}{a} \right) + B_n \cos \left( \frac{a x}{a} \right) \\
y &= A_n \sin \left( \frac{a y}{a} \right) + B_n \cos \left( \frac{a y}{a} \right) \\
z &= A_n \sin \left( \frac{a z}{a} \right) + B_n \cos \left( \frac{a z}{a} \right) \\
\theta &= \exp \left( - \left( \frac{u_n^2 + v_n^2 + w_n^2}{u_n^2} \right) \right)
\end{align*}
\]

where \( u, v \) and \( w \) are values to be determined by the boundary condition. Equation 3 must satisfy the boundary condition of Equation 2. Thus,

\[ \frac{dX}{dx} = -aX, (x = a); \frac{dX}{dx} = aX, (x = 0) \]

where \( a \) is thickness of a welded plate (mm).

Therefore, \( X \) is given as follows:

\[ X = A_n \left( \cos \left( \frac{a x}{a} \right) + \frac{B_n \sin \left( \frac{a x}{a} \right)}{u_n} \right) \]

where \( u_n \) : \( n \)-th root (characteristic value)

\[ u_n = \left( \frac{2B_n}{B_n^2 - u_n^2} \right)^{1/2} \]

where \( B_n \) is a dimensionless Biot modulus (\( = a \alpha /\lambda \)).

\( A_n \) in Equation 6 is given by the initial condition. The symmetry of heat flow gives the coefficients as:

\[ A_n = A_n = 0 \]

\( B_n \) and \( B_z \) are also given by the initial condition. Let us suppose that \( f(x), g_1(y) \) and \( g_2(z) \) are arbitrary functions of \( x, y \) and \( z \) coordinates, respectively. If the temperature distribution at the start of...
welding is assumed as a function, \( f(x)g_1(y)g_2(z) \), the following relations are given:

\[
A_y = \frac{2a^2}{(u_x^2 + B_y^2)a + 2B_y a} \\
\int_0^\infty \left( \frac{u_y}{a} x + \frac{B_y}{a} \sin \left( \frac{u_y}{a} x \right) \right) f(x)dx
\]  
(8a)

\[
\int_0^\infty B_1(v) \cos(vy)dv = g_1(y) \quad (8b)
\]

\[
\int_0^\infty B_2(w) \cos(wz)dw = g_2(z) \quad (8c)
\]

Equation 8 is valid only when a body is infinite in the y and z directions. In the case of a small test piece, \( B_y \) and \( B_z \) must be given in a form of Equation 8a.

Equation 3, along with Equations 6, 7 and 8, is the comprehensive solution for the analyses of heat flow in Models I, II and III. This solution has been applied to Models I, II and III, as shown below.

Model I

From Equation 8, relative temperature, \( \theta \), is given as follows:

\[
\theta = \sum_n A_n \left[ \frac{\cos \left( \frac{u_y}{a} x \right) + \frac{B_y}{a} \sin \left( \frac{u_y}{a} x \right) }{ \shortint_0^\infty B_1(v) \cos(vy)dv } \right] \exp \left( -\frac{\pi^2}{a^2} x \right) \shortint_0^\infty B_2(w) \cos(wz)dw \exp \left( -\pi^2 \nu^2 \right) dv
\]  
(9)

Assuming a small cubic instantaneous heat, an instantaneous heat source, \( Q \) (J) is given at \( x = d, y = 0 \) and \( z = 0 \) when \( t = 0 \) and the cubic heat source with a length of \( 2\ell_1 \) is \( d \) (mm) under the surface, as shown in Fig. 2, the initial condition becomes

\[
\theta = \frac{Q/\rho c_p(2\ell_1)}{\pi^2 a^2} \frac{\cos \left( \frac{u_y}{a} x \right) + \frac{B_y}{a} \sin \left( \frac{u_y}{a} x \right) }{ \shortint_0^\infty B_1(v) \cos(vy)dv } \exp \left( -\frac{\pi^2}{a^2} x \right) \shortint_0^\infty B_2(w) \cos(wz)dw \exp \left( -\pi^2 \nu^2 \right) dv
\]  
(10a)

where \( \rho \) is density (kg/mm\(^3\)) and \( c_p \) is specific heat (kJ/kg°C).

The initial condition of Equation 10a can be rewritten by using \( f(x) \), \( g_1(y) \) and \( g_2(z) \) as follows:

\[
\theta = f(x)g_1(y)g_2(z) = \begin{cases} \frac{Q/\rho c_p(2\ell_1)}{\pi^2 a^2} \frac{\cos \left( \frac{u_y}{a} x \right) + \frac{B_y}{a} \sin \left( \frac{u_y}{a} x \right) }{ \shortint_0^\infty B_1(v) \cos(vy)dv } \exp \left( -\frac{\pi^2}{a^2} x \right) \shortint_0^\infty B_2(w) \cos(wz)dw \exp \left( -\pi^2 \nu^2 \right) dv, & \text{if } x = d \pm \ell_1, \\ 0, & \text{otherwise}. \end{cases}
\]

Substituting Equation 10b into Equations 8a, 8b and 8c, the coefficients in Equation 9 are given as follows:
Suppose that a point heat source starts at \( t = -\tau_0 \) and that its \( z \) coordinate at \( t = 0 \) is zero. In the light of Equation 12, a solution at \( t = \tau \) for a point heat source moving at constant velocity of \( v_s \) (mm/s) can be obtained by integrating \( \theta \) with respect to \( t \) from \(-\tau_0\) to \( \tau \). This kind of solution is valid because the linearity in temperature holds in the heat conduction equation of Equation 1, and the boundary condition of Equation 2. Thus, \( \theta \) is given by the summation of the temperature increment caused by point heats from \( t = -\tau_0 \) to \( t = \tau \). When total heat input, \( Q (J) \) is considered to be heat input given at the time interval, \( dt \), then \( Q \) is expressed as \( q dt \), where \( q \) is welding heat input (J/s). In this case, \( \theta \) in Equation 12 can be considered as the temperature increment, \( d\theta \), and temperature, \( \theta \) at \( t = \tau \) is given as follows:

\[
\theta = \frac{q}{2\pi \alpha \lambda} \sum_{m=1}^\infty A_m \int_0^\infty \exp \left( -\frac{y^2 + z^2}{4\kappa} \right) \exp \left( -\frac{u_m^2}{a^2} (\tau - t) \right) \frac{\sin \left( \frac{u_m}{a} y \right)}{\sin \left( \frac{u_m}{a} \right)} \frac{\sin \left( \frac{u_m}{a} z \right)}{\sin \left( \frac{u_m}{a} \right)} \frac{\sin \left( \frac{u_m}{a} \right)}{\sin \left( \frac{u_m}{a} \right)} \frac{\exp \left( -\frac{u_m^2}{a^2} (\tau - t) \right)}{y^2 + \left( z - v_s t \right)^2} \frac{1}{4 \kappa (\tau - t)} \, dt
\]

where, \( A_m \) is given in Equation 12 and \( K_0 \) is 0 order modified Bessel function of the second kind, and \( r = \sqrt{y^2 + (z - v_s t)^2} \).

In Equation 14, \( \tau \) has been replaced with \( t \). When \( d \) is zero, \( A_m \) is \( u_m^2/(u_m^2 + B_i^2 + 2B_i) \), i.e., Equation 14 becomes Tanaka's solution of a surface point heat source.

**Model II**

A solution for Model II is used to predict welding cooling times of a small test piece. From Equation 6, the solution for this case is as follows:
Fig. 6 — Thermal histories of SAW and EGW using the groups of point heat sources shown in Fig. 4. The thermal properties used are shown in Table 2.

\[ \Theta = \sum_{a=1}^{n} \left\{ \cos \left( \frac{u_a}{a} \right) + \frac{B}{u_a} \sin \left( \frac{u_a}{a} \right) \right\} \exp \left( -\kappa \frac{u_a^2}{a^2} \right) \]

where \( B_a = \frac{b}{a} \lambda \) and \( B_c = \frac{c}{a} \lambda \) are Biot moduli.

The coefficients, \( A_n, B_m \) and \( C_s \) in Equation 15 must be determined for an instantaneous line heat source and uniform preheating. First, the line heat with a length of \( 2L \) has been considered. When the edge length of its cross-section is \( 2l \), the initial condition becomes

\[ \Theta = Q / (8 \pi c, f^2 L) \]

\[ d - \ell_1 < x < d + \ell_1; \]

\[ b/2 - \ell_2 < y < b/2 + \ell_2; \]

\[ c/2 - L < z < c/2 + L \]

\[ \Theta = 0, \text{ the other region} \]  

The initial condition of Equation 16 can be rewritten by using \( f(x), g_1(y) \) and \( g_2(z) \) in the same fashion as Equation 10b. Hence, when \( \ell_2 \) becomes zero, \( A_n, B_m \) and \( C_s \) are given from Equation 16 and Equation 8a as follows:

\[ A_n = \frac{Q}{\rho c_p \alpha} \left( \frac{2u_n^2}{u_n^2 + B_n^2 + 2B_n} \right) \]

\[ B_m = \frac{1}{c^2} \left( \frac{2v_m^2}{v_m^2 + B_m^2 + 2B_m} \right) \]

\[ C_s = \frac{1}{c^2} \left( \frac{2w_s^2}{w_s^2 + B_s^2 + 2B_s} \right) \]
Fig. 8 — $t_{95%}$ of experiments and calculations. The welding conditions and the thermal properties used are shown in Tables 1 and 2, respectively.

$$C_i = \frac{1}{L} \frac{2w_i}{w} + B_i^2 + 2B_i \frac{\sin \left( \frac{w_i}{2} \right)}{c}$$

$$\theta = T_p - T_{\infty} \begin{cases} \begin{array}{l} \left( 0 < x < a, \ 0 < y \right) \\ \left( b < x, \ 0 < z < c \right) \end{array} \end{cases}$$

The initial condition of Equation 18 can be also rewritten by using $i(x)$, $g_2(y)$ and $g_2(2)$ in the same fashion as Equation 10b. In most welding cases, $B_a$, $B_b$, and $B_c$ are order of 0.1 and thus $B_a$, $B_b$, $B_c < \pi/2$. Therefore, $A_n$, $B_n$, and $C_n$ become from Equation 8a as follows:

$$A_n = \frac{2B_a (T_p - T_{\infty})}{u_i^2 + B_i + 2B_i} \left\{ 1 + (-1)^{n+1} \right\}$$

$$B_n = \frac{2B_b}{v_n^2 + B_n^2 + 2B_n} \left\{ 1 + (-1)^{n+1} \right\}$$

$$C_n = \frac{2B_c}{w_n^2 + B_n^2 + 2B_n} \left\{ 1 + (-1)^{n+1} \right\}$$

Second, a uniform preheating case is considered. This initial condition for uniform preheating is as follows:

$$\theta = T_p - T_{\infty} \begin{cases} \begin{array}{l} (0 < x < a, \ 0 < y) \\ (b < x, \ 0 < z < c) \end{array} \end{cases}$$

Fig. 9 — Comparison of calculated and experimental $t_{95%}$. The welding conditions and thermal properties used are shown in Tables 1 and 2, respectively.

Fig. 9 — Comparison of calculated and experimental $t_{95%}$. The welding conditions and thermal properties used are shown in Tables 1 and 2, respectively.

Fig. 10 — $t_{100}$ calculation of actual welding by the linear combination of Model I and Model III. The thermal properties used are shown in Table 2.
Fig. 11 — Temperature distribution at preheat temperature = 200°C 
(bw, half width of preheating, = 100 mm) calculated by Model III. 
The thermal properties used are shown in Table 2.

Fig. 12 — Chart to estimate preheat power, qh (J/mm²s). The lines 
show temperature increases at the center of preheating which are 
calculated by Model III. The thermal properties used are shown in 
Table 2.

When a uniformly preheated test 
piece is welded, cooling times such as 
t₁₀₀₀ can be calculated by the linear com­
bination of Equation 15 along with Equa­
tions 17 and 19.

Model III

Model III is a two-dimensional (x and y direction) heat flow problem of locally 
preheated plates. In light of Equation 6, the solution for this case is as follows:

\[
\theta = Q_h / (4pc\ell_v b_w) \\
\theta = 0, \ the \ other \ region
\]

The initial condition of Equation 21 is separable like Equation 10, hence, Aₙ and Bₙ(v) in Equation 20 can be determined from Equations 8a and 8b. When the depth of the instantaneous heat 
source, f₃, becomes zero, the following equation is given from Equation 8:

\[
\theta = \frac{Q_h}{pc} \sum A_n \left[ \cos \left( \frac{u_n}{a} x \right) + \frac{B_n}{u_n} \right] \\
\sin \left( \frac{u_n}{a} x \right) \\
\exp \left( -\frac{u_n^2}{a^2} \right) x \exp \left( -\frac{v^2}{v} \right) \\
\int_{-\infty}^{\infty} \exp \left( -Kt - \frac{v^2}{v} \right) dv
\]

(22)

A solution for local preheating is ob­tained by integrating θ in Equation 22 with regard to t from t = 0 to t = τ. When qₜₗ (J/mm²s) is the power of preheat added on the unit area, then Qₜₗ =

\[
\frac{2b_w q_h dt}{2b_w q_h dt}
\]

and thus:

\[
\theta = \frac{Q_h}{pc} \sum A_n \left[ \cos \left( \frac{u_n}{a} x \right) + \frac{B_n}{u_n} \right] \\
\sin \left( \frac{u_n}{a} x \right) \\
\exp \left( -\frac{u_n^2}{a^2} \right) x \exp \left( -\frac{v^2}{v} \right) \\
\int_{-\infty}^{\infty} \exp \left( -Kt - \frac{v^2}{v} \right) dv
\]

(23)

where Aₙ is given in Equation 22.

Welding is, in general, started when 
temperature at the surface becomes a 
certain level of preheating temperature. 
Supposing τₚ is given as the time neces­
ary for θ of Equation 22 to reach the re­
quired preheating temperature, then the 
termination of preheating can be re­
garded as the onset of an addition of an­
other heat source with power of -qₜₗ at 
t = τₚ. Thus, the following equation gives 
the thermal history after the completion 
of preheating:

\[
\theta = \theta(t) - \theta(t - \tau_p)
\]

when t > τₚ

(24)
Calculation for Models I, II and III

Application of Model I to High Heat Input Welding

Single pass submerged arc welding (SAW) with three electrodes in tandem is typical of high heat input welding. Figure 3A shows a macrophotograph of a SAW weld section made under conditions shown in Table 1. In order to simulate a thermal history and to predict a HAZ shape for large heat input welds as shown in Fig. 3A, the point heat source of Model I was arbitrarily distributed in a manner shown in Fig. 4A by comparing its distribution with the shape of the molten weld pool.

Figure 5A shows the comparison of the experiment with the weld shape predicted from the calculation of the heat conduction for distributed point heat sources. The thermal properties used are in Table 2. The contour lines of the weld interface, the line of the phase transformation from ferrite to full austenite, and the subcritical (ferritic and austenitic) transformation line, A<_3, have been determined metallurgically, while the computed lines have been obtained by searching points whose peak temperatures are equal to the melting temperatures, A<_3 and A<_1, respectively. These temperatures have been obtained from empirical formulas into which the chemical compositions of the welded steel have been substituted (Ref. 19). Figure 6A shows thermal histories at the weld midthickmess, where the solid line is obtained by the computation of heat flow and the dotted line is obtained experimentally. Fairly good agreement between the experiment and the calculation is recognized from Figs. 5A and 6A.

Another typical high heat input process is electrogas welding (EGW). Contrary to SAW, the surface heat transfer in EGW is accelerated by copper scraps cooled by water. Model I can also analyze the heat flow of EGW by selecting proper thermal properties. As seen in Table 2, the surface heat transfer coefficient in EGW is much higher than that in SAW. Figure 3B shows a macrophotograph of an EGW weld section made in the welding condition shown in Table 1. Figures 5B and 6B show the comparison of the experimental results with the HAZ shape and the thermal history at midthickmess calculated by the distributed point heat sources given in Fig. 4B. The weld interface, A<_3, and the subcritical transformation lines were determined in the same manner as the SAW. The experiments and the calculation are also in fairly good agreement with EGW. Therefore, Equation 14, which is the solution of Model I, can simulate the HAZ shape and thermal history of high heat input welding by properly distributing the point heat sources and selecting thermal properties.

Application of Model I to the Prediction of t_g/5

The solution for Model I differs from Tanaka’s solution (Ref. 3) in the possibility of an arbitrary selection of a point heat source location. Figure 7 shows the calculated thermal histories of SAW under the welding conditions in Table 1. As seen in Fig. 7, the location of the point heat source affects the thermal history. However, a cooling time between 800°C and 500°C, t_g/5, which welding engineers often use to assess transformation phenomena of ferritic steels, does not differ. This implies that t_g/5 can be predicted from the heat flow of a surface point heat source. Figure 8 shows t_g/5 calculated from computation and experiments of shielded metal arc welding (SMAW). The welding conditions are shown in Table 1. Since t_g/5 is less influenced by the preheating method than t_100, uniform preheating has been assumed in the calculation for Fig. 8. The following equation is a solution for heat conduction of a plate of finite thickness preheated to T_w without neglecting surface heat transfer (Ref. 20):

\[
\theta = 2(T_p - T_w) \sum_{n=1}^{N} \frac{A_n}{u_n^2} \left[ \frac{\cos \left( \frac{u_n x}{a_n} \right)}{a_n^2} + \frac{B_n \sin \left( \frac{u_n x}{a_n} \right)}{a_n^2} - \frac{\exp \left( -a_n^2 k t \right)}{a_n^2} \right] \]

where \( A_n = \frac{1}{u_n^2} \left( 1 + B_n^2 + 2B_n \right) \)

t_g/5 in Fig. 8 has been calculated by the linear combination of Equation 14 (heat flow by a moving point heat source) and Equation 25 (heat flow of preheating).

The values of thermal properties vary as temperature changes, but they cannot be changed in the analytical calculation in this study, unlike numerical analyses. Hence, effective thermal properties are desirable to be introduced to fit calculated results with experimental results. In the calculation of t_g/5, the values of thermal properties have been changed depending on welding heat input energy, \( E \) (kJ/mm), because of the difference in the retention time at high temperatures. The thermal conductance, \( \lambda \) (W/mm²°C), used in Model I is

\[
\lambda = 0.02633 + 2.82 \times 10^{-2} \eta E \]  (26)

And diffusivity, \( \kappa \) (mm²/s), in Model I is

\[
\kappa = 4.0 + 1.23 \times 10^{-2} \eta E \]  (27)

where \( \eta \) is the thermal efficiency of welding. The values of \( \lambda \) and \( \kappa \) used in this calculation are shown in Table 2.

Application of Model II to the Prediction of t_100 in the Test Piece

Model II is a problem of an instantaneous line heat source. This model is applicable to a uniformly preheated plate. The assumption of a line heat is not valid for the entire case. In fact, Model II is limited to analyzing welding cooling times of lower temperatures, such as t_100 for a small test piece. Figure 9 shows t_100 calculated by Equation 15 along with Equation 17 (heat flow of line heat) and Equation 15 along with Equation 18 (heat flow of a uniformly preheated test piece). It is seen that t_100 is influenced considerably by preheating temperature although t_100 is almost the same at preheating temperatures over 150°C (302°F) even under different welding heat inputs. As seen in Table 2, the values of \( \lambda \) and \( \kappa \) used in the calculation of t_100 have differed from those in the computation for the weld shape and t_g/5 because the temperature range of interest differs in the two cases.

Application of Model III to the Prediction of t_100 in Wide Plate Welding

A welded plate, in actual welding, can be considered to be infinitely wide and long. When preheating is conducted locally, t_100 is influenced by the preheating width and preheat power. Model III has been introduced to analyze heat conduction in actual welding with local preheating. Cooling times such as t_100 can be calculated by the linear combination of Model I and Model III. Figure 10 shows t_100 calculated under different preheating temperatures, which are assumed to be measured at the center of the preheated area, that is, point P in Fig. 1C. It is understood from Fig. 10 that the higher preheat power causes the shorter time in t_100. This is because the higher preheat power raises the surface temperature in a more concentrated region around point P while the surrounding area still remains cool. Thermal properties used in the computation for Fig. 10 are the same as those in Fig. 9.

It is seen in Fig. 10 that t_100 differs...
depending on preheat power, \( q_b \) (J/mm²s), even though the same preheating temperatures are employed. This is caused by nonuniform distribution at the start of welding. Figure 11A and 11B shows temperature distribution in a plate preheated to 200°C (392°F) in the surface temperature at the center of a preheated zone with different preheat powers. As preheat power is higher, temperature at the surrounding area is seen to become lower, and shorter \( t_{100} \) results.

Preheat power, which is an important parameter for heat conduction of preheating in Equation 23, is difficult to measure directly. Figure 12 shows a temperature increase at the center calculated from Equation 23. This figure provides a chart to estimate preheat power, \( q_b \), from the time period of preheating until its termination. For instance, 200 s is supposed to be required as the time necessary to attain a preheating temperature of 75°C (167°F) under the ambient temperature of 0°C (32°F). In this case, \( T \) is zero and \((T_p - T)/q\) is found to be 1500 from the chart, and thus \( q_b \) must be 0.05 J/mm²s.

Conclusion

A comprehensive solution of heat flow in welding has been found. This has been developed to analyze the heat conduction of three models: a moving heat source inside the plate without neglecting surface heat transfer (Model I); a instantaneous line heat source (Model II); and local preheating (Model III). Using these models, it becomes possible to predict HAZ shapes, welding thermal histories and cooling times in welding. On the basis of comparison of the calculation with experiments, the following conclusions are drawn:

1) An expanded heat source encountered in high heat input welding is modified by the distributed point heat sources of Model I. The computation of heat conduction by the distributed heat sources satisfactorily predicts contours of fusion line and phase transformation lines as well as thermal histories in SAW and EGW with high heat input.

2) Welding cooling time, \( t_{w5} \), necessary for phase transformation and HAZ hardness can be predicted by a point heat source of Model I on the plate surface. In this case, the values of thermal properties used in the computation depend on the heat input employed.

3) Welding cooling time, \( t_{100} \), for small test pieces can be predicted by Model II because the type of heat source has no effect on the cooling time to lower temperatures. The values of thermal properties used for \( t_{100} \) differ from those for \( t_{w5} \).

4) Welding cooling time, \( t_{100} \), for wide plates is influenced by preheat power which affects temperature distribution at the start of welding. The preheat power can be found from a chart obtained by computation of Model III.

Appendix

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>T_\text{amb}</td>
<td>Ambient temperature (°C)</td>
</tr>
<tr>
<td>T_p</td>
<td>Preheating temperature (°C)</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>Temperature relative to ambient temperature (( = T - T_\text{amb} )) (°C)</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Thermal diffusivity (m²/s)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density (kg/m³)</td>
</tr>
<tr>
<td>( c_p )</td>
<td>Specific heat (J/kg °C)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Surface heat transfer coefficient (J/m²°C s)</td>
</tr>
<tr>
<td>( t )</td>
<td>Time (s)</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Thermal conductivity (W/m°C)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Welding velocity (mm/s)</td>
</tr>
<tr>
<td>( b_w )</td>
<td>Width of specimen in Model II (mm)</td>
</tr>
<tr>
<td>( c )</td>
<td>Length of specimen in Model II (mm)</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>Biot modulus (( \Lambda / h_c ))</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>Biot modulus (( \Lambda / h_b ))</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>Biot modulus (( \Lambda / h_c ))</td>
</tr>
<tr>
<td>( K_0 )</td>
<td>Depth of heat source (mm)</td>
</tr>
<tr>
<td>( r )</td>
<td>Order modified Bessel function of the second kind</td>
</tr>
<tr>
<td>( q )</td>
<td>Welding heat input in Model I (J/s)</td>
</tr>
<tr>
<td>( q_h )</td>
<td>Power of preheat in Model III (J/mm²)</td>
</tr>
<tr>
<td>( Q )</td>
<td>Instantaneous heat source in Model I (J/mm² s)</td>
</tr>
<tr>
<td>( Q_{10} )</td>
<td>Instantaneous line heat source in Model II (J/mm²)</td>
</tr>
<tr>
<td>( Q_{10} )</td>
<td>Instantaneous heat source of pre-heating in Model III (J/mm²)</td>
</tr>
<tr>
<td>( \ell_1 )</td>
<td>Half edge length of an instantaneous heat source in Model II (mm)</td>
</tr>
<tr>
<td>( \ell_2 )</td>
<td>Half edge length of an instantaneous heat source in Model II (mm)</td>
</tr>
<tr>
<td>( \ell_3 )</td>
<td>Half edge length of an instantaneous heat source in Model II (mm)</td>
</tr>
<tr>
<td>E</td>
<td>Welding energy (kJ/mm²)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Thermal efficiency of welding</td>
</tr>
<tr>
<td>( t_{w5} )</td>
<td>Welding cooling time between 80°C and 500°C (s)</td>
</tr>
<tr>
<td>( t_{100} )</td>
<td>Welding cooling time from solidification to 100°C (s)</td>
</tr>
</tbody>
</table>

References