Carbon Equivalent and Multiplying Factor for Hardenability of Steel

The correlation of the carbon equivalent and the multiplying factors are clarified by a heat conduction analysis

BY T. KASUYA AND N. YURIOKA

ABSTRACT. The carbon equivalent and the multiplying factor are indexes for hardenability of steels. The carbon equivalent is used generally in welding and is related to the critical cooling time for the full martensite structure in the HAZ. The multiplying factor is used in heat treatment of hardenable steels and is related to the critical diameter for the full martensite. A heat conduction in a round bar has clarified that the ideal critical diameter should be expressed in a product form of alloy elements as long as the carbon equivalent is expressed in a linear combination of elements. Coefficients of the elements in the carbon equivalent have been calculated from their multiplying factors reported in the literature in light of the result of a heat conduction analysis. The coefficients of Mn, Cr, Mo and Cu are in fairly good agreement with the experimental results, while the coefficients of Si and Ni are in poor agreement.

Introduction

Dearden and O'Neill (Ref. 1) first proposed the carbon equivalent to assess the hardness of steel's weld heat-affected zone (HAZ). This carbon equivalent is expressed as a linear combination of the concentration of each alloy in a steel and has long been used to assess steel's weldability and its HAZ hardenability. Meanwhile, Grossmann, *et al.* (Refs. 2, 3), introduced the concept of expressing the hardenability of heat-treatable steels in terms of a multiplying factor, according to which the effect of steel composition on hardenability is given in a product form.

Although the carbon equivalent is related to welding and the Grossmann factor is related to heat treatment, both factors describe steel hardenability. However, they differ in mathematical expression for assessing the effect of steel composition on hardenability: the carbon equivalent is a linear combination and the Grossmann factor is a multiplying combination.

The objective of the present study is to assist welding metallurgists to better understand the meaning of the carbon equivalent by clarifying the relationship between the carbon equivalent and the Grossmann factor.

KEY WORDS

Hardenability
Steel
Carbon Equivalent
Multiplying Factor
Heat Conductance
Hardenability Index
Heat Flow
Ideal Quenching
Ideal Critical Diameter

Hardenability Indexes

Carbon Equivalent

Dearden and O'Neill (Ref. 1) used Reeve weldability tests to examine the relationship between steel's chemical composition and its maximum HAZ hardness.

They proposed the following carbon equivalent to express HAZ hardness:

$$CE = C + \frac{P}{2} + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr+V}{5} + \frac{Ni}{15} \quad (1)$$

This carbon equivalent was revised slightly to equal the IIW (International Institute of Welding) carbon equivalent:

$$CE_{IIW} = C + \frac{Mn}{6} + \frac{Cu+Ni}{15} + \frac{Cr+Mo+V}{5} \quad (2)$$

Welding engineers have used the IIW carbon equivalent for assessing not only HAZ hardness, but also steel's susceptibility to hydrogen-assisted cracking or cold cracking. Moreover, metallurgists in steel industries have used the carbon

T. KASUYA and N. YURIOKA are with the Joining Technical Development Bureau, Nippon Steel Corp., Shintomi, Futtso, Chiba-ken, Japan.

Paper presented at the 72nd annual AWS meeting, held April 15–19, 1991, in Detroit, Mich.

equivalent in designing the chemical composition of high-strength steels. Hardenability denotes how readily a martensitic structure is obtained in a heat-treated steel matrix or in a steel HAZ. Therefore, the steel strength and the HAZ hardness are not necessarily related to hardenability. For example, an alloyed steel with low carbon produces low HAZ hardness mainly because of its low carbon content, although it shows a wider HAZ due to its higher hardenability.

Maynier, *et al.* (Ref. 4), showed that the critical cooling rate for obtaining a full martensitic structure can be expressed as a linear combination of alloy elements. The equation of the critical rate is shown in Equation A7 in Appendix B. The critical cooling rate for a fully martensitic structure is obviously a representative of hardenability. Therefore, Maynier's carbon equivalent is one of the hardenability indexes. However, his formula is not related to welding but to heat treatment of hardenable steels.

Since HAZ hardness, especially maximum HAZ hardness, is an important factor for steel weldability, many formulas have been proposed (Refs. 5–14) to describe it as a function of a steel's chemical composition and welding conditions. Most formulas assume a cooling time between 800°C (1472°F) and 500°C (932°F), $t_{8/5}$, as a requisite welding condition, because $t_{8/5}$ denotes the duration from the start to the end of phase transformation of most ferritic steels.

One formula (Ref. 14) of the HAZ hardness estimation includes an equation that makes it possible to predict the longest cooling time, $t_{8/5}$ necessary to provide HAZ with a full martensitic structure. This critical cooling time, T_m (s) is given as:

$$\ln T_m = 10.6\, CE_{HARDEN} - 4.8 \qquad (3)$$

$$CE_{HARDEN} = C + \frac{Si}{24} + \frac{Mn}{6} + \frac{Cu}{15} + \frac{Ni}{12} + \frac{Cr}{8} + \frac{Mo}{4} + \Delta H \qquad (4)$$

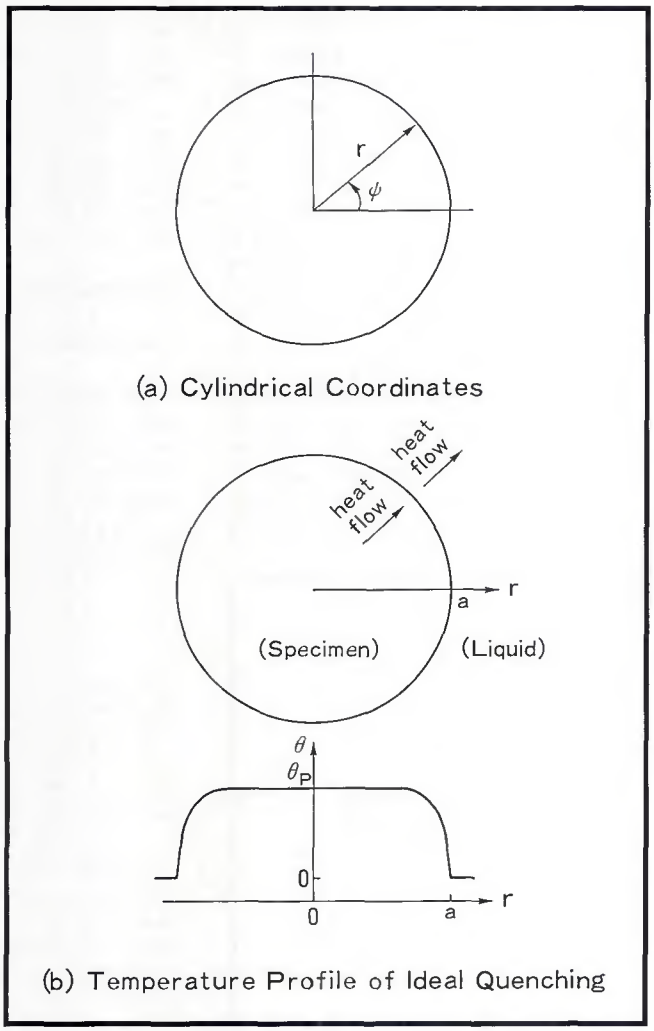

Fig. 1 — Heat conduction for ideal quenching. A — Cylindrical coordinates; B — temperature profile of ideal quenching.

CE_{HARDEN} is one type of carbon equivalent and also one of the hardenability indexes. It is similar to the IIW carbon equivalent, although CE_{HARDEN} includes a ΔH term to describe the effect of unspecified elements and impurities such as boron, sulfur, oxygen, and nitrogen (Ref. 15). In particular, even a very low level of boron significantly influences HAZ hardenability. Its effect in low-nitrogen steels is as follows:

$$\begin{aligned} \Delta H &= 0, \text{ when } B \le 1\,ppm \\ \Delta H &= 0.03, \text{ when } B = 2\,ppm \\ \Delta H &= 0.06, \text{ when } B = 3\,ppm \\ \Delta H &= 0.09, \text{ when } B \ge 4\,ppm \end{aligned} \qquad (5)$$

Multiplying Factor

Grossmann, *et al.* (Refs. 2, 3), introduced an ideal critical diameter, D_I to describe hardenability of heat-treatable steels. D_I is determined as the maximum diameter of a cylindrical bar that shows a 50% martensitic structure at its center under the ideal quench condition. Grossmann, *et al.* (Ref. 3), described the effect of alloying elements as the following multiplying factor:

$$D_I = D_{I0} f_C f_{Si} f_{Mn} f_{Cu} f_{Cr} f_{Mo} \cdots \qquad (6)$$

where, D_{I0} is a coefficient determined by an austenite grain size.

The multiplying factor of each element is simply expressed as

$$f_x = \begin{cases} X^{1/2} & \text{for carbon} \\ 1 + A_x X & \text{for elements other than carbon} \end{cases} \qquad (7)$$

where, f_x is a multiplying factor of element X, A_x is a coefficient, and X is the weight percentage of element X.

Although the Grossmann factor deals with 50% martensite specimens, Hodge, *et al.* (Ref. 16), found that a critical diameter for a full martensitic structure is proportional to that for a 50% martensite structure. Therefore, the critical diameter for a full martensitic structure is given by the same multiplying factor as Grossmann's.

Relationship between the Carbon Equivalent and Critical Diameter

The carbon equivalent of hardenability such as CE_{HARDEN} is directly related to the cooling time or cooling rate at which a 100% martensitic structure is obtained. Meanwhile, D_I is a critical diameter for obtaining a 50 or 100% martensitic structure at the center of a cylindrical specimen. As the diameter of a specimen increases, the cooling time at the center increases or the cooling rate decreases. Therefore, there is a possibility that an analysis of heat flow

clarifies the mathematical relation between the two indexes.

Heat Flow of Ideal Quench

If the temperature of a liquid medium used to quench a cylindrical bar is assumed to be zero, the heat conduction equation for two-dimensional cylindrical coordinates (Fig. 1) is

$$\frac{\partial\theta}{\partial t} = \kappa\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial\theta}{\partial r}\right) \quad (8)$$

where κ = thermal diffusivity, θ = temperature, and t = time.

Because of its symmetry with respect to angle ψ, temperature θ is a function of radius r and time t alone. When a bar is heated up to θ_p before quench, then the initial condition is

$$\theta = \theta_p, \quad 0 \le r \le a \quad (t = 0) \quad (9)$$

If a cylindrical bar is ideally quenched, the surface temperature of the bar instantly becomes zero. Hence, the boundary condition becomes

$$\theta = 0, \quad r = a \quad (10)$$

The solution that satisfies Equations 8, 9 and 10 is given as follows:

$$\theta = 2\theta_p \sum_{n=1}^{\infty} \frac{1}{j_n}\cdot\frac{J_0\left(j_n\frac{r}{a}\right)}{J_1(j_n)} exp\left(-j_n^2\kappa\frac{t}{a^2}\right) \quad (11)$$

where, j_n is a n-th root of the following Eigen value equation:

$$J_0(x) = 0, \quad J_0 \text{ is 0-order Bessel function} \quad (12)$$

At the center (r = 0), Equation 11 becomes

$$\theta = 2\theta_p \sum_{n=1}^{\infty} \frac{1}{j_n}\cdot\frac{1}{J_1(j_n)} exp\left(-j_n^2\kappa\frac{t}{a^2}\right) \quad (13)$$

where J_1 is a first-order Bessel function. By substituting 800° and 500°C into θ in Equation 13, respectively, cooling times to 800°C, t_{800}, and to 500°C, t_{500}, from θ_p at the center of a quenched bar can be calculated.

A study is presently being conducted for cases in which a full martensite or a 50% martensite is obtained at the center of cylindrical bar specimens. Hence,

$$a = \frac{D_I}{2} \quad (14)$$

Table 1 — Carbon Coefficients in the Taylor Series

B_C[(a)]	CE_{HARDEN}	CE_{HARDEN}	Ref. 4
8.3	10.6	8.4	9.8

(a) Calculated by Equation 26.

Meanwhile, the logarithm of $t_{8/5}$ is:

$$ln(t_{8/5}) = ln(t_{500} - t_{800}) \quad (15)$$

$t_{8/5}$ in Equation 15 becomes T_M only when the radius of a quenched bar is half of the ideal diameter (Equation 14). In this case, $t_{8/5}$ is the longest cooling time to obtain full martensite. The empirical relation of Equation 3 implies that $ln(T_M)$ is a function of many variables (C, Si, Mn, P, S, etc.). Hence, $ln(T_M)$ can be expressed in the manner of the Taylor series. The Taylor series of a multivariable function is given as follows:

$$f(x_1, x_2, \ldots, x_n) = f(a_1, a_2, \ldots, a_n) + (x_1 - a_1)\frac{\partial f(a_1, a_2, \ldots, a_n)}{\partial x_1} + (x_2 - a_2)\frac{\partial f(a_1, a_2, \ldots, a_n)}{\partial x_2} + (x_n - a_n)\frac{\partial f(a_1, a_2, \ldots, a_n)}{\partial x_n} + \sum_{m=2}^{\infty}\frac{1}{m!}\left((x_1 - a_1)\frac{\partial}{\partial x_1} + (x_2 - a_2)\frac{\partial}{\partial x_2} + (x_n - a_n)\frac{\partial}{\partial x_n}\right)^m f(a_1, a_2, \ldots, a_n) \quad (16)$$

When $ln(t_{8/5})$ in Equation 15 is expressed in the Taylor series, the second and higher order terms and cross-terms from Equations 3 and 4 can be neglected. Therefore,

$$ln(T_M[C, Si, Mn \cdots]) = ln(T_M[C_0, Si_0, Mn_0, \cdots]) + B_C(C - C_0) + B_{Si}(Si - Si_0) + B_{Mn}(Mn - Mn_0) + \cdots \quad (17)$$

where, B_X (e.g., B_C, B_{Si}, B_{Mn}, ...) is a first-order coefficient of element X (e.g., C, Si, Mn,.....)

$$B_x = \frac{\partial ln(T_M)}{\partial x}\Big|_{C=C_0,\ Si=Si_0,\ Mn=Mn_0 \cdots} \quad (18)$$

The following correlation between the critical cooling time and critical diameter is given in Appendix A:

$$\frac{\partial ln(T_M)}{\partial x} = 2\frac{\partial ln(D_I)}{\partial x} \quad (19)$$

From Equations 18 and 19, the following relation is obtained:

$$2\frac{\partial ln(D_I)}{\partial x} = B_x \quad (20)$$

Integration of Equation 20 with respect to X gives

$$ln(D_I) = \frac{B_x}{2}X + G \quad (21)$$

where, G is a function of elements other than X.

Equation 20 is valid for any element, therefore

$$ln(D_I) = B_0 + \frac{B_C}{2}C + \frac{B_{Si}}{2}Si + \frac{B_{Mn}}{2}Mn + \cdots \quad (22)$$

where, B_0 is a constant.

From Equation 22,

$$D_I = exp(B_0)\cdot exp\left(\frac{B_C}{2}C\right)\cdot exp\left(\frac{B_{Si}}{2}Si\right)\cdot exp\left(\frac{B_{Mn}}{2}Mn\right)\cdots\cdots = D_{I0}f_C f_{Si} f_{Mn}\cdots\cdots$$

$$\text{where } D_{I0} = exp(B_0),\ f_C = exp\left(\frac{B_C}{2}C\right)\cdots\cdots \quad (23)$$

Equation 23 is virtually equal to the multiplying factor of Equation 6. Equation 23 has been derived from the condition that $ln(T_M)$ is expressed as a linear combination of the concentration of each alloy, *i.e.*, carbon equivalent. Hence, it is concluded that D_I should be expressed by multiplying factors as long as the carbon equivalent is expressed as a linear combination of each element. Furthermore, the critical cooling time T_M is given from Equation 17,

$$ln(T_m) = B_0' + B_C\left\{C + \left(\frac{B_{Si}}{B_C}\right)Si + \left(\frac{B_{Mn}}{B_C}\right)Mn + \cdots\cdots\right\}$$

$$\text{where } B_0' = ln(T_m[C_0, Si_0, Mn_0, \cdots]) - (B_C C_0 + B_{Si}Si_0 + B_{Mn}Mn_0 + \cdots\cdots) \quad (24)$$

The second term in the right side in Equation 24, $\{C+(B_{Si}/B_0)Si+(B_{Mn}/B_C)Mn+\ldots\ldots\}$, is considered to be the carbon

Table 2 — Multiplying Factors of Alloy Elements

Element	Ref. 19	Ref. 20	Ref. 21[(a)]	Ref. 22[(a)]	Ref. 23[(a)]
Si	1+0.64Si	1+0.67Si	1+0.55Si	1+1.25Si	1+0.85Si
Mn	1+4.1Mn	1+4.8Mn	1+1.2Mn	—	1+3.3Mn
Cr	1+2.33Cr	1+2.16Cr	1+1.73Cr	—	1+2.3Cr
Ni	1+0.52Ni	1+0.74Ni	1+0.42Ni	1+0.3Ni	1+0.36Ni
Mo	1+3.14Mo	1+2.53Mo	1+1.78Mo	1+5.0Mo	1+3.2Mo
Cu	1+0.27Cu	—	1+0.45Cu	—	—

(a) Determined by the authors using the graphic data.

equivalent for hardenability, CE_{HARDEN}.

Calculation of the Carbon Equivalent from Multiplying Factors

A carbon coefficient in the carbon equivalent is a unit from the concept of carbon equivalency. From Equation 24, a coefficient of an element X other than carbon, C_X, when $\ln(T_M)$ is expanded around $X = X_0$ in the Taylor series, is given as B_X/B_C. In order to calculate a coefficient of element X in the carbon equivalent, it is necessary to expand $\ln(T_M)$in the Taylor series. The composition range for the expansion should be within the compositions by which CE_{HARDEN} in Equation 3 was found, so that the second and higher order terms and cross terms in the Taylor series can be neglected. This study has used the following approximate mean values of the contents previously examined (Ref. 14) as X_0 ($X_0 = C_0$, Si_0, Mn_0, ...)

$$C_0 = 0.12\%,\ Si_0 = 0.6\%,\ Mn_0 = 1.0\%,\ Cu_0 = 0.4\%,\ Ni_0 = 2.5\%,\ Mo_0 = 0.5\%,\ Cr_0 = 0.5\% \quad (25)$$

Then, from Equations 7 and 20, B_C is given as follows:

$$B_C = 2\frac{\ln(C^{1/2})}{C}\Big|_{C=C_0} = \frac{1}{C_0} = 8.3 \quad (26)$$

B_C is a coefficient of the carbon equivalent when $\ln(T_M)$ is expressed by a form of carbon equivalent such as CE_{HARDEN}. Then, B_C is compared with the experimental results shown in Equations 3, A8 and A12. The results shown in Table 1 are in fairly good agreement. Using Equation 26, C_X is calculated as follows:

$$C_x = \frac{B_x}{B_C} = \frac{B_x}{8.3} = \frac{\partial \ln(D_I)}{\partial x}\frac{1}{4.15}\Big|_{X=X_0} \cong \frac{A_x}{4.2(1+A_x X_0)} \quad (27)$$

Multiplying factors have been experimentally determined by many researchers (Refs. 19–23) and reported as equations (Equation 7) or figures. When multiplying factors were graphically reported, the authors determined the factors in the form of Equation 7. The multiplying factors used for the calculation are listed in Table 2. Using these multiplying factors, coefficients for the carbon equivalent were determined by Equations 25 – 27. The results are shown in Table 3. To compare the experimentally determined carbon equivalents, their coefficients are also listed in Table 3. $CE_{HARDEN'}$ in Table 3 is the carbon equivalent on a 50% martensite basis. The carbon equivalent in Ref. 4 and $CE_{HARDEN'}$ have been calculated from the original results, as shown in Appendix B. Table 3 shows that the Mn, Cr , Mo and Cu coefficients are in fairly good agreement, while the Si and Ni coefficients differ significantly.

A coefficient of element x in the carbon equivalent, $C_X(= B_X/B_C)$is significantly dependent on X_0 at which the Taylor expansion is made. Furthermore, X_0 must be selected so that the higher order terms can be neglected in the Taylor series. This study has tentatively used the values from the previous experiment (Ref. 14) as X_0. The authors, however, believe that further discussion of X_0 in Equation 22 will be necessary.

Metallurgical Aspects of Coefficients

The main factors that affect the hardenability of steel are:

1) Austenite grain size.
2) Elements that interact with carbon.
3) Elements that reduce the austenite-ferrite transformation temperature.
4) Elements that segregate into the austenite grain boundary.
5) Inclusions that act as an intragranular ferrite nucleation site.

During cooling after welding or heat treatment, proeutectoid ferrite is likely to nucleate at prior austenite grain boundaries. It follows that larger austenite grains provide fewer ferrite nucleation sites, resulting in higher hardenability. D_I considers the effect of the austenite grain size, but the carbon equivalent for HAZ hardenability does not. In the case of heat treatment, the austenization temperature often differs from experiment to experiment. Since the austenization temperature is one of the most important parameters for the austenite grain size, D_I considers the grain size effect.

In the case of welding, austenization temperature is considered to be the peak temperature at the vicinity of the fusion zone where maximum HAZ hardness is obtained. The peak temperature is almost equal to the melting temperature of a steel, *i.e.*, the austenization temperature scarcely varies in welding of steels. Furthermore, the following experimental fact is important in assessing the significance of the austenite grain size in welding. High heat input in welding increases the retention time for the austenization, resulting in coarser austenite grain sizes. However, HAZ hardness never rises with increasing heat input (Ref.14). Higher heat input increases the cooling time and always reduces HAZ hardness. This implies that the welding cooling time is a more important factor to HAZ hardenability than the austenite grain size. These facts are a presumable reason why CE_{HARDEN} and CE_{IIW} do not take into account the austenite grain size effect.

As transformation from austenite to ferrite proceeds, carbon must diffuse from ferrite to austenite. Cr, Mo and Mn retard carbon diffusion to suppress ferrite transformation, resulting in the formation of a lower temperature transformation product, that is, martensite. Mn, Ni and Cu reduce the transformation temperature to promote martensite formation. As a result, Mn, Mo, Cr, Ni and Cu in that order influence hardenability as shown in Tables 2 and 3.

Boron tends to segregate at prior austenite grain boundaries to suppress ferrite nucleation from the boundaries, and thus significant increases in hardenability result. The boron effect has been considered in the multiplying factor reported by Grossmann (Ref. 23), in which D_I suddenly increased 1.4 times over the original value when boron became effective. This trend is similar to the abrupt increase in ΔH of Equation 5 due to a few ppm of boron. Boron is likely to be combined with nitrogen, which serves to negate the effect of boron. Therefore, the boron effect is indirectly affected by the amount of nitro-

Table 3 — Coefficients in the Carbon Equivalent from Multiplying Factors and Experiments

	Experimental Results		Calculations from Multiplying Factors					
Element	CE_{HARDEN}	CE_{HARDEN}	Ref. 4	Ref. 19	Ref. 20	Ref. 21	Ref. 22	Ref. 23
Si	1/24	1/38	—	1/9.1	1/8.8	1/10.2	1/5.9	1/7.5
Mn	1/6	1/4.8	1/4.1	1/5.2	1/5.1	1/7.7	—	1/5.5
Cr	1/8	1/6.5	1/8.5	1/3.9	1/4.0	1/4.5	—	1/3.9
Ni	1/12	1/10.7	1/7.9	1/18.6	1/16.2	1/20.5	1/24.5	1/22.2
Mo	1/4	1/4	1/6.5	1/3.4	1/3.8	1/4.5	1/2.9	1/3.4
Cu	1/15	1/16.5	—	1/17.2	—	1/11.0	—	—

gen (Ref.15) as well as Al, Ti and Zr, elements that have a high affinity for nitrogen. The fact that ΔH of Equation 5 is not a linear function of boron means that the assumption of linear carbon equivalent used in this study is not valid, as long as boron is concerned. Therefore, to compare ΔH of Equation 5 with the multiplying factor of boron, further investigation is necessary.

Some inclusions, such as MnS, complex sulfides and some oxides (Refs. 17, 18), act as intragranular ferrite nucleation sites, resulting in decreased hardenability. Therefore, it is considered reasonable that the coefficient of sulfur in the multiplying factor is negative and that ΔH due to sulfur and oxygen in CE_{HARDEN} is also negative (Ref. 17).

CE_{HARDEN} is, in fact, not a linear function with regard to carbon when more than 0.3% carbon is present (Ref.14). This means that the second and higher order terms of carbon in Equation 16 cannot be neglected within a range of C ≥0.3%. This is partly because the multiplying factor for carbon is different in mathematical form from those of the other elements, as seen in Equation 7.

Conclusion

The relationship between the two different hardenability indexes, carbon equivalent and ideal critical diameter, has been investigated by a heat conduction analysis. This investigation supports the following conclusions:

1) Heat conduction analysis has verified that the ideal critical diameter should be expressed by multiplying factors as long as the carbon equivalent is expressed as a linear combination of each element.

2) The coefficient of each element in the carbon equivalent is calculated from a multiplying factor through a simple relation as shown in Equation 27.

3) Coefficients for Mn, Cr, Mo and Cu in the carbon equivalent calculated from multiplying factors reported in the literature have been found to match fairly well those acquired through experiments. However, the coefficients for Si and Ni did not match experimental results.

Acknowledgments

The authors would like to express sincere gratitude to Dr. Takamura, emeritus professor of Kyoto University; and Dr. M. Okumura, senior researcher at Nippon Steel Corp., for their helpful discussions.

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Appendix A

Derivation of Equation 19

The cooling time until temperature reaches 500°C, t_{500}, is given by substituting 500°C into Equation 13 as follows:

$$500 = 2\theta_p \sum_{n=1}^{\infty} \frac{1}{j_n} \cdot \frac{1}{J_1(j_n)} exp\left(-j_n^2 \kappa t_{500} / a^2\right) \quad \text{(A1)}$$

t_{500} is a function of the radius, which is now assumed to be $D_I/2$. Since D_I is a function of each element, t_{500} is also a function of each element. By differentiating both sides of Equation A1 with regard to element X, Equation A1 becomes:

$$0 = 2\theta_p \left\{ \sum_{n=1}^{\infty} \begin{array}{l} \frac{1}{j_n} \cdot \frac{1}{J_1(j_n)} \left(-j_n^2 \cdot \frac{\kappa}{a^2} \cdot \frac{\partial t_{500}}{\partial x} \right) \\ exp\left(-j_n^2 \kappa t_{500} / a^2\right) + \sum_{n=1}^{\infty} \frac{1}{j_n} \\ \cdot \frac{1}{J_1(j_n)} \left(2 j_n^2 \cdot \frac{\kappa}{a^3} \cdot t_{500} \frac{\partial a}{\partial x} \right) \\ exp\left(-j_n^2\right) \kappa t_{500} / a^2 \end{array} \right\} \quad \text{(A2)}$$

Therefore,

$$\left(\frac{\partial t_{500}}{\partial x} - 2 \frac{t_{500}}{a} \cdot \frac{\partial a}{\partial x} \right) \sum_{n=1}^{\infty} \frac{j_n}{J_1(j_n)} exp\left(-j_n^2 \kappa t_{500} / a^2\right) = 0 \quad \text{(A3)}$$

When the temperature is 800°C, a similar equation is obtained.

$$\left(\frac{\partial t_{800}}{\partial x} - 2 \frac{t_{800}}{a} \cdot \frac{\partial a}{\partial x} \right) \sum_{n=1}^{\infty} \frac{j_n}{J_1(j_n)} exp\left(-j_n^2 \kappa t_{800} / a^2\right) = 0 \quad \text{(A4)}$$

From Equations A3 and A4,

$$\frac{\partial t_{500}}{\partial x} = 2 \frac{t_{500}}{a} \cdot \frac{\partial a}{\partial x}, \frac{\partial t_{800}}{\partial x} = 2 \frac{t_{800}}{a} \cdot \frac{\partial a}{\partial x} \quad \text{(A5)}$$

From Equation A5,

$$\frac{\partial ln(t_{8/5})}{\partial x} = \frac{\partial ln(t_{500} - t_{800})}{\partial x}$$
$$= \frac{1}{t_{500} - t_{800}} \cdot \frac{2}{a} \cdot \frac{\partial a}{\partial x} \cdot (t_{500} - t_{800})$$
$$= \frac{2}{D_I} \cdot \frac{\partial D_I}{\partial x} = 2 \frac{\partial ln(D_I)}{\partial x} \qquad \text{(A6)}$$

When $t_{8/5}$ is the critical time for a full martensite, $t_{8/5}$ can be substituted by T_M in Equation 3. Then Equation A6 is equal to Equation 19.

Appendix B

Carbon Equivalents in the Literature

Carbon equivalent from Ref. 4

From Ref. 4, the critical cooling rate during 800° to 500°C for full martensite, V_1 (°C/h) is:

$$log(V_1) = 9.81 - \begin{pmatrix} 4.24C + 1.05Mn + 0.54Ni \\ +0.5Cr + 0.66Mo + 0.00183 \cdot Pa \end{pmatrix} \qquad \text{(A7)}$$

Pa: austenization parameter

Substituting $V_1 = 3600 \cdot 300/(t_{8/5})$ into Equation A7,

$$ln(t_{8/5}) = 9.81 \cdot CE + 0.00421 \cdot Pa - 8.70 \qquad \text{(A8)}$$

where

$$CE = C + \frac{Mn}{4.1} + \frac{Ni}{7.9} + \frac{Cr}{8.5} + \frac{Mo}{6.5} \qquad \text{(A9)}$$

CE in Equation A9 is considered to be a carbon equivalent for hardenability on a full martensite basis.

Carbon equivalent on a 50% martensite basis

Carbon equivalent for a 50% martensitic structure, $CE_{HARDEN'}$ in Table 3 is calculated from Ref. 14. The critical cooling time for 50% martensite, T_{50}, is given as:

$$T_{50} = (T_m \cdot T_b)^{1/2} \qquad \text{(A10)}$$

T_m is the critical cooling time for a 100% martensite and T_b is for 0% martensite. T_m and T_b are:

$$T_m = exp(10.6\ CE_{HARDEN} - 4.8)$$
$$T_b = exp(6.2\ CE_{III} - 0.74)$$
$$CE_{HARDEN} = C + \frac{Si}{24} + \frac{Mn}{6} + \frac{Cu}{15} + \frac{Ni}{12} + \frac{Cr}{8} + \frac{Mo}{4} + \Delta H$$
$$CE_{III} = C + \frac{Mn}{3.6} + \frac{Cu}{20} + \frac{Ni}{9} + \frac{Cr}{5} + \frac{Mo}{4} \qquad \text{(A11)}$$

Then

$$T_{50} = (T_m \cdot T_b)^{1/2} = exp\begin{pmatrix} 5.3\ CE_{HARDEN} \\ +3.1\ CE_{III} - 2.03 \end{pmatrix}$$
$$= exp(8.4\ CE_{HARDEN}' - 2.03) \qquad \text{(A12)}$$

From Equations A11 and A12, the carbon equivalent for 50% martensite is given as follows:

$$CE_{HARDEN}' = C + \frac{Si}{38} + \frac{Mn}{4.8} + \frac{Cu}{16.5} + \frac{Ni}{10.7} + \frac{Cr}{6.5} + \frac{Mo}{4} + \frac{\Delta H}{1.6} \qquad \text{(A13)}$$

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