ABSTRACT. The evolution of the buckling phenomenon starts during the weld cooling cycle caused by an onset inelastic strain incompatibility condition. This initial bifurcation phenomenon may continue to grow until the completion of the cooling cycle, which results in the final buckling distortion of the plate. With lower heat input and/or smaller plate dimensions, this initial instability may stop during the cooling cycle due to diminishing strain incompatibility and recovering of the plate rigidity. The buckling evolution process is complex due to the highly nonlinear nature of the welding problem. This paper studies this buckling evolution process using an integrated experimental and numerical approach. Bead-on-plate welds of AH36 steel were experimentally studied. The welding process was numerically simulated and analyzed using a 3D large deformation thermal-elastic-plastic (3D EP) model. The phenomenon of the onset of bifurcation and instability was investigated by monitoring the transient stress and displacement. It was found the longitudinal shrinkage strain distribution can be uniquely determined by the maximum peak temperature profile characterized by “nil-plasticity peak temperature,” which is influenced by welding heat input and plate size. The eigenvalue analysis using the longitudinal inherent shrinkage strain distribution and 3D EP analysis predicted the same critical welding conditions.

Introduction

Buckling is one of the most prevalent types of welding-induced distortion in fabricating thin plate panel structures. This distortion problem is of a particular concern when dealing with higher strength materials, which design tends to use thinner plates while the welding-induced compressive stress in the plate is also higher. The common engineering approach to this problem has been to apply the eigenvalue analysis to the compressive residual stress in the plate resulting from welding (Refs. 1–5). This compressive residual stress is dependent upon material, joint configuration, and welding parameters.

Once the characteristic residual stress is obtained for the given welding conditions, the critical dimensions (width and length) can be estimated using eigenvalue analysis from the design of experiments, or the stability of the specific dimensions under the given welding condition can be checked. This approach postulating the buckling instability under the residual stress state may inaccurately estimate the buckling strength of the panel due to the observation in this study of incipient bifurcation in the weldment during the weld cooling cycle. If this phenomenon continues until completion of the weld cooling cycle, the panel would buckle at lower threshold stress values than that predicted by the stress-based eigenvalue solution.

In this study, an integrated experimental and numerical approach was used to investigate the mechanics of welding-induced buckling phenomenon. In the experimental study, bead-on-plate welding was performed along the middle line of AH36 steel plates using the submerged arc welding process. The effect of welding heat input on distortion was evaluated with heat input ranging between 560 and 1280 J/mm. The plate size effect was also studied for a constant heat input (1097 J/mm). The purpose of this experimental study was to establish the baseline data for calibration and comparison with the numerical analysis.

In the numerical studies, three-dimensional, thermoelastic-plastic, large deformation analyses were performed on the weld experimental models to understand the distortion process observed in the experiments. The distorted shape and the magnitude of vertical displacements obtained from the numerical analyses were compared with those obtained from experiments. The numerical models were extended to analyze the transient evolution of the bifurcation phenomenon during welding and after completion of the weld thermal cycle for the buckling criterion.

Experimental Study

Figure 1 shows the test specimen configuration. All the specimens were square shaped of various sizes, made of 6-mm-thick AH36 (i.e., 345 MPa yield stress) steel plates. The submerged arc welding process and welding consumables in accordance with AWS A5.17 (F7A4-EL8) were used for welding. End tabs of 120 × 160 mm were attached to both ends of the specimen to ensure a continuous bead cross section in the ends of the weld plate. The electrode extension of 25 mm and direct current electrode positive (DCEP) arc polarity were used in all welding procedures.

Experimental Setup

Table 1 summarizes three plate sizes (300, 400, and 1000 mm) and welding parameters for the ten weld models investigated. The welding heat input varied from 558 to 1280 J/mm by setting the proper combinations of wire diameter, current, arc voltage, and travel speed. All experiments were carried out at an ambient temperature (25.8°C). The specimens were

KEYWORDS

Buckling
Bifurcation
Plasticity
Welding Distortion
Longitudinal Inherent Shrinkage
Strain
Finite Element Analysis
Temperature and strain histories during welding were monitored at several instrumented locations on the top surface of specimen 5E. ANSI K-type (Chromel/Alumel) thermocouples were used for temperature measurement. For strain measurement, stacked 45-deg-rosette strain gauges with self-temperature compensation corrections (gauge factor: 2.09 and gauge resistance: 350 ohms) were used. High-temperature epoxy adhesive was used to attach the strain gauges. A data acquisition system was used to read out temperature and strain measurements at a sampling rate of 0.5 Hz.

After preparing the specimens to the required dimensions, a grid with lines at intervals of 50 mm were made on the back surface of the specimens to measure the welded without any external constraints.

<table>
<thead>
<tr>
<th>Weld Metal</th>
<th>Plate Size (mm)</th>
<th>Wire Diameter (mm)</th>
<th>Current (Ampere)</th>
<th>Voltage (Volts)</th>
<th>Speed (mm/min)</th>
<th>Heat Input (J/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A</td>
<td>500</td>
<td>2.0</td>
<td>300</td>
<td>31</td>
<td>1000</td>
<td>558</td>
</tr>
<tr>
<td>5B</td>
<td>500</td>
<td>2.0</td>
<td>300</td>
<td>31</td>
<td>750</td>
<td>744</td>
</tr>
<tr>
<td>5C</td>
<td>500</td>
<td>2.0</td>
<td>300</td>
<td>31</td>
<td>650</td>
<td>859</td>
</tr>
<tr>
<td>5D</td>
<td>500</td>
<td>2.0</td>
<td>300</td>
<td>31</td>
<td>550</td>
<td>1015</td>
</tr>
<tr>
<td>5E</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>700</td>
<td>1097</td>
</tr>
<tr>
<td>5F</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>650</td>
<td>1182</td>
</tr>
<tr>
<td>5G</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>600</td>
<td>1280</td>
</tr>
<tr>
<td>5H</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>700</td>
<td>1097</td>
</tr>
<tr>
<td>5I</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>700</td>
<td>1097</td>
</tr>
<tr>
<td>5J</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>700</td>
<td>1097</td>
</tr>
<tr>
<td>5K</td>
<td>500</td>
<td>3.2</td>
<td>400</td>
<td>32</td>
<td>700</td>
<td>1097</td>
</tr>
</tbody>
</table>

\( L = \) Plate length; \( 2w = \) Plate width
displacement at intersection points of the grid lines. The flatness or normal deformation due to welding was calculated taking the differential value between the two normal displacements at the grid points measured before and after welding. The measured initial flatness of the plates was not considered in the following numerical simulations.

The displacement measurement equipment was set up and fixed to the ground plate with the horizontal guide bar running parallel to a grid line. A digital dial gauge slides on the horizontal bar reading the height of the distorted plate at the grid points from reference ground plate surface. The maximum deflection of the horizontal bar was approximately 0.2 mm at the mid-span of the bar. This initial bar deflection condition was subtracted from the measured displacement data. The deformation profile of the weld specimens were constructed by connecting the corrected displacements at the grid points.

### General Experimental Observations

#### Macrographs and Transient Temperature Profiles

Figure 2 shows the typical macrographs of the weld cross section cut from the specimens 5A (heat input 558 J/mm) through 5G (heat input 1280 J/mm). These profiles were used to characterize the heat-input effect on bead width, penetration depth, and weld nugget area. Weld nugget information of specimen 5E (heat input 1097 J/mm) was used to calibrate the heat source model used in the numerical modeling and analyses in this study.

The general observation of weld heat-input effect on weld nugget shape is consistent to the expectation. The weld nugget area increases with increasing welding heat input. When the heat input increases, the effect of wider heat distribution is shown to increase bead width instead of weld penetration, which results in various weld nugget shapes. The fusion boundary of the weld nugget shows an inherent melting peak temperature boundary established during welding, which provides a natural calibration condition for the numerical heat flow analysis.

Figure 3 shows temperature histories measured from the welding test on specimen 5E. Transient heat and cool cycles during welding were obtained from each measurement location. Since the steel starts to soften at approximately 500°C, it shows a softening zone width of approximately 20 mm in the weld area. This width indicates the inherent shrinkage zone width, within which zone the compressive plastic strains contribute to the majority of weld residual stresses and distortion in the weldment. Explanation of this shrinkage behavior is presented in the numerical analysis section.

#### Effect of Heat Input and Plate Size on Longitudinal Distortion

The welded plates are distorted in both longitudinal (along weld axis) and transverse (normal to weld axis) directions, forming a saddle shape. The plates are downward bent in the longitudinal direction with an opposite curvature in the transverse direction. Transverse angular distortion is dominant in specimens subjected to lower heat input. Longitudinal angular distortion shows a single curvature in the welding direction, which gradually becomes more pronounced as heat input increases.

Distorted shapes obtained experimentally from the specimens of different sizes subjected to constant heat input (1097 J/mm) were analyzed. For a small plate of 300 mm, transverse angular distortion is dominant. With increasing the size of the plate, longitudinal distortion increases, which shows a saddle shape.

### Numerical Modeling and Analysis

#### Process Model

The three-dimensional finite element analyses were carried out to numerically investigate the welding-induced distortion in plates. The decoupled thermomechanical analysis procedure was utilized in the simulation. In order to enhance the simulation accuracy for nonlinear geometrical behaviors, the large deformation theory was incorporated into the thermomechanical-plastic finite element analysis procedure.

Figure 4 shows an example of the finite element mesh model used in the welding simulation of the weld model 10E. Since the welded specimen is symmetric about the weld centerline (y = 0), a half plate model was considered in the finite element analysis. Element meshes are denser in the vicinity of the weld centerline where the arc heat impinges, but meshes become coarser away from the weld centerline. The length of elements in the welding direction near the weld centerline was uniform for a constant arc scanning time over each element.

#### Thermal Model

Welding heat transfer analyses with given sets of welding variables were performed on the three-dimensional plate models. Temperature histories at element nodes were computed at each incremental time step. Three-dimensional, 8 nodes, linear brick, and heat transfer elements [DC3D8 in ABAQUS (Ref. 6)] were chosen. Free convection boundary condition
The nodal temperature histories obtained from the thermal analysis were given as input loading to the mechanical analysis. Eight nodes, linear brick elements with the reduced-integration [C3D8R in ABAQUS (Ref. 6)] were selected for the mechanical analysis. The reduced-integration element was chosen to avoid shear locking that occurs in fully integrated linear elements undergoing bending deformations (Ref. 6).

Since experimental specimens were plates without boundary constraint, the mechanical model consisted of free boundaries as well. Temperature-dependent mechanical properties were incorporated in the mechanical analyses (Ref. 7).

Solution Sensitivity

The solution sensitivity test was performed to check the adequacy of the element size and mesh layout by comparing the predicted results with the experimental measurements. Figure 6 shows peak temperature distributions along the traverse direction (y-direction) on the top surface of the weld plate, obtained from both the experiment and the simulation. The predicted peak temperature distribution fits well with the experimental curve.

Figure 7A–C shows the comparison of calculated stress evolutions with the strain gage data at two gauge locations (x = 250 mm and y = ±100 mm, ±150 mm, and ±200 mm) in weld model 5E. The predicted stresses are element stresses. The experimental stresses were converted from the strain data. In the strain/stress conversions, room-temperature elastic modulus of steel was used. The comparisons show reasonable correlation at the 200-mm gauge location; however, they also show major deviation at lower temperatures during cooling cycle at the 100- and 150-mm gauge locations. This deviation starts sooner with greater magnitude when the gauge location is closer to the weld, but magnitudes of deviation at those locations appear the same after completely cooling. One reason for this deviation is due to the data location. The predicted stress is at the centroid of the surface element, while the measured strain location is at the plate surface. This location difference has a magnitude of half element thickness. Because of the plate bending due to longitudinal distortion, through-thickness stress gradients exist, which may contribute to the difference in the stress values.

The other reason may be associated with apparent thermal strains resulting from temperature changes in the strain
gauge in the experimental measurements during welding. Since the strain gauge data are only corrected based on the expansion of the strain gauge itself without considering the apparent expansion difference between the strain gauge material and the adhesive, this may have contributed to data deviation.

Despite the deviation, the characteristic shapes of all stress curves are similar. The other observation from these comparisons is good agreement shown between the predicted and the measured stresses within 200 (at \( y = 100 \) mm) or 450 (at \( y = 150 \) mm) seconds after the arc crossing the cross section where the strain gauges were placed. The predicted bifurcation phenomenon (Fig. 11B) shows an onset instance at approximately 200 s after the arc crossing the cross section. This coincides with the stress deviation moment as shown in Fig. 7B. It is suspected that buckling instability might have contributed to the stress deviation. It was our judgment that the tested finite element model could reasonably simulate the transient buckling behaviors of the weldment for investigation of the buckling evolution process.

**Parametric Study**

The parametric study simulated all ten weld models as shown in Table 1 to establish the baseline information using the numerical scheme. Despite the stress deviation shown in comparisons for weld model 5E, the predicted distortion shapes of all weld models investigated agree well with the experimental measurements. Figure 8 shows the worse distortion scenario (weld model 10E) that included high heat input (1097 J/mm) and large dimension (1000 mm). The maximum vertical displacements appear at the center of the plate, which are 6.35 mm from experiment and 6.95 mm from numerical analysis. The experimental deformation shape shows a reverse curvature region in the middle of the plate indicating a buckling phenomenon, although this reverse curvature zone is not clearly shown in the predicted deformation shape. However, the later stress and displacement analyses of the 10E model demonstrated such a buckling behavior.

Figure 9 compares the predicted and measured vertical displacements \((U_z)\) at the center of the 5E plate with various heat inputs. Even though the measured data show some experimental scatter, the general relationship between heat input and distortion in both curves is similar; displacement increases with increase of heat input.

Figure 10 shows the effect of plate size on the longitudinal distortion under constant heat input (1097 J/mm). The variations in vertical displacements \((U_z)\) are plotted against a normalized spatial distance along the weld centerline of three weld models (3E, 5E, and 10E). The predicted results agree reasonably well in distortion shape to the experimental curves. The numerical analyses overestimated the distortion magnitude for larger plates (5E, 10E), but underestimated the distortion magnitude for the small plate (3E). The differences are expected due to lack of the exactness of the numerical model simulating the complex welding behaviors. Two generic trends are observed from the plots: 1) single curvatures along the weld centerline are shown for all three models; and 2) the magnitude of distortion increases with the plate size.
Distortion Evolution

Bifurcation Phenomenon

Figure 11 shows the evolution of vertical displacements ($U_z$) and longitudinal stresses ($\sigma_{x_0}$) on top and bottom surfaces of three simulated weld models (3E, 5E/10E, respectively), which have the same 1097 J/mm heat input. The vertical displacements are taken from the node located at the center of the plate. The stresses are those at the centroid of the outermost elements on the top and bottom surfaces, located approximately quarter width of the plate from weld centerline (e.g., $y = 70$ mm for 3E, $y = 175$ mm for 5E, and $y = 270$ mm for 10E) in the mid-length section.

The bifurcation is shown when the stresses on the top and bottom surfaces start to deviate; meanwhile, the vertical displacement curve shows a sudden jump in the growth rate. The elastic-plastic, large deformation FEA model depicted the aggravation of the bending curvature effect and predicted the bifurcation phenomenon. As shown in Fig. 11, the onset bifurcation and stress deviation at the top and bottom surfaces are more obvious in large plates; the weld model 3E, which is relatively a small size, does not show stress deviation as much as the weld model 5E and 10E, so that buckling may not be the critical issue in weld model 3E.

Therefore, the following discussion regarding the onset bifurcation is focused on the 5E and 10E models. The onset bifurcation occurs approximately 200 and 500 s after the weld heat source crossing the middle cross section for small (5E) and large (10E) plates, respectively. Both indicate the onset occurrence starting during the weld cooling cycle 3 and 7 min after completion of the entire weld length in 5E and 10E, respectively. These moments correspond to the evolution of the plastic shrinkage strains accumulated over the weld thermal cycles, which will be explained in the latter section of this article.

By investigating the bifurcation growth of the small weld model (500 mm), it appears that stress deviation and displacement growth hindered at approximately 2000 s. Longitudinal bending occurs in the plate without showing the buckling mode, which shows simple longitudinal and
transverse bending curvatures in the final saddle distortion shape. For the larger weld model (1000 mm), the onset bifurcation continues to grow without any indication of convergence one hour after completion of the weld. Buckling is predicted, which coincides with the experimental observation (Fig. 8).

Figure 12 shows the effects of heat input and plate size on vertical displacement at the plate center. All weld models investigated (Table 1), except the largest plate (1000 mm) with 1097 J/mm heat input and plate size on vertical displacement (Fig. 8).

Investigation of the Onset Bifurcation Behavior

Figure 13 shows longitudinal plastic strains and thermal strain at the onset bifurcation for the cases with same heat input but different plate sizes. Considering the fact that the transient stress depends on the distribution pattern of inelastic strains (thermal strain and plastic strain), Fig. 13 shows the different type of mechanism developing the transient stress. For weld model 3E (small plate), the tensile inelastic strain (thermal > plastic) is localized only the region away from the weld where thermal and shrinkage plastic strains are cancelled out at the weld region. For weld model 10E (large plate), the compressive inelastic strain (plastic > thermal) exists at the weld, and the tensile inelastic strain at the region away from the weld. Even though the inelastic strain distributions of two cases are different, both induce a significant amount of transient compressive stress field enough to cause the onset bifurcation.

Figure 14 shows the transient stress distributions at a different time for weld model 3E and 10E, respectively. At onset bifurcation, both weld models have the well-developed compressive stress field.

As the weld cooling process continues, the final residual stress distribution shows a uniformly distributed compressive stress zone away from the weld. The tensile stress zone near the weld region, in fact, represents the region where the inherent shrinkage plastic strain is located (Ref. 11). The compressive stresses are caused by the compressive force due to the shrinkage of the tensile zone. Depending upon what occurs between the onset moment and completion of the weld thermal cycle, as well as the geometric instability of the plate (i.e., plate size), the final buckling instability may occur if the bifurcation phenomenon grows continuously to divergence.

Researches on buckling behaviors due to thermal loading dubbed "thermal buckling" have been reported (Refs. 12-14), which may be appropriate to describe the onset phenomenon during the weld. "Thermal buckling" refers to the distortion phenomenon occurring under the nonlinear temperature field in thin strips subjected to thermal heating. Thermal buckling is regarded as a thermoelastic instability behavior, under which the axial membrane force driven from the thermal strains causes the buckling instability. A wide heated region and a critical temperature differential are generally needed to produce a sufficient compressive stress field for the buckling instability. Thermal buckling is hindered if the temperature differential is less than the critical value for the given set of heating and geometrical parameters. A wide thermal field in welded plates can be expected upon cooling to start the bifurcation based on the thermal buckling theory.

As the weld plate cools to lower temperatures, thermal strain effects gradually
diminish; instead, the shrinkage strains take over as the driving force for continuous growth of the stress deviation between the top and bottom of the plate. The effect of the shrinkage strains on buckling is the same as that caused by the thermal strains, except that recovering of the plate rigidity at lower temperatures may halt further growth of the stress deviation in the plate. A hypothesis on the halting mechanism of the buckling aggravation may be assessed based on the critical state of the inherent shrinkage strains after the weld plate returns to room temperature. This hypothesis is investigated in the following sections.

The welding-induced incompatible inelastic strains in the weldment during the heating and cooling weld cycles include transient thermal strains, cumulative plastic strains, and the final inherent shrinkage strains. At any instant during welding, the incompatible thermal strains resulting from the nonlinear temperature distributions generate the mechanical strains, which lead to the incremental plastic strains in the weldment if yielding occurs. The incremental plastic strains accumulate over the periods of heating and cooling. Upon completion of the welding cycles, the cumulative plastic strains interact with the weldment stiffness and the joint rigidity resulting in the final state of residual stresses and distortion of the weldment. This final state of the incremental plastic strains, which are always compressive, is referred to as the “inherent shrinkage strains.”

Ueda (Ref. 15) first presented this inherent shrinkage concept to predict weld residual stresses and distortion in 1979, followed by many publications demonstrating the numerical procedures for weld residual stresses and distortion prediction using the inherent shrinkage concept (Refs. 16–19). In 1993 through 1995, Tsai (Refs. 20, 21) used spring elements to describe the inherent shrinkage strains in the numerical simulation and analysis of angular weld distortion of tubular joints in automotive frames. Jung and Tsai (Refs. 22, 23) developed the plasticity-based distortion analysis (PDA), which enables the direct mapping of the inherent shrinkage strains (plastic strains) into elastic models, and investigated the contribution of each inherent strain component to distortions, and evaluated the validity of elastic-based distortion and residual stress prediction approach.

The inherent shrinkage strains are primarily caused by material softening and nonlinear thermal gradients in the cooler areas. The inherent shrinkage strains are uniform along the weld length, except in the areas of arc start and stop. They are nearly uniform within the softening area in the direction transverse to the weld. The strain magnitude decreases at a steep slope to zero within a short distance from the edges of the softening zone. The peak temperatures attained in the weldment are uniquely related to the longitudinal inherent shrinkage strains due to a relatively large stiffness ratio between the soften zone and its cooler surroundings. The peak temperature distribution can be used to estimate the longitudinal inherent shrinkage strains with good accuracy. The longitudinal residual stresses and the longitudinal cambering can, therefore, be determined from the peak temperature distributions. Buckling can also be predicted in large thin-plate weldments (Ref. 7).

The peak temperatures alone are in-
sufficient to determine the transverse inherent shrinkage strains because of the smaller stiffness ratio, and its sensitivity to the joint thickness and the external constraint conditions. Nevertheless, with a correction procedure considering the material incompressibility, the modified transverse inherent shrinkage strains can be used to estimate with good accuracy the transverse residual stresses and angular distortion of the weldment.

Figure 15 shows the longitudinal inherent shrinkage strain distribution along the middle plate cross section in weld model 10E. The two-dimensional (2D) curve as indicated in the figure was investigated to develop a 2D engineering approach for distortion analysis, which is beyond the scope of current study and is not discussed further in this article. The inherent shrinkage strain area determines the average shrinkage force acting on the weld plate.

Figure 16 plots the longitudinal shrinkage strains, $e_{xx}$, vs. the peak temperatures at the corresponding locations during welding for the 500-mm plate with heat inputs from 558 to 1280 J/mm. Regardless of the heat-input magnitudes, the longitudinal shrinkage strain distributions correlate well with the peak temperatures. Figure 16 also plots the longitudinal shrinkage strains vs. peak temperature correlation for four different plate sizes with the same heat input of 1097 J/mm. The predicted results of two plate dimensions (2000 and 3000 mm) in addition to those listed in Table 1 are shown in this figure. Although the figure shows some deviations at high temperatures, the differences are insignificant in determining the shrinkage strain areas.

The characteristic inherent shrinkage strain distributions are shown in Fig. 17, which shows the predicted results of a weld model of 2000 mm width and 1280 J/mm heat input. The figure shows a plot of numerical results and correlation functions for four characteristic regions as follows:

- **Zone “0”** — Shrinkage strains are not produced in this zone, where the peak temperatures are less than the threshold, dubbed “nil-ductility peak temperature, $T_h$.”
- **Zone “I”** — Peak temperatures are between $T_h$ and $T_c$ that is a characteristic transition temperature. Within this zone, the inherent shrinkage strain increases with the peak temperature. The shrinkage strains are primarily generated during the weld heating cycle without reverse yielding during the cooling cycle. Incompatible thermal strains are responsible for the plastic strains. $T_c$ is approximately equal to twice $T_h$, subtracting the reference temperature ($T_0$). This approximation is practically true due to high stiffness of the plate in the longitudinal direction as compared with the softening zone in the weld area.
- **Zone “II”** — The peak temperatures are between $T_c$ and the mechanical melting temperature (MMT) (i.e., beyond this temperature the shrinkage strain becomes maximum and constant). The inherent shrinkage strains become nonlinearly distributed in the peak-temperature spectrum. Material softening is largely responsible for the shrinkage strains, and reverse yielding during the cooling cycle offsets some of the compressive plastic strains as the peak temperature increases.
- **Zone “III”** — When the peak temperatures exceed the MMT, complete material softening or melting occurs that relaxes all plastic strains accumulated.
During the weld heating cycle, the inherent shrinkage strains are independent of the peak temperature. They are caused by weld shrinkage and shrinkage of the softened base metals upon recovery of the mechanical strength and the rigidity of the weld plate during the cooling cycle.

The characteristic inherent shrinkage strain and peak temperature correlations may be written as follows:

1. Zone "0" where peak temperatures, \( T_{\text{max}} \leq T_h \)

\[
\varepsilon_{xx} = 0 \quad (2)
\]

2. Zone "I" where \( T_h \leq T_{\text{max}} \leq T_c \)

\[
\varepsilon_{xx} = \frac{T_{\text{max}}}{T_h} \left( T_{\text{max}} - T_h \right) \quad (3)
\]

3. Zone "II" where \( T_c \leq T_{\text{max}} \leq \text{MMT} \)

\[
\varepsilon_{xx} = \frac{T_{\text{max}}}{T_c} \left( T_{\text{max}} - T_c \right) + R_H \frac{T_{\text{max}}}{\text{MMT}} - 1
\]

4. Zone "III" where \( T_{\text{max}} > \text{MMT} \)

\[
\varepsilon_{xx} = \text{MMT} (1 + R_H) \left( T_{\text{max}} - T_o \right)
\]

where \( \alpha_{\text{max}} \) and \( \alpha_{\text{MMT}} \) are thermal expansion coefficients taken at the peak
temperature and the mechanical melting temperature, respectively. \( R_H \) is the slope of the shrinkage strain vs. peak temperature relationship, which is a function of \( T_h \). The nil plasticity peak temperature (\( T_h \)), the characteristic transition temperature (\( T_c \)), and the mechanical melting temperature (MMT) are influenced by heat input and plate size. They can be determined from the parametric numerical results. Figure 18 shows the effect of heat input and plate size on the nil-plasticity peak temperature relationships. Increase in heat input or decrease in plate size will increase \( T_h \).

The material factor (\( R_H \)) can be determined from regression analysis of the numerical results to best fit the populated data of all weld models investigated. Figure 19 shows the relationship between \( R_H \) and \( T_h \) for the Zone “II” shrinkage strains. Figure 20 shows a comparison between the predicted shrinkage strains using Equation 5 and the numerical simulation results of the Zone “III” shrinkage strains. The figure shows a data scatter within one standard deviation band.

The inherent shrinkage strains in different zones can be normalized by the maximum value in Zone “III,” which is a constant value. The critical buckling threshold can therefore be determined using \( (\varepsilon_{\text{III}})^{\text{II}} \) as the single parameter for the buckling criterion of weld plate.

**Critical Buckling Instability by Eigenvalue Analysis**

The eigenvalue buckling analysis was...
carried out to check the stability of the tested plates. The equivalent thermal strain was used as the perturbation. Each case with a different plate size and heat input has a different distribution pattern that is defined by the nil plasticity temperature where the plastic strain is zero, and MNT where the plastic strain is uniform and maximum. In the eigenvalue buckling analysis, the maximum plastic strain is 1.0E-5, so that the critical buckling strain will be determined by multiplying 1.0E-5 to the eigenvalue for the specific eigenmode, especially the first eigenmode. The tested plate size is 500 to 3000 mm. For this analysis, shell element-based FE models were developed, and the thermal strain associated with the plate size and heat input was applied by the distributed temperature and antistipper thermal expansion coefficient.

The calculated critical buckling strains for each case are plotted in Fig. 21 with solid lines. The maximum shrinkage strains (driving force) shown in Fig. 16 are plotted with dashed lines. Using this graph, the stability of each case can be checked by comparing the critical buckling strains and the maximum shrinkage strain for the cases. When the maximum shrinkage strain is greater than the critical buckling strain in terms of magnitude, buckling occurs.

Figure 22 summarizes the critical buckling conditions with respect to heat input and plate size. The solid line represents the situation when the critical buckling strain and maximum shrinkage strain are the same in Fig. 21, so that the upper-right corner space above the solid line indicates the buckling regime and the space below the solid line represents the stable regime without buckling.

Conclusions

This paper demonstrates a numerical procedure to determine the peak temperature-based buckling criterion for welding thin steel plates. Several other key conclusions from the discussions of the analyses presented in this paper can be summarized as follows:

1. A bifurcation phenomenon starts in the weld plates during the weld cooling cycle regardless of heat input and plate size. However, this phenomenon stops when the weld plate cools to lower temperatures for smaller plates and lower heat inputs.

2. The longitudinal weld distortion is primarily caused by the inherent longitudinal shrinkage strain accumulated during the weld thermal cycles.

3. The inherent longitudinal shrinkage strains can be uniquely determined based on the peak temperatures in the weld plate. This is due to high stiffness of the weld plate in the longitudinal direction.

4. The characteristic peak temperature that determines the formation of plastic strains is defined as “nil-plasticity peak temperature,” which is influenced by welding heat input and plate size. There exists a bilinear relationship as function of heat input and plate size.

5. The critical buckling threshold can be determined using the eigenvalue FEA analysis procedure using the maximum longitudinal inherent shrinkage strain for a given heat input as the perturbation strain load applied using the equivalent thermal loading temperatures. The threshold value is the first-mode eigenvalue that decreases with increasing weld heat input or plate size.

6. The buckling driving parameter (\(\varepsilon_{p}\)) is proportional to the nil-plasticity shrinkage temperature \(T_{b}\), the material factor \(R_{m}\), and the thermal expansion coefficient evaluated at material’s mechanical melting temperature.

Acknowledgments

The authors acknowledge Daewoo Shipbuilding & Marine Engineering Co., Ltd. in Korea for financial support and assistance in the experimental investigations.

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