Approximate Stress Formulas for a Multiaxial Spot Weld Specimen

The global force applied to a test specimen is related to the local stress at the spot weld

BY S. ZHANG

ABSTRACT. Based on the fracture mechanics analysis of spot welds, approximate stress formulas of structural stress, notch stress and stress-intensity factors are obtained for a newly proposed multiaxial spot weld specimen that enables a spot weld to be tested under combined loads ranging from shear to tension. The formulas were validated by finite element simulations.

Introduction

Recently, a novel test setup was proposed by Lee, *et al.* (Ref. 1), for spot weld testing. The test specimen is illustrated, in principle, in Fig. 1. The test fixture enables a spot weld to be tested under combined loads of shear and tension. In analysis of the test results, it may be necessary to convert the global force applied to the specimen into local stress parameters (structural stress, notch stress and stress-intensity factor) at the spot weld. This can be done by refined finite element analysis, but it is a costly process and special expertise is needed for the notch stress and stress-intensity factor. This paper gives a number of approximate formulas that relate the global force applied to the specimen directly to the local stress parameters at the spot weld. The formulas were obtained on the basis of fracture mechanics analysis of spot welds (Refs. 2–5) and validated by finite element simulations.

Derivation of Stress Formulas

The spot weld in the specimen (Fig. 1) was subjected to combined shear and tension. The shear and tension forces (F_s and F_t) were actually two projections of the total applied force F, *i. e.*, $F_s = F\cos\theta$ and $F_t = F\sin\theta$. The contribution of shear and tension forces to complete notch stress σ_k is designated as σ_{ks} and σ_{kt}, respectively. Local stress parameters at the spot weld are to be derived from superposing portions contributed by shear and tension forces. Taking notch stress as an example, notch stress at the spot weld caused by shear force is as follows:

$$\sigma_{ks} = \frac{4F_s}{\pi dt}\left(1 + \frac{1}{2\sqrt{\pi}}\sqrt{\frac{t}{\rho}}\right) \tag{1}$$

with nugget diameter d, sheet thickness t and notch root radius ρ at the spot weld. The above equation is based on the notch stress formula for the tensile-shear specimen (Ref. 2). The difference between shear (mode I stress-intensity factor $K_I = 0$) and tensile-shear ($K_I \neq 0$) has been taken into account. Notch stress caused by tension force is given by (Ref. 2)

$$\sigma_{kt} = \frac{\eta(b-d)F_t}{\pi dt^2}\left(1 + \frac{2}{\sqrt{3\pi}}\sqrt{\frac{t}{\rho}}\right) \tag{2}$$

where a correction factor η is additionally introduced to account for the influence of the specimen width w. The η-value is given in Table 1 as a function of e/d with e = (w – d)/2. The length b of the specimen is assumed to be larger than its width w. The complete notch stress at the spot weld is then superposed from the two portions σ_{ks} and σ_{kt}

$$\sigma_k = \frac{4F}{\pi dt}\left[\left(1 + \frac{1}{2\sqrt{\pi}}\sqrt{\frac{t}{\rho}}\right)\cos\theta + \frac{\eta(b-d)}{4t}\left(1 + \frac{2}{\sqrt{3\pi}}\sqrt{\frac{t}{\rho}}\right)\sin\theta\right] \tag{3}$$

Similarly, structural stress ($\sigma_r = \max\{\sigma_{ui}, \sigma_{li}\}$, *i.e.*, maximum radial stress on the nugget periphery at the inner side of the overlap sheets) and stress-intensity factors (K_I and K_{II}) are derived as follows:

$$\sigma_r = \frac{4F}{\pi dt}\left[\cos\theta + \frac{\eta(b-d)}{4t}\sin\theta\right] \tag{4}$$

$$K_I = \frac{\eta(b-d)F}{\sqrt{3\pi dt}\sqrt{t}}\sin\theta \tag{5}$$

$$K_{II} = \frac{2F}{\pi d\sqrt{t}}\cos\theta \tag{6}$$

The stress state at a spot weld is usually multiaxial. The multiaxial stress state is reflected in the structural and notch stresses whereas an equivalent stress-intensity factor should be introduced for K_I and K_{II} to consider the multiaxial stress state. The equivalent stress-intensity factor is defined as (Ref. 3)

$$K_{eq} = \pm\sqrt{K_I^2 + \alpha K_{II}^2 + \beta K_{III}^2} \tag{7}$$

where the square root takes the plus sign when $K_I \geq 0$ and the minus sign when $K_I < 0$. The sign of the equivalent stress-intensity factor accounts for the opening (+) and closure (–) of a crack tip. The two parameters α and β are weight factors of the mode II and III stress intensities. The value of α and β is normally not far from 1.0. For the current specimen, K_{III} vanishes due to the symmetry conditions and α = 1.0 is introduced, namely,

$$K_{eq} = \pm\sqrt{K_I^2 + K_{II}^2}. \tag{8}$$

Above equation 8 actually measures the strain energy release rate. Substituting Equations 5 and 6 into Equation 8, the equivalent stress-intensity factor for the specimen is explicitly expressed by

$$K_{eq} = \frac{2F}{\pi d\sqrt{t}}\sqrt{\cos^2\theta + \frac{\eta^2(b-d)^2}{12t^2}\sin^2\theta} \tag{9}$$

where the sign of the square root in Equation 8 is directly determined by the sign of the applied force F.

KEY WORDS

Spot Weld
Specimen
Structural Stress
Notch Stress
Stress-Intensity Factor
Fatigue Strength

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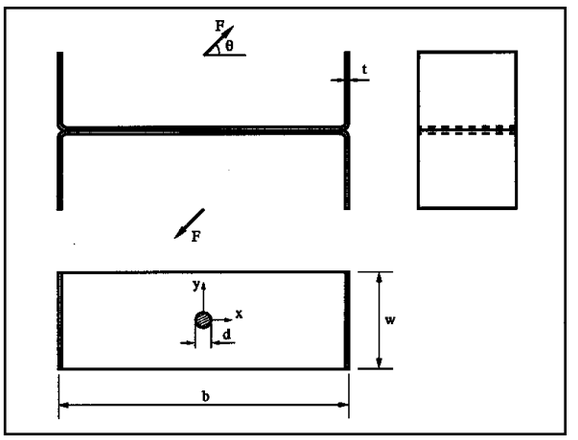

Fig. 1 — A multiaxial spot weld specimen in principle (Ref. 1).

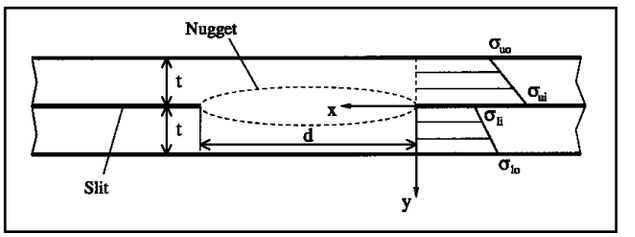

Fig. 2 — Structural stresses around a spot weld.

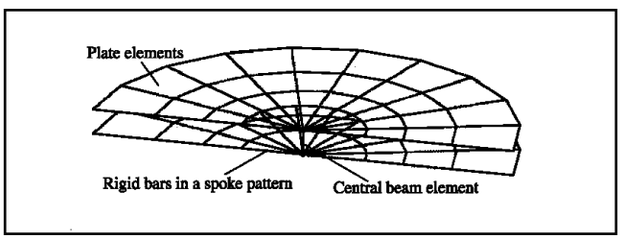

Fig. 3 — Modeling of a spot weld.

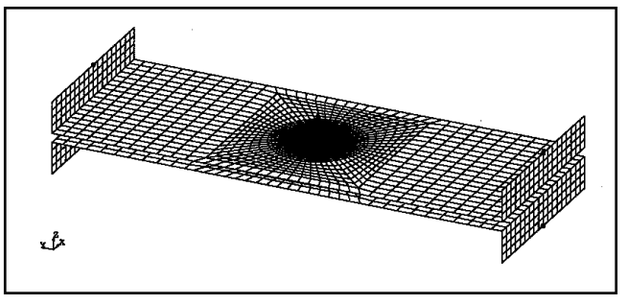

Fig. 4 — Finite element model of the specimen.

Finite Element Validation

The stress formulas derived in the last section cannot be checked by experiments because there is no experimental method available to measure structural stress, notch stress and stress-intensity factors at a spot weld. Finite element simulations are, instead, conducted to validate the formulas. If structural stresses σ_{ui}, σ_{uo}, σ_{li} and σ_{lo} around a spot weld (Fig. 2) are available, structural stress σ_r at the spot weld is directly given by the larger of σ_{ui} and σ_{li}. Notch stress σ_k, and stress-intensity factors K_I and K_{II} at the spot weld can also be determined by the structural stresses around the spot weld (Refs. 2–5). The corresponding equations are listed as follows:

$$\sigma_k = \max\{\sigma_{ui}, \sigma_{li}\} \tag{10}$$

$$\sigma_k = \sigma_n + \frac{1}{4\sqrt{3\pi}}\sqrt{\frac{t}{\rho}}\left\{\sigma_{ui} - \sigma_{uo} + \sigma_{li} - \sigma_{lo} \pm \left[2\left(\sigma_{ui} - \sigma_{li}\right)^2 + \left(\sigma_{uo} + \sigma_{lo}\right)^2\right]^{1/2}\left[\sigma_{ui}^2 + \sigma_{li}^2 - \sigma_{ui}\sigma_{uo} + 2\left(-\sigma_{ui}\sigma_{lo} - \sigma_{uo}\sigma_{li}\right) + \left(-\sigma_{li}\sigma_{lo}\right)\right]^{1/2}\right\} \tag{11}$$

$$K_I = \frac{\sqrt{3}}{12}\left(\sigma_{ui} - \sigma_{uo} + \sigma_{li} - \sigma_{lo}\right)\sqrt{t} \tag{12}$$

$$K_{II} = \frac{1}{4}\left(\sigma_{ui} - \sigma_{li}\right)\sqrt{t} \tag{13}$$

where $\sigma_n = \sigma_{ui}$ if $(\sigma_{ui} - \sigma_{li}) / (\sigma_{ui} - \sigma_{uo} + \sigma_{li} - \sigma_{lo}) \geq 0$ and $\sigma_n = \sigma_{li}$ if $(\sigma_{ui} - \sigma_{li}) / (\sigma_{ui} - \sigma_{uo} + \sigma_{li} - \sigma_{lo}) < 0$. The combined sign ± in Equation 11 takes the plus if $\sigma_{ui} - \sigma_{uo} + \sigma_{li} - \sigma_{lo} \geq 0$ and otherwise the minus. As seen from the above equations, the main task for getting the local stress parameters at the spot weld is to determine the structural stresses around the spot weld. To obtain the structural stresses reliably, the spot weld is particularly modeled by a spoke pattern (Ref. 4). The pattern is shown in principle in Fig. 3. The central beam in the spoke pattern is actually a cylindric bar element with a diameter of the nugget diameter. The diameter of the pattern is also equal to the nugget diameter. Heavy mesh refinements (much finer than that shown in Fig. 3) are introduced around the spoke pattern in order to obtain the structural stresses accurately. The finite element model for the specimen is shown in Fig. 4.

The specimen is loaded at different angles of θ = 0, 30, 60 and 90 deg to cover the whole loading range from shear to tension. The simulations are conducted at two nugget diameters of d = 5.4 and 8.0 mm and at two specimen lengths of b = 79.6 and 49.6 mm, while the other dimensions of the specimen are kept constant at w = 31.0, t = 1.6 and ρ = 0.2 mm. The notch root radius ρ = 0.2 mm correspond to ferritic steels (Ref. 6). What matters is actually the ratio of the different geometric parameters. The variation of d and b results in significant variations in all the ratios such as d/t, w/d and b/w. Therefore, the finite element simulations are representative for the specimen. The correction factor, interpolated from Table 1, takes different values as well, *i.e.*, η = 1.11 at d = 5.4 mm and η =1.50 at d = 8.0 mm. Young's modulus of E = 210,000 MPa and Poisson's ratio of ν = 0.3 for common steels are generally introduced for all the variants.

Table 1 — Correction Factor η with ***e = (w – d)/2***

e/d	1.0	1.5	2.0	2.5	≥3.0
η	1.68	1.47	1.26	1.06	0.94

On the one hand, the structural stress, notch stress and equivalent stress-intensity factor at the spot weld in the specimen are determined by finite element analyses and Equations 10–13 and 8. On the other hand, they are predicted by the formulas 4, 3 and 9, respectively. The results for all the variants are compared in Tables 2 through 5. As seen from the tables, the values predicted by the stress formulas agree well with those from the fi-

nite element analyses for all variants and loading angles. The discrepancy of the stress parameters, as shown in the tables, should be acceptable for most engineering applications. The discrepancy results mainly from the analytic assumptions behind the formulas. Analytically, b and w should be infinitely larger than d and d infinitely larger than t. Obviously, the specimen cannot fully meet these conditions.

Equations 4, 3 and 9 can be directly utilized for determining the static ultimate strength of the spot weld in terms of structural stress, notch stress and stress-intensity factor. In the case of fatigue testing, the stress formulas take the following form accordingly:

$$\Delta\sigma_r = \frac{4\Delta F}{\pi d t}\left[\cos\theta + \frac{\eta(b-d)}{4t}\sin\theta\right] \tag{14}$$

$$\Delta\sigma_k = \frac{4\Delta F}{\pi d t}\left[\left(1+\frac{1}{2\sqrt{\pi}}\sqrt{\frac{t}{\rho}}\right)\cos\theta + \frac{\eta(b-d)}{4t}\left(1+\frac{2}{\sqrt{3\pi}}\sqrt{\frac{t}{\rho}}\right)\sin\theta\right] \tag{15}$$

$$\Delta K_{eq} = \frac{2\Delta F}{\pi d\sqrt{t}}\sqrt{\cos^2\theta + \frac{\eta^2(b-d)^2}{12t^2}\sin^2\theta} \tag{16}$$

with load range of ΔF, structural stress range of $\Delta\sigma_r$, notch stress range of $\Delta\sigma_k$ and equivalent stress-intensity factor range of ΔK_{eq}. For impact loads, the load and stress ranges in the above equations are to be replaced by the rate-dependent ones (strain rate of $\dot{\varepsilon}$) such as $F(\dot{\varepsilon})$, $\sigma_r(\dot{\varepsilon})$, $\sigma_k(\dot{\varepsilon})$ and $K_{eq}(\dot{\varepsilon})$.

Table 2 — Stress Parameters Predicted by the Formulas and by the Finite Element Simulations

	σ_r (N/mm^2)		σ_k (N/mm^2)		K_{eq} (N/mm$^{3/2}$)	
θ (degree)	Eq. 4	FEM	Eq. 3	FEM	Eq. 9	FEM
0	14.7	14.4	26.5	25.9	9.3	9.1
30	107.6	103.4	292.3	265.2	69.7	64.4
60	171.6	175.7	479.8	475.1	120.0	118.8
90	189.6	193.0	538.7	537.5	138.5	136.5

d = 5.4 mm, t = 1.6 mm, b = 79.6, w = 31.0 mm, ρ = 0.2 mm; the specimen is loaded by a total force of F = 100 N at different angles (θ)

Table 3 — Stress Parameters Predicted by the Formulas and by the Finite Element Simulations

	σ_r (N/mm^2)		σ_k (N/mm^2)		K_{eq} (N/mm$^{3/2}$)	
θ (degree)	Eq. 4	FEM	Eq. 3	FEM	Eq. 9	FEM
0	9.9	9.5	17.9	17.0	6.3	6.0
30	92.1	91.1	252.6	237.9	61.2	58.4
60	149.5	156.1	419.6	424.6	105.6	106.5
90	166.9	172.2	474.2	478.7	121.9	121.5

d = 8.0 mm, t = 1.6 mm, b = 79.6, w = 31.0 mm, ρ = 0.2 mm; the specimen is loaded by a total force of F = 100 N at different angles (θ)

Table 4 — Stress Parameters Predicted by the Formulas and by the Finite Element Simulations

	σ_r (N/mm^2)		σ_k (N/mm^2)		K_{eq} (N/mm$^{3/2}$)	
θ (degree)	Eq. 4	FEM	Eq. 3	FEM	Eq. 9	FEM
0	14.7	13.8	26.5	24.8	9.3	8.7
30	69.2	64.9	183.4	157.7	42.0	37.3
60	105.2	111.0	291.2	292.8	71.6	72.2
90	113.0	118.5	320.9	329.8	82.5	83.7

d = 5.4 mm, t = 1.6 mm, b = 49.6 mm, w = 31.0 mm, ρ = 0.2 mm; the specimen is loaded by a total force of F = 100 N at different angles (θ)

Table 5 — Stress Parameters Predicted by the Formulas and by the Finite Element Simulations

	σ_r (N/mm^2)		σ_k (N/mm^2)		K_{eq} (N/mm$^{3/2}$)	
θ (degree)	Eq. 4	FEM	Eq. 3	FEM	Eq. 9	FEM
0	9.9	8.5	17.9	15.4	6.3	5.4
30	57.1	54.8	153.2	136.9	35.8	32.8
60	88.9	95.3	247.5	253.4	61.4	62.8
90	97.0	102.1	275.5	283.4	70.8	71.9

d = 8.0 mm, t = 1.6 mm, b = 49.6 mm, w = 31.0 mm, ρ = 0.2 mm; the specimen is loaded by a total force of F = 100 N at different angles (θ)

Conclusions

Approximate stress formulas were obtained for the multiaxial spot weld specimen. What is given by the formulas is the hot-spot stresses, *i.e.*, the maximal of the structural stress, notch stress and stress-intensity factor on the nugget periphery. The stress formulas are validated by finite element simulations. Given the good approximation shown in Tables 2 through 5, the formulas can be utilized, without sophisticated finite element analysis, to convert the global force applied to the specimen into the local structural stress, notch stress and stress-intensity factor at the spot weld. The formulas are linear solutions and therefore bound to elastic and small-deformation behavior of the material. The results may be applicable to brittle fracture, high-cycle fatigue and high-speed impact failure of the spot weld where plasticity is contained by a large elastic stress field.

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